Information Revelation and Bid Shading in Sequential Auctions with Price-proportional Benefits to Bidders: Theory and Evidence from Rotating Savings and Credit Associations

January 2004

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Abstract:

We analyze auctions in which the price obtained does not go to a third-party seller, but is instead shared by all bidders. It is shown that a one-shot second price or oral ascending auction of this kind generates a higher expected price than a first price auction. In a sequence of such auctions, however, information about the valuation of bidders revealed in early auctions has an opposite effect on bids when a second price or oral ascending instead of a first price auction format is used. In data from 94 Rotating Savings and Credit Associations from South India, where such auctions determine allocations, we find evidence for bid shading in early auctions of sequences with oral ascending bids.

Keywords: Sequential Auctions; Information; Financial Institutions; Roscas

JEL categories: D44; G29; O16; O17

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1 Introduction

In usual auctions, the price the winner of the auction pays for the item auctioned goes to a seller, who is not one of the bidders in the auction. This paper, in contrast, is concerned with auctions where the price remains within the group of bidders. Such auctions have been called "auctions with price-proportional benefits to bidders" (or PPB auctions, for short) and have first been analyzed by Engelbrecht-Wiggans (1994). They are of immediate interest to policy makers since they play an important role both in knockout auctions, a form of collusion in bidding rings (e.g. Deltas, 2002), and in toehold situations, where several partners who own an enterprise jointly stage an auction among themselves to determine which of them buys out the remaining other partners. The auction then determines the buyout price, which the winner and future sole owner pays to his former partners. Such a scenario is commonly encountered when firms whose majority share holder is a public entity are to be privatized or in takeover battles where those interested in taking over already own part of the shares of the company in question.

One result of the existing literature is that revenue equivalence of different auction protocols fails to hold in such auctions, even when bidders are symmetric and risk neutral. Instead, the expected revenue of a second price sealed bid (SP) auction is higher than revenue of a first price sealed bid (FP) auction. To provide an intuition for this result, suppose there are two partners, each of whom owns half of the enterprise that is to be allocated to one of them. Each partner has a private valuation for the enterprise, which is drawn from the same continuous probability distribution and privately observed. The dominant strategy equilibrium of the corresponding standard SP auction, where the price goes to a seller who is not a bidder, is well known to be truth-telling. In a standard SP as well as in a SP-PPB auction, the losing bidder determines the price obtained in the auction. In contrast to the standard SP auction, however, it is that losing bidder who receives the price determined by her bid in the SP-PPB auction. On the other hand, this effect is not present in a FP-PPB auction, where it is the winner whose bid determines the benefit accruing to the losing bidder. Therefore, the SP format creates an incentive to overbid relative to the FP format and this effect drives the revenue comparison obtaining in the Bayesian Nash equilibria of FP and SP-PPB auctions.

To the best knowledge of the author, all of the existing literature on such auctions has been concerned with only a one-shot scenario. The present paper expands the existing framework by studying an institution where there is a sequence of PPB auctions. More specifically, there is single unit demand and there are as many identical items auctioned as there are bidders. Bidders have a preference, however, for obtaining an item early.

As is well-known from the literature on sequential standard (not PPB) auctions with multi-unit demand, information revealed through the price the winner of an early auction pays introduces an asymmetry in the distribution of information in later auctions, thus complicating the analysis considerably (e.g. Donald et al., 2002). In sequential PPB auctions, moreover, the feature of price-proportional benefits makes the issue of information revelation in early auctions a particularly delicate one because a bidder with zero valuation for the item who has learned about the (higher than his own) valuation of another bidder in a previous auction has an incentive to raise the price up to the valuation of that other bidder. This is, as explained above for the one-shot situation, because a higher price paid by the winner implies a higher benefit for the losers of each auction.

The impact of information revelation on bidding in sequences of PPB auctions with different auction protocols is the topic of this paper. We compare the FP and oral ascending (OA) protocol for such auctions and find, first, that the incentive to conceal one's valuation can induce bidders to shade their bids in early auctions when an OA protocol is in place, and second that, in early auctions of a sequence of OA-PPB auctions, the incentive to shade bids can outweigh the incentive to overbid.

A widespread institution where such auctions frequently occur and which underlies the empirical analysis of this study is the Rotating Savings and Credit Association (Rosca) with an auction allotment mechanism. A unique set of data from 94 such Roscas with heterogeneity with respect to the auction format has been used to compare prices fetched in sequences of 38 PPB auctions. The empirical findings support the theoretical result for bidding in sequential PPB auctions according to which bidders in OA auctions occurring early in the sequence shade bids while bidders in FP auctions have no incentive to do so.

While we use a specific economic institution, the Rotating Savings and Credit Association (Rosca), to study the inter-relation of information revelation and strategic behavior in repeated strategic situations, the insights obtained are of broader interest for the study of sequential auctions as well as for the discipline of mechanism design in general. This paper, moreover, adds to the empirical literature on auctions by providing the first data analysis of auctions with price-proportional benefits to bidders.

The remainder of this paper is organized as follows. Section 2 gives an introduction to the institution and the data set underlying this study. In Section 3, a theoretical model that emphasizes the role of the auction format for the impact of information revelation on bidding in sequential PPB auctions is developed. In Section 4 I conduct a data analysis of auction outcomes with a sample of 94 Roscas and relate the findings to the theoretical results. The final section summarizes the findings and concludes.

2 Auctions in Rotating Savings and Credit Associations

2.1 Institutional Background

The Rotating Savings and Credit Association (Rosca) plays an important role as a financial intermediary in many parts of developing countries. It flourishes in both urban and rural settings, especially where formal financial institutions seem to fail to meet the needs of a large fraction of the population. Bouman (1979), for example, estimates that, in central African countries, about 20% of household savings are accumulated in informal Roscas. In the South-Indian state of Tamil Nadu with a population of 62 million, the turnover in registered Roscas has been estimated at 100 billion Rupees, about 2.5 billion US dollars, in 2001 (Ganga-Rao, 2001), while bank deposits averaged 55 billion Rupees during the same period. Compared to the extensive economic literature on financial transaction in rural areas of developing economies and the recently flourishing interest in microfinance institutions such as the Grameen Bank, Roscas have received little attention by economists.

In broad terms, a Rosca can be defined as ‘a voluntary grouping of individuals who agree to contribute financially at each of a set of uniformly-spaced dates towards the creation of a fund, which will then be allotted in accordance with some prearranged principle to each member of the group in turn’ (Calomiris and Rajaraman, 1998) define. Once a member has received a fund, in South India also called a ‘chit’, she is excluded from the allotment of future chits until the Rosca ends. The term ‘chit’ stems from the Tamil word ‘chitty’, meaning a written piece of paper, which points to one important allotment mechanism where a lot determines each date’s ‘winner’ of the chit. In contrast to such a random Rosca, in a bidding Rosca, an auction is staged among the members who have not yet received a chit. The highest bid wins the chit and the price the winner pays is distributed among the other participants.

Many empirical studies report that the funds obtained from a Rosca are often used to purchase a lumpy good whose cost cannot be covered by a member's current income. In this connection, a random Rosca has the merit of allocating the full amount of the chit to one of the members each time the group meets. The bidding allotment mechanism, on the other hand, allows members to obtain a chit when an unforeseen opportunity or emergency arises, albeit at the cost of a discount. Even in a world without uncertainty, the more flexible bidding arrangement allows members with more profitable investment opportunities to obtain funds earlier by compensating the other members through the price determined in the auction. Thus, in a certain world, a random Rosca is preferred by identical individuals desiring to overcome indivisibilities in consumption or investment, while a bidding Rosca is superior in responding to heterogeneity among its members (Besley et al., 1993; Kovsted and Lyk-Jensen, 1999). For example, an individual who is following a long-term savings objective may benefit from the impatience of other members in a bidding Rosca by taking the last chit and earning an implicit interest through the price in each round, while, compared to saving autarkically, such an individual does not benefit from joining a random Rosca. Thus, unlike random Roscas, bidding Roscas are capable of providing mutually beneficial intermediation between heterogeneous individuals and thus compete with more familiar intermediaries such as banks and credit cooperatives. While banks provide impersonal intermediation on a large scale, Rosca groups typically do not exceed 100 members, with often no more than ten. Bidding Roscas thus face an issue of strategic interaction as well as one of volatility if the world is not perfectly certain. On the other hand, their limited size makes Roscas particularly suited for settings in which financial intermediation through banks often fails. In an agricultural village, for example, where enforcement is limited or costly for outsiders, Roscas exploit social ties among the group's members to overcome such problems.

While originating in the informal financial sector, it may come as a surprise that Roscas have become a big business in urban settings where no social collateral among the members exists. Since the beginning of the twentieth century, so-called “chit-fund companies” have evolved throughout all urban areas of South India. Since, apart from the auctions, customers of chit-fund companies typically do not interact with the other members of the Rosca group they belong to, the role of the organizer, or foreman, is more important in such Roscas than in a setting where members can mutually enforce contributions. Instead, the company collects the contributions and is liable for paying the winner of each chit and members are not even aware of defaults or late payments of other group members.

The steady growth of commercial chit funds, which are almost exclusively of the bidding kind, has induced Indian legislators to pass specific legislation ruling that part of the financial sector. As the first of its kind, the Travancore Chitties Act went into effect in Kerala in 1945. The Tamil Nadu Chit Fund Act was passed in 1961. Among other things, the Act requires each chit-fund company to obtain a license from the state government and rules minimum capital requirements.

2.2 Description of the Data

This subsection describes the price data obtained from one branch of a chit-fund company that operates throughout the South of India. Unlike in bigger cities such as the state capital Chennai, the company has only one branch in this comparatively small urban agglomeration (the Census of 1991 reports a population of about 285,000).

All the company’s groups meet once per month. In total, the company operates no less than 100 different denominations, that is combinations of the number of members per group and the amount of the contribution paid by each member in each round. In this paper, we focus on one particular denomination, 40 members who contribute Rs. 250 at each meeting, which amounts to a chit of Rs. 10,000, which, in turn, equaled about 12 US Dollars in 1992. For means of comparison, the daily minimum wage for an agricultural laborer in the state of Tamil Nadu, at that time, was equal to Rs. 30. The 94 groups in our dataset were started between August 1990 and June 1993. With each Rosca spanning over 40 months, the data thus cover a time period of six years. Later in 1993 the federal Chit Funds Act of 1982 was enforced and ruled that bidding in each auction may not exceed 30% of the chit's value. While this is an interesting issue in its own right (see Eeckhout and Munshi, 2003; Klonner and Rai, 2004), in this paper we focus on Roscas with unrestricted bidding.

We now turn to the institutional details of the groups administered by the company. The company itself acts as a special member of each group and is entitled to obtain the first chit at a price of zero. This practice is in accordance with the provisions of the Chit Fund Act and has a tradition in the rural history of Roscas, where the organizer gets the first chit to have hot cash for settling potential defaults or late payments later in the life of the Rosca. Each Rosca in the sample thus involves 38 auctions. On top of receiving one 40'th of the price in each round, the company also deducts six percent commission from each round's chit, in our data Rs. 600 per group and month. In each auction, the minimum bid is Rs. 600 and if no bid is received at that price, a lottery among the active members[1] determines the recipient of that round's chit, who then, in our case, receives Rs. 9,400.

The bulk of the 94 groups in our data involve oral ascending auctions, which is the traditional auction format of village Roscas in South Asia. Eight groups, however, involve sealed-bid auctions, where the highest bid determines the price as well as the winner. The auction

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[1] In accordance with the literature, we refer to those members as 'active' who, in a given round, have not yet received a chit and are thus eligible to receive the current or a future chit.

protocol in each of the sample Roscas is exogenous in the sense that it is the company which decides on the features of each Rosca before members can join a group. The data, moreover, resemble a controlled natural experiment in that, at common significance levels, the distribution of starting dates of groups with FP auctions is not different from the distribution of starting dates of groups with OA auctions.[2] For each of the eight FP Roscas in the sample, there is an OA Rosca that starts no more than four weeks earlier or later.

To give a flavor of price determination in the sample Roscas, Figure 1 depicts the price path of three randomly chosen Roscas in the sample. While prices are declining on average, there is no single Rosca in the sample where the price decreases in each round. On average, in 38% of the auctions a higher price obtains than in the round before. The kinky line in Figure 2 depicts the path of the median price.

Turning to the aspect of financial intermediation in the sample Roscas, according to the medians the recipient of the second chit[3] lends at an interest rate of 1.6% while the recipient of the last chit earns an interest of 0.7% per month.[4] This compares to lending rates of pawn brokers who require gold as collateral of 1.5% percent and bank deposit rates between 0.5% on a savings

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[2] Against a two-sided alternative, Wilcoxon's two-sample rank-sum test statistic is significantly different from zero only at the 64%-level. Similarly, the Kolmogorov-Smirnov two-sample test statistic is significantly different from zero only at a level of 76%.

[3] Recall that the company receives the first chit without a discount.

[4] Calculating implicit interest rates for intermediate recipients is flawed by the fact that they are neither pure borrowers nor lenders, so that, for many of the intermediate rounds, the corresponding polynomial does not have a real-valued root.

account and 0.8% for fixed deposits of 36 months.[5] Taking into account the comparatively low collateral requirements of the company,[6] Roscas are a comparatively cheap source of funds for potentially credit-constrained customers and offer an attractive return for savings-oriented customers who value the flexibility of a Rosca compared to fixed-term bank deposits.

3 The Bidding Rosca as a Sequence of Auctions with Price Proportional Benefits to Bidders: Theory

Unlike standard auctions, where a seller who is not one of the bidders extracts the price obtained in the auction, the price remains within the group of bidders in a Rosca auction, which makes such an auction one with price-proportional benefits to bidders. As mentioned in the introduction, one result of this literature is that, as long as bidders are symmetric and play a one shot game, the expected price in a second price (SP) PPB auction is higher than in a first price (FP) PPB auction. This is because, in the equilibrium of a SP-PPB auction, any bidder who does not observe the highest possible value faces a positive probability of determining the side payment occurring to her. In the simplest case of only two bidders and continuously distributed valuations, if bidder 2, say, turns out not to be the winner of the auction, her bid determines the price paid by bidder 1 and thus the benefit accruing to her, bidder 2. Compared to a standard auction, where this effect is not present, all bidders who face a positive probability of not winning the auction therefore, somewhat loosely speaking, overbid in a SP-PPB auction. On the other hand, a bidder with the highest possible valuation bids truthfully because, in equilibrium, the probability that she does

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[5] Informal money-lenders charge 3% and more.

[6] The recipient of a chit before the 20th round has to provide three grantors with a monthly wage income of at least Rs 1,500. After the 20th round, two such grantors have to be provided. No physical collateral has to be furnished.

not win the auction is equal to zero. In a first price PPB auction, on the other hand, the winner himself determines the price and thereby the benefit accruing to the losing bidder and there is thus not incentive to 'overbid'. In principle, this argument applies also to a Rosca auction, irrespective of the fact that, in such auctions, a losing bidder does not only obtain a share of the winning bid but also the privilege of obtaining a chit later in the Rosca cycle.

We now turn to the relationship between the auction format and the impact of information revealed through bidding in an early auction of the sequence on bids in consecutive auctions. Suppose there are n individuals who jointly own $m < n$ identical items, which are to be auctioned consecutively. In each auction, the winning bid is equally shared by all n individuals. The winner of the t'th auction cannot bid in the remaining $m - t$ auctions. Each bidder demands a single unit of the item, for which she has a privately observed valuation which is independently drawn from some continuous probability distribution. We thus have a sequence of PPB auctions.

Consider first such a sequence with FP auctions, where, after each auction, only the price paid by the winner, which is the bid submitted by the high bidder, becomes public information. When a bid submitted in any of the auctions is related to the valuation of the bidder, then, after each auction, information on the valuation of the winning bidder becomes available to the bidders in the next auction. Since this winning bidder does not bid in any of the remaining auctions, however, the distribution of information remains symmetric among the bidders of the following auction. With the rule that after each auction the winner of that auction is excluded from the group of bidders in the remaining auctions, if follows that information among the bidders is held symmetrically throughout all auctions of the sequence. As a consequence, equilibrium bid functions for each auction can be obtained recursively by backward induction and the allocation induced is efficient in that the individuals with the m highest valuations obtain items.

Consider now the first auction in a sequence of two SP-PPB auctions with three bidders, where, as before, the price paid by the winner of each auction becomes public information after the auction. With an SP rule, this price is determined by the second highest bidder, *A* say, who is also a bidder in the following (and final) auction. Thus, when bids submitted in the first auction are related to the valuation of each bidder, the other losing bidder, *B* say, receives more information about the valuation of *A* than *A* has learnt about *B*'s. This creates an asymmetric distribution of information in the second auction. If *A*'s bid in the first auction is fully informative about her valuation and *B*'s valuation is in fact lower than *A*'s, it is straightforward that the equilibrium of the second auction has bidder *B* bid *A*'s valuation minus some small increment. Bidder *A*, in turn, will bid her valuation and thus be left with a surplus of almost zero. As a consequence, fully informative bids, that is bids that reveal the valuation of the bidder, cannot be an equilibrium of the first auction of this sequence of SP-PPB auctions. If, on the other hand, there is no one-to-one mapping between the bid and valuation in the first auction, inefficient allocations occur with positive probability.

In the remainder of this section, I develop a model of sequential auctions in bidding Roscas. Section 3.1 analyzes a simple model of a Rosca with only one auction. In accordance with the argument developed at the beginning of this section, it is found that the price obtaining in such an auction is higher with the SP or OA than with the FP format. Allocations, on the other hand, are efficient with all three auction protocols. Section 3.2.1 models a Rosca involving a sequence of two auctions. It is shown that there exists an equilibrium where, irrespective of realized valuations, bidders do not bid in the first auction to hide information about their valuations from other bidders. As a consequence, inefficient allocations occur.

3.1 Bidding in a One Shot Rosca Auction

We consider a Rosca with two members and thus two rounds. In each round, each of them contributes $1 to a fund, also called chit. An auction determines the order of receipt of chits. The winner of the first auction receives the first chit minus half the price obtained in the auction. If that price is b, say, the winner's relevant payoffs (that is leaving the fixed contribution in each round aside) are thus 2-b/2 in the first round and zero in the second. The other bidder receives b/2 in the first and 2 in the second round. Assume that member I has access to a divisible investment opportunity that yields R_i, which is an independently distributed dichotonomous random variable with realizations $r > 0$ and 0, which occur with probability p and 1-p, respectively. Members maximize their expected wealth at the end of the second period. For the winner of the first auction, this is $(1+R_i)(2-b/2)$ and for the loser $(1+R_i)b/2 + 2$.

When bidders can submit any non-negative bid, both the FP and SP protocol have only a mixed strategy equilibrium, which is analogous to the equilibrium of a standard FP auction where valuations are drawn from a discrete distribution. Suppose, in the FP Rosca auction, a bidder who observes zero submits a bid of zero (which is her valuation for the first chit) and a bidder who observes r submits a random bid that she draws from a probability distribution with cdf G. In equilibrium, her expected wealth at t = 2 before the auction when bidding b is

$$\Pi(b) = (1+r)(2-b/2)(1-p+pG(b)) + ((1+r)\mathrm{E}[B/2|B>b]+2)p(1-G(b)).$$

The equilibrium condition is that G is such that, for all b in the support of G, $\partial\Pi(b)/\partial b = 0$, which gives

$$G(b) = \frac{1-p}{p}\left(\left(\frac{2r}{(1+r)(2-b)-2}\right)^{\frac{1}{2}} - 1\right) \tag{1}$$

with support $[0,\ 2(1-(1-p)^2)r/(1+r)\,]$.

Similar reasoning gives the equilibrium of the SP Rosca auction. Here a bidder who observes r bids her valuation, which is obtained by equaling wealth at $t = 2$ when receiving chit one or two:

$$(1+r)(2-b/2) = (1+r)b/2 + 2,$$

which yields $b^{SP}(r) = 2r/(1+r)$. A bidder who observes zero, in contrast, randomizes her bid according to the following distribution function,

$$H(b) = 1 - \frac{p}{1-p}\left(\frac{1}{1-r+(1+r)b/2} - 1\right), \tag{2}$$

which has support $[\,2(r-(1-p))/(1+r)\,, 2r/(1+r)\,]$. To rule out negative bids, we shall assume that $r \geq 1\text{-}p$. Notice that bidders with a zero valuation overbid as argued in the introduction to this section. When an oral ascending bid Rosca auction is modeled as a clock auction, it is, moreover, straightforward to show that the payoffs occurring in the OA equilibrium are equal to those in the SP Rosca auction just derived.

With these bidding equilibria, allocations induced by the auctions are, of course, efficient and it is easy to show that the expected price obtaining in the auction is strictly bigger with the SP or OA format conditional on any configuration of realized valuations. This is summarized in the following proposition.

Proposition 1

i) In the symmetric equilibrium of a Rosca with one first-price auction,

a) bidders who observe r randomize their bids according to (1);

b) bidders who observe zero submit a bid of zero;

c) the equilibrium induces efficient allocations.

ii) In the symmetric equilibrium of a Rosca with one second-price or oral ascending auction,

a) bidders who observe r bid their valuation;

b) bidders who observe zero randomize their bids according to (2);

c) the equilibrium induces efficient allocations.

iii) The first-price auction results in a lower expected price than the second-price or oral ascending auction.

3.2 Sequential Rosca Auctions

In this subsection, I develop a model that captures the effect of information revelation in early auctions. Consider a Rosca with three members and thus two auctions. One of the members is known to have no profitable investment opportunity outside the Rosca. Index this member by zero. Before the beginning of the Rosca, each of the two other members indexed by 1 and 2 privately observe an interest rate R_i, $i = 1, 2$, of either r or 0 with probability p and 1-p, respectively. As in Section 3.2 , each member contributes one Rupee per round. Notice that member 0 never gains from winning a chit, but only from positive prices in the first two rounds.

In accordance with the rules of the company managing the Roscas of our sample, bidding occurs in discrete increments (in our sample groups it has to be a multiple of 50). In this model, we assume that bids belong to some countable infinite, ordered set $\hat{B} = \{\hat{b}_0, \hat{b}_1, ...\}$ of equally spaced points in the set of non-negative real numbers, where $\hat{b}_0 = 0$. For analytical convenience, we consider sufficiently small increments. We denote the element of $\hat{B}$ that is closest to, but not bigger than, the real-valued number x by $\hat{b}(x)$.

3.2.1 Equilibrium with in a Rosca with First-Price Auctions

In such a Rosca, before each auction each bidder submits a bid in a sealed envelope. The highest bid determines the winner and the price. In case of a tie, a fair lottery between the tying high-bids determines the winner.

We solve the auction game backwards. For the time being, assume an efficient outcome in the first round. Without loss of generality, assume that member 1 received the chit in the first round and that thus, in the second round, members 0 and 2 remain as bidders. In this situation, it is a weakly dominating strategy for 0 to submit a bid of zero. If $R_2 = 0$, member 2 also submits zero, while it is his best response to 0's strategy to bid $\hat{b}_1$ if $R_2 = r$.

Going back to the first round, for 0, again, it is a weakly dominating strategy to submit a bid of zero. The same line of reasoning applies to each of the other two bidders, whenever he observes $R = 0$. On the other hand, as is know from the theory of auctions with discrete valuations, when bidder i observes r, an equilibrium only exists in mixed strategies (see, e.g. Wolfstetter, 1995), where i is indifferent between submitting any bid that is an element of some set $\tilde{B}(a) \equiv \{\hat{b}_1,...,\hat{b}(a)\}$ and, in equilibrium, submits each element of $\tilde{B}(a)$ with a certain probability. If the strategy space were continuous, which is a reasonable approximation for determining the equilibrium randomization distribution if the bid increments are sufficiently small, we can write the wealth of bidder i in $t = 3$ conditional on $R = r$ as

$$\Pi_1^i(b\,|\,r) \equiv (1+r)^2\left(3-\frac{2}{3}b\right)\big((1-p)+pG(b)\big)+p\big(1-G(b)\big)\left((1+r)^2\frac{1}{3}E[B\,|\,B>b]+3(1+r)\right), \quad (3)$$

where $G(\ )$ is the cumulative distribution function according to which the bidder randomizes his bid. $(1+r)^2\left(3-\frac{2}{3}b\right)$ is his payoff if the wins the first auction. This event occurs whenever the other bidder, j say, has observed zero, which has probability 1-p, and also when j has observed r

but *j*'s random bid is smaller than *b*, the probability of this event being $pG(b)$. If, on the other hand, *j* has submitted a higher random bid then *i*, *i* receives a third of the winning bid in $t = 1$, in expectation $\frac{1}{3}E[B \mid B > b]$, and, as he can project, a payoff of roughly 3 in $t = 2$ because, as argued above, there is no competition in the second auction and, provided bid increments are small, $\hat{b}(3)$ is arbitrarily close to 3. Multiplying those payoffs with $(1+r)^2$ and $(1+r)$, respectively, gives the term in the last bracket on the RHS of (3). The probability of this event is, of course, $p(1-G(b))$.

Differentiating (3) with respect to *b* together with the boundary condition $G(0) = 0$ gives the equilibrium distribution function

$$G(b) = \frac{1-p}{p}\left(\left(\frac{3r}{(1+r)(3-b)-3}\right)^{\frac{2}{3}} - 1\right), \tag{4}$$

which has support $(0, \bar{b})$, where

$$\bar{b} \equiv \frac{3r}{1+r}\left(1-(1-p)^{\frac{3}{2}}\right).^{7} \tag{5}$$

Notice that ex ante, that is before the first auction, the expected wealth in $t = 3$ with such a Rosca for a bidder who observes *r* is

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[7] Strictly speaking, in the case of discrete valuations the mixed-strategy equilibrium is given by a probability measure $\tilde{G}_n$, say, where $n = \hat{b}(\bar{b})/\hat{b}_1$. $\tilde{G}_n$ assigns a positive probability to each element in $\tilde{B}(\bar{b})$ and zero otherwise. It can be shown that $\tilde{G}_n$ converges to $G(\,)$ as bidding increments become arbitrarily small, i.e. as *n* approaches infinity.

$$EU(r) = 3 + r\left(4 + 2(1-p)^{\frac{3}{2}}\right) + r^2\left(1 + 2(1-p)^{\frac{3}{2}}\right). \tag{6}$$

Defining efficiency as allotting chits first to members who observe *r*, we can summarize the results derived thus far.

Proposition 2

In the symmetric sequential equilibrium of a Rosca with first-price auctions,

a) bidders who observe *r* randomize their bids according (approximately) to (4) in the first auction and bid $\hat{b}_1$ in the second auction;

b) bidders who observe zero submit a bid of zero in both auctions;

c) the sequential equilibrium generates an efficient allocation;

d) the expected price in the first auction is approximately $\frac{r}{1+r}\left(1 + 3(1-p)^2 - 4(1-p)^{\frac{3}{2}}\right)$ and $p^2\hat{b}_1$ in the second auction.

Uniqueness of the equilibrium characterized in Proposition 2 within the class of symmetric equilibria can be proven by contradiction.

3.2.2 Equilibrium in a Rosca with Oral Ascending Auctions

In this subsection, under some further assumptions, I establish an equilibrium where bidders do not bid at all in the first round. Roughly speaking, this is because, if a member who observes *r* starts to bid, he reveals this information and bidder 0 will exploit this to extract higher benefits for himself. In the equilibrium, a bidder with *r* prefers to incur a lottery instead of being bid up in the way just described. Consequently, there is a positive probability of inefficient allocations.

We first need to specify how the ascending auction proceeds. In the sample Roscas, the rules of the Rosca company limit the bidding in each auction to five minutes. As in Roth and Ockenfels (forthcoming) on ascending ebay auctions I assume that during these 300 seconds, each bidder always has time to react to the bid of each other bidder. Thus, while the reaction time, γ say, is strictly positive, it is sufficiently small to ensure that when a bid is submitted at instant $\psi < 300$ (measured in seconds), $\psi + \gamma < 300$. Thus, as long as $\psi < 300$, infinitely many reactions and re-reactions can occur. After this process has come to an end, at the last instant, i.e. at $\psi = 300$, each bidder can submit one more bid. We allow for one-time reactions in that, if bidder 1, say, bids $\hat{b}_n$ in the last instant, each of the other bidders can give one more bid bigger than $\hat{b}_n$, where, again, if 2, say, outbids $\hat{b}_n$ by $\hat{b}_m$, say, where $m > n$, bidder 3 can outbid $\hat{b}_m$. Then the auction is over. Thus, in this example, bidder 1 cannot react to bidder 2 and 3 in the last instant.

To provide some intuition for modeling last-minute bids this way, consider the auction in round 36 in one of the sample Roscas. Bidder one says ‘3000’ in the last second of that auction. Saying ‘3000’ takes the rest of that second. Bidder 2, when hearing 1 starting to say ‘3000’ rushes to say ‘3500’ before 1 has finished saying ‘3000’. When bidder 3 hears 2 starting to say ‘3500’, he in turn rushes to say ‘3700’. There is no time for any of the bidders to start saying another number before the auction is over.

Without going into the details of the derivation we state the equilibrium in the following proposition.

Proposition 3

In a Rosca with oral-ascending auctions, if $1/2 < p < 2/3$, there exists a sequential equilibrium where

a) a bidder who observes r does not bid in the first auction. In the second auction, he bids $\hat{b}_1$ when the other bidder is member 0, and he bids zero when the other bidder is not member 0;

b) bidders who observe zero do not bid in any of the two auctions;

c) the sequential equilibrium generates an inefficient allocation with positive probability;

d) the expected price in the first auction is zero and in the second auction $2p\hat{b}_1/3$.

Proof:

Assume that, in round one, there has been no bidding. Then, in round two, the bidders are either 1 and 2, 0 and 1, or 0 and 2. If 0 and 1 are the bidders, first note that, since there was no information revealed in $t = 1$, before that auction 0's prior probability about 1 having observed r is the same as ex ante, i.e. p. Assume that 1 follows the following strategy: if $R_1 = 0$, do not bid; if $R_1 = r$ and before the last instant the standing bid is some $\hat{b}_m < \hat{b}(3r/(1+r))$, bid $\hat{b}_{m+1}$ at the last instant.

What is 0's best response to that? He will certainly not outbid 1 in the last instant since this implies a loss for him. Suppose, however, before the last instant 0 submits some bid $\hat{b}_m < \hat{b}(3r/(1+r))$, say, such that, provided 1 had observed r, he would have a higher utility from obtaining chit two at $\hat{b}_{m+1}$ than from not reacting. In this case, 1's best response is to bid $\hat{b}_{m+1}$ at the last instant because, by doing so, 0 cannot bid him further up. On the other hand, if 1 had observed 0, his best response is not to react to $\hat{b}_m$ at all, in which case 0 ends with a loss. In expectation, 0's gain from rising the bid is positive if, and only if, $p > 2/3$. It is further easily verified that 1's strategy posited above is a best response to such behavior of 0. Thus, whenever $p < 2/3$, this equilibrium implies no bidding when $R_1 = 0$ and 1 winning the auction at price $\hat{b}_1$ when $R_1 = r$. The case where 0 and 2 are the bidders in the second auction is analogous.

What if 1 and 2 are the bidders in the second round? If both of them remain silent, the payoff for a bidder who has observed r is $\Pi_2^i(0\,|\,r) = (3(1+r)+3)/2 = 3+3r/2$ because, in this case, a lottery determines whether he receives chit 2 or 3. If an r-type follows this strategy, a zero-type's best response is, of course, to remain silent as well as long as $p < 2/3$; see the argument above. What if an r-type, i say, does bid instead? The best he can do is bid $\hat{b}_1$ at the last instant. In this case j will outbid him with $\hat{b}_2$ resulting in a round 3 payoff for i of roughly 3. If, instead, j is a zero type and i bids $\hat{b}_1$ at the last instant, j will not react, which implies round 3 wealth of roughly 3+3r. Thus i's expected payoff from bidding $\hat{b}_1$ at the last instant is 3 + 3(1-p)r, which is smaller than $\Pi_2^i(0\,|\,r)$ whenever $p > ½$. Focusing on the case $1/2 < p < 2/3$ and provided the first auction was non-informative, the following strategies thus constitute an equilibrium. For an r-type: bid $\hat{b}_1$ at the last instant when the other bidder is member 0; do not bid when the other bidder is $j \in \{1,2\}$. For any zero type: do not bid.

Turning to the first round, when all bidders do nothing, anticipate the above-derived outcome of round 2, and a lottery determines the recipient of chit 1, the payoff to an r-type is

$$\Pi_1^i(0\,|\,r) = \frac{1}{3}\Big((1+r)^2 3 + (3+3r/2) + (3+3r)\Big) = 3 + 7r/2 + r^2 . \tag{7}$$

If an r-type bids and $p > 1/2$, basically the same happens as in $t = 2$, where he is outbid in the last instant. On the other hand, for an r-type, it is not profitable to overbid 0 in case the latter starts to bid because by doing so he is either bid up to a level where he is indifferent between obtaining chit 1 or not obtaining it and being 'exploited' in $t = 2$ because his type has been revealed. Clearly the latter scenario is worse than a lottery. Thus if 0 starts to bid, he ends in loss, so this will not happen. □

To summarize, in this model, outcomes of the FP and OA protocol differ in two respects. First, the expected price obtaining in the first auction is bigger with the FP than with the OA format. This challenges the result of Section 3.1, where the expected price obtaining in an OA Rosca auction is always bigger. In the second auction of the sequence, however, the revenue result of the model with only one auction is confirmed – which comes as no surprise given that information revealed in that auction cannot be used by other bidders afterwards. Second, as a consequence of hiding information on valuations, the OA regime generates an inefficient allocation with positive probability when there is a sequence of Rosca auctions. This is again in stark contrast to the one-shot Rosca auction, where both the FP and OA format generate an efficient allocation.

4 Empirical Results

In the previous section two effects with opposite impacts on prices in early auctions of a sequence of OA-PPB auctions have been highlighted. While price-proportional benefits create an incentive to 'overbid' relative to one's valuation, information on one's valuation revealed through a bid can create an incentive to shade bids in early rounds of the auction sequence. It has also been shown that neither of the two effects is present when first price auctions are used. In this section we compare prices obtained in the 94 Rosca groups in our data set. As mentioned earlier, eight of these groups use FP and 86 OA auctions.

The sequence of medians of prices for the entire sample of 94 groups is depicted in Figure 2, while Figure 3 graphs the same statistic for the two auction regime specific sub-samples separately. As is seen from these latter graphs, the paths of median prices with FP and OA auctions are similar in shape for auctions occurring during the second half of the Rosca. For earlier rounds, however, the graph of OA auction prices bends back more strongly than that of FP prices, resulting in median prices up to 15% lower during initial rounds.

Table 1 makes this observation more precise. It sets out the Wilcoxon two-sample rank sum and median two-sample test statistics by round, where a positive value obtains when, according to the respective test criterion, the prices in first-price auctions are bigger than those in OA auctions. Working backwards, for rounds 18 to 39, these test statistics do not provide significant evidence against the hypothesis that the location of the distribution of prices is different under the two auction regimes.

At the 10 per cent level, the median test rejects only for the round 20 data while the Wilcoxon test rejects only for the data from round 27. The directions of these rejections are, moreover, opposite. Taking into account that more than 40 realizations of test statistics are considered here (twice 22 rounds), the *p* values corresponding to these two rejections of .027 and .057 cannot serve as evidence against the hypothesis that prices under the two regimes are drawn from the same distribution.

For rounds 2 to 17, in contrast, all realizations of the test statistics are bigger than zero and significantly so, at the 10% level, for eleven and nine rounds with the two different test criteria, respectively. For the first five auctions, moreover, the Wilcoxon test rejects in favor of the hypothesis that prices in FP auctions are higher at a significance level of five per cent.

To summarize, I do not find significant evidence that prices under the two auction formats are different in auctions occurring during later rounds (18 through 39) of the Roscas, while prices in FP auctions are systematically higher in early (the first 17) rounds and in particular in the initial five auctions. There is thus no evidence for the theoretical result of Section 3.1 for one-shot PPB auctions, according to which the average price in an OA auction is higher than in a FP auction. The empirical findings, in contrast, support the result of Section 3.2 for bidding in sequential PPB auctions, according to which bidders in OA auctions occurring early in the sequence shade bids to avoid being bid up in later rounds while bidders in FP auctions have no incentive to do so. Accordingly, in the model of Section 3.2, it was found, first, that the expected price in the early auction of a sequence of OA auctions is lower than in a sequence of FP auctions and, second, that the expected price in the late auction is almost identical under the two auction formats. This qualitative relationship of early to late prices is precisely what the statistical tests of the empirical exercise reflect.

5 Concluding Remarks

In this paper, we investigate sequential auctions with prices-proportional benefits (PPB) to bidders. For a one-shot PPB auction, it has been established theoretically that, within a symmetric independent private value bidder framework, revenue equivalence of the first price sealed bid and oral ascending auction does not hold when bidders receive price-proportional benefits. Instead, the oral ascending format yields a higher expected price because a bidder has an incentive to raise the price to increase the benefit accruing to her in case she does not win the auction. This result has been challenged with a model of sequential PPB auctions where the incentive just mentioned has the opposite effect on a bidder with a high valuation. Such a bidder shades his bid in early auctions of the sequence in order to hide his valuation from other bidders, who would exploit such information to raise the price in later auctions of the sequence, which in turn would decrease her expected surplus. For a sequence of first price auctions, in contrast, it has been shown that these incentives are not present. A revenue comparison of the two formats has shown that the expected price obtained in an early auction of a sequence of OA-PPB auctions can be lower than when FP auctions are in place, thus reversing the revenue ranking of the one-shot PPB auction.

A widespread institution where such auctions frequently occur and which underlies the empirical analysis of this study is the Rotating Savings and Credit Association (Rosca) with an auction allotment mechanism. A unique set of data from 94 such Roscas with heterogeneity with respect to the auction format has been used to compare prices fetched in sequences of 38 PPB auctions. I have not found statistically significant evidence that prices in sequences of first price and oral ascending auctions are different in auctions occurring later in the sequence, while prices in sequences of FP auctions are systematically higher in early auctions. These empirical findings are at odds with the revenue ranking for a one-shot PPB auction and support the theoretical result for bidding in sequential PPB auctions according to which bidders in OA auctions occurring early in the sequence shade bids while bidders in FP auctions have no incentive to do so.

Taken together with the empirical evidence that systematical hiding of valuations in early auctions occurs in my data, the theory developed in this paper suggests that inefficient allocations are to be expected more often when oral ascending instead of first price auctions are used. This challenges a common wisdom of auction theory that, when affiliation or asymmetric bidders are an issue, open bidding generally creates more efficient allocations and higher revenue. In this paper I have found empirical evidence and theoretically shown within a symmetric independent private value bidder framework that the opposite can be true when items are auctioned sequentially and there are price-proportional benefits to bidders. These results also provide an insight into agents' ability to process information in repeated strategic situations. By comparing outcomes of two auction formats, we have found indirect evidence that Rosca members do use information on other bidders' valuations revealed in previous auctions of the sequence. Given that each Rosca in the sample spans over 40 months with only one auction per month and that, by the design of the institution, individual returns to exploiting this information are rather small (one

40th), Rosca members appear to store and process sequentially revealed information fairly efficiently.

The findings of this paper highlight the importance of institutional design in environments with scope for strategic behavior. According to our findings, models of repeated interaction with oblivious actors have to be discarded. Moreover, when there is repeated interaction, incentives inherent in a mechanism carefully have to be designed such that information revealed by an agent cannot put her into a disadvantage later on and thus counter the common wisdom that mechanisms which reveal information to agents induce more efficient allocations.

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Round	z^W	$P(\|Z^W\|>z^w)$	z^M	$P(\|Z^M\|>z^M)$
2	2.12	0.034	1.53	0.125
3	2.87	0.004	2.32	0.020
4	2.66	0.008	2.25	0.025
5	3.15	0.002	2.26	0.024
6	2.25	0.024	2.29	0.022
7	0.61	0.542	0.00	1.000
8	2.65	0.008	2.21	0.027
9	1.74	0.081	1.49	0.135
10	0.57	0.569	1.13	0.260
11	2.74	0.006	2.21	0.027
12	1.18	0.237	0.75	0.455
13	1.57	0.117	1.47	0.141
14	1.83	0.068	1.73	0.083
15	1.49	0.135	2.21	0.027
16	2.20	0.028	1.47	0.141
17	1.66	0.098	1.87	0.062
18	0.11	0.913	0.09	0.925
19	-0.56	0.573	-1.49	0.136
20	-1.35	0.177	-2.21	0.027
21	-0.75	0.456	-0.74	0.462
22	0.82	0.412	0.74	0.462
23	-0.57	0.569	-0.75	0.455
24	0.22	0.823	0.75	0.456
25	0.99	0.322	0.74	0.457
26	0.55	0.582	0.00	1.000
27	1.91	0.057	1.47	0.141
28	1.03	0.303	0.93	0.352
29	0.43	0.664	0.74	0.462
30	0.69	0.493	0.74	0.457
31	-0.69	0.493	-0.83	0.405
32	-0.75	0.454	-0.74	0.462
33	0.50	0.620	0.00	1.000
34	0.45	0.653	0.89	0.373
35	0.16	0.870	0.00	1.000
36	-0.64	0.520	-0.78	0.436
37	1.55	0.121	1.14	0.254
38	-0.59	0.554	-0.56	0.572
39	-0.68	0.497	-0.70	0.486

z^w: Wilcoxon's two-sample rank sum test statistic; a positive (negative) value obtains if, within the pooled sample of a round, the mean rank-sum score is higher (lower) for observations from FP auctions; *p*-value based on asymptotic T-approximation

z^M: Median two-sample test statistic; a positive (negative) value obtains if more (less) than four observations from FP auctions are bigger than the median of the pooled sample of a round; *p*-value based on asymptotic normal approximation

Table 1. Wilcoxon and median two-sample test statistics for sample Roscas with FP and OA auctions

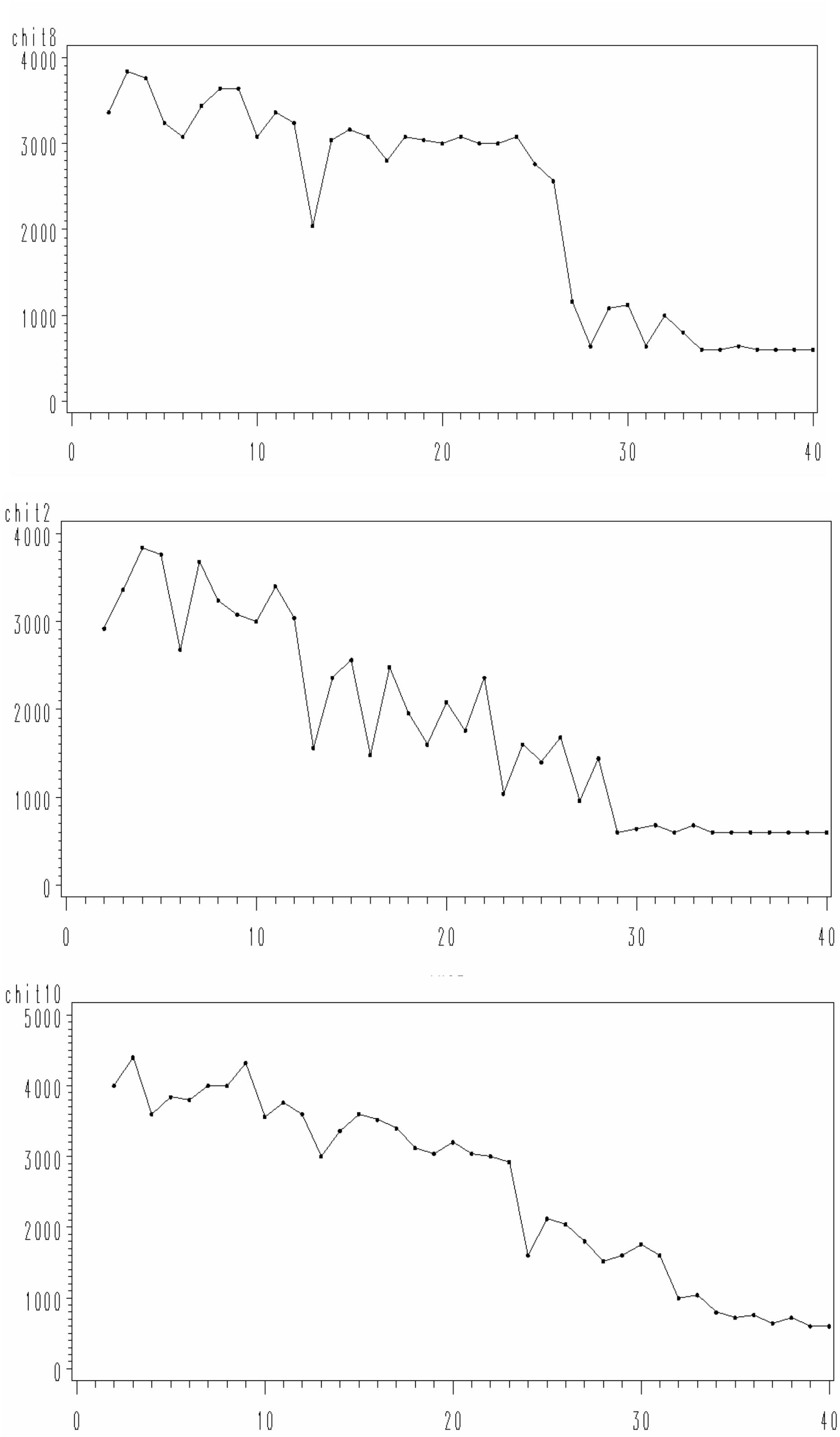

Figure 1. Auction outcomes in three sample Roscas

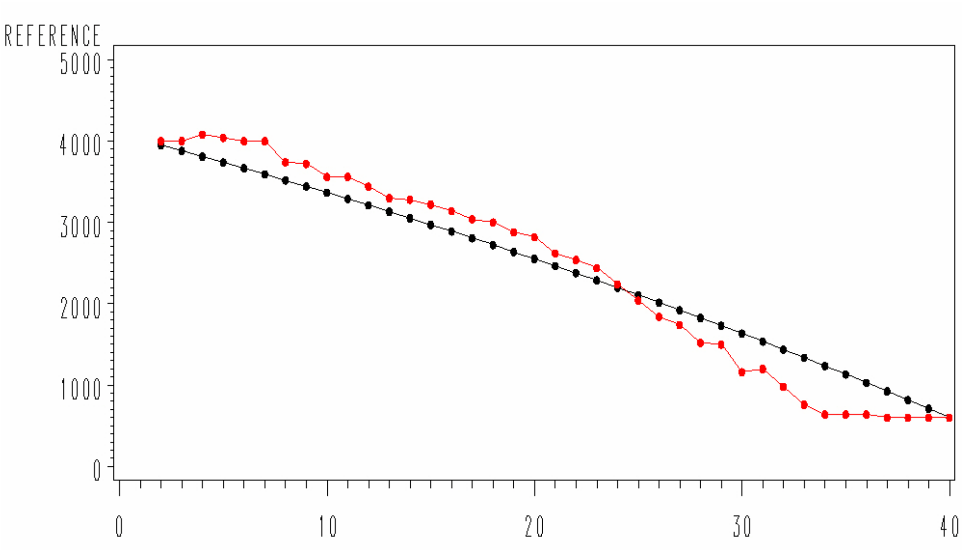

Figure 2. Median price path for all Roscas in the sample

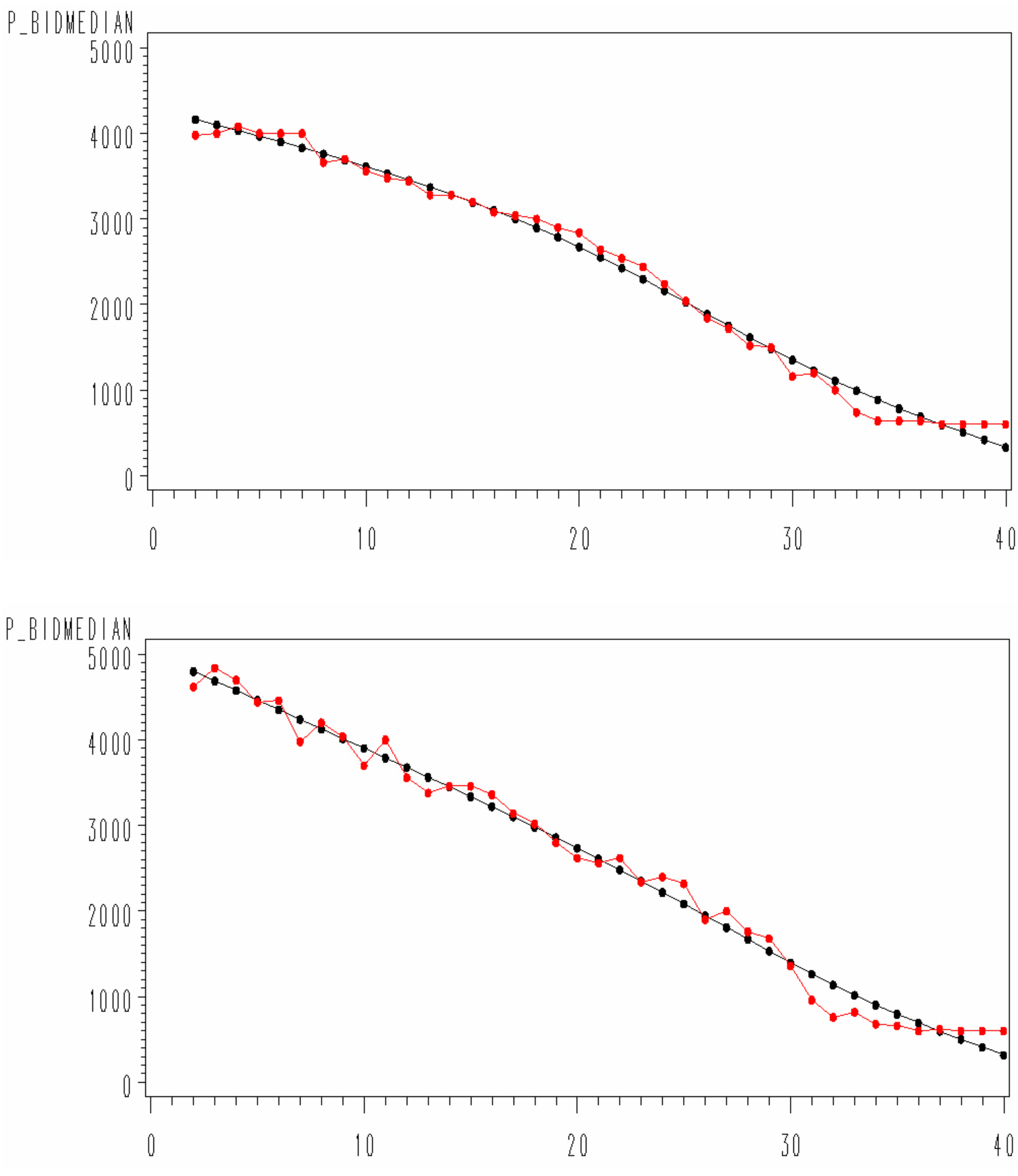

Figure 3. Median prices in sample Roscas with OA (upper) and FP auctions (lower panel), actual and smoothed