How Roscas Perform as Insurance

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Abstract:

Recent theoretical literature has explained the prevalence of rotating savings and credit associations (Roscas) exclusively with the desire to finance an indivisible good. In this paper, in contrast, Rosca participants' incomes are independently distributed and privately observed, and Rosca funds are used for consumption. We classify a bidding Rosca as a simple intertemporal trading mechanism, which provides insurance against idiosyncratic income shocks. For a large economy, we compare the risk-sharing performance of a credit market with a fixed interest rate and a network of bidding Roscas with non-overlapping membership. For the case of CARA preferences and uniformly distributed incomes, we find that the auction allocation mechanism of a bidding Rosca limits the set of feasible allocations in such a way that there exists no network of bidding Roscas which provides more efficient risk sharing than a credit market with a fixed interest rate.

Keywords: Roscas; Auctions; Risk Sharing; Insurance

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1 Introduction

It is widely recognized that in low-income countries risk plays a crucial role in people's everyday life. In the agricultural sector, there is uncertainty about rainfall and crop damage, in the cities, casual labourers face employment uncertainty. In both sectors, the prevalence of infectious diseases makes labourers' ability to generate income uncertain. At the same time, poor public infrastructure, illiteracy and an inefficient legal system impose limits on the functioning of formal market institutions that may insure such risks (see Besley, 1995). While some governments try to mitigate aggregate shocks, e.g. by accumulating and releasing food stocks, the absence of formal health and unemployment insurance often leaves individuals alone when they are affected by idiosyncratic shocks. Because of this lack of formal insurance markets, however, numerous nonmarket risk-sharing institutions are observed. The basic idea underlying all these institutions is that, if shocks are not perfectly correlated across individuals, transfers contingent on each individual's shock improve each individual's situation, at least from an ex ante perspective.

The analysis of such institutions has a long history in development economics dating back to Cheung's (1968) contribution on risk sharing in sharecropping contracts. More recently, economists' interest in this field has grown rapidly. In an empirical investigation, Udry (1990) finds that informal credit in rural Nigeria serves as insurance against idiosyncratic risks. In a theoretical paper, Coate and Ravallion (1993) characterize optimal risk sharing between two households when contractual claims cannot be enforced. In both studies, each household observes not only his own but also his contract partner's income. This assumption may be reasonable in the context of rural villages where information flows freely. In urban settings, where income is generated mostly outside the residential neighbourhood (or slum), individuals may only observe their own incomes. Eswaran and Kotwal (1989) allow private information on incomes but exclude any enforcement problems. They find that, in a two-period world, a market for consumption credit facilitates higher investment than autarky because individuals can smooth their consumption streams by lending and borrowing instead of putting money aside unproductively. In a multi-period world, the analysis of risk sharing with private information becomes rather complicated. Green (1987), Phelan and Townsend (1990), and Atkeson and Lucas (1992) consider a principal and one or many risk-averse agents and characterize incentive-compatible allocations. 'Incentive-compatible' in this context means that, for each agent, reporting the realization of his income truthfully, constitutes a Nash equilibrium. In all of these papers, there is neither individual borrowing nor saving and aggregate consumption does not need to equal aggregate income. In Wang (1995), in contrast, there is no principal and an aggregate budget-balancing constraint is imposed. He analyses the constrained efficient, incentive-compatible insurance contract among two infinitely lived, ex-ante identical agents, when incomes are privately observed and enforcement problems are absent.

There are very few papers addressing the performance of existing nonmarket institutions in developing countries when incomes are privately observed. The reason for this is likely that the mainstream of micro-development economics has focused on the theory of contracts and institutions in the agricultural sector and, as argued above, within a village, information on individual states is often public knowledge. An exception is Udry (1994) who considers idiosyncratic income shocks which are privately observed. Klonner (2000) analyzes Rotating Savings and Credit Associations (Roscas) in a private-information environment among individuals who are exposed to idiosyncratic income shocks and use Rosca funds for consumption only.[1] He finds that, under reasonable assumptions on individual preferences,

[1] An introduction to and further literature on Roscas can be found in that paper.

participation in one bidding Rosca is preferred to autarky. The reason is that the auctions, which take part in the course of a bidding Rosca, identify the bidder with the highest current need for funds and thus generate gains from intertemporal trade.

While Klonner (2000) analyses the functioning of and preferences for Roscas when either one or several Roscas are run within a small group of individuals, the present paper takes a more general perspective and considers an infinitely large two-period economy where individuals are exposed to privately observed income shocks and engage in bilateral intertemporal trade that does not necessarily take the form of a Rosca. We analyze the efficiency of allocations generated by different incentive-compatible trading mechanisms and argue that, because of their complexity, first-best mechanisms are in general not feasible in an environment where formal financial intermediaries are absent and low levels of human capital prevail. We develop a formal categorization of the complexity of intertemporal trading mechanisms and characterize mechanisms which, within our framework, are simpler than first-best mechanisms. Examples for such simple mechanisms are a credit contract with a predetermined interest rate and a bidding Rosca. Both of these mechanisms are frequently observed in informal markets.

The remainder of the paper is organized as follows. In Section 2, we characterize the problem of risk sharing with private information, define intertemporal bilateral trading, and introduce the benchmark of an information-constrained optimal credit contract (ICOC), which essentially involves lending and borrowing at a state-dependent interest rate. Section 3 develops a simple theory of complexity of credit contracts and identifies two groups of contractual arrangements, which, within the said framework, are one category simpler than the ICOC. The first one involves credit contracts with a fixed interest rate and a state-independent principal, while, in the second group, the principal is fixed but the interest rate is state-dependent. Also in the third section, we characterize efficient fixed-interest rate arrangements. Section 4 characterizes fixed-principal arrangements and introduces bidding Roscas as a real-world example. In Section 5, we consider an economy with a network of bidding Roscas with non-overlapping membership and analyze the bidding equilibrium. The sixth section compares fixed-interest and fixed-principal arrangements for a specific family of utility functions and income shocks and investigates whether a network of bidding Roscas can be more efficient than a trading mechanism involving a fixed interest rate. The final section summarizes the findings and offers conclusions.

2. Risk-sharing and Credit Contracts with Private Information

We consider an economy with n identical individuals that ends after two periods. To set out the analytical framework, assume that each individual evaluates first and second-period consumption levels c_1 and c_2 with a bivariate von-Neuman-Morgenstern utility function, $u(c_1, c_2)$ which is strictly increasing and concave in each argument, and that, in period t, her income y_t[2] is drawn from a distribution characterized by the smooth distribution function F on the domain $I \equiv [y_l, y_u]$. All Y_{it}, $i = 1,..., n$, $t = 1, 2$, are assumed to be independently and identically distributed according to F. Each individual observes only her own income and has no further information on other individuals' incomes except that they are drawn from F. In a state of autarky, an individual's expected utility after observing her first period income y is thus $\tilde{u}(y,Y)$, where $\tilde{u}(\cdot, X) \equiv E_X[u(\cdot, X)] = \int_{y_l}^{y_u} u(\cdot,x)dF(x)$. For any two individuals whose first period incomes are not identical, there is scope for intertemporal trade because, under weak assumptions, there is an interest rate at which the better off of the two is willing to hand out a loan to the worse off. There

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[2] Throughout the paper, random variables are denoted by capital letters, while lower case letters denote realizations.

is, however, the problem of privately observed incomes, so a mechanism is needed that induces both individuals to indeed engage in a credit contract.

<u>*Definition 1:*</u> *Consider a set of individuals* $S = \{1,\ldots, n\}$. *For any pair* $(i, j) \in S$, *we define a bilateral trading mechanism* Π_n*:* $(\sigma_i, \sigma_j) \in \mathbb{R}^2 \rightarrow (\kappa, m/(n-1), \psi) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \{i, j\}$, *which assigns a credit contract to the two announcements* σ_i, σ_j, *where* κ *is the discount factor implicit in the loan, m is the principal, and* ψ *is equal to i* (*j*), *if i* (*j*) *is the borrower in the resulting credit contract. A symmetric mechanism, denoted by* Λ_n, *satisfies* $\kappa(\sigma_i, \sigma_j) = \kappa(\sigma_j, \sigma_i)$, $m(\sigma_i, \sigma_j) = m(\sigma_j, \sigma_i)$, *and* $\psi(\sigma_i, \sigma_j) = i$ *if, and only if,* $\psi(\sigma_j, \sigma_i) = j$. Λ *denotes the set of mechanisms* $\{\Lambda_n\}_{n=2}^{\infty}$.

To illustrate this definition, when there are two individuals, i.e. $n = 2$, the payoffs for individual 1 resulting from a credit contract implied by Λ_2 after σ_1 and σ_2 have been announced, are

$$\kappa(\sigma_1, \sigma_2)\, m(\sigma_1, \sigma_2)\,(3-2\psi(\sigma_1, \sigma_2)) \tag{1}$$

in the first period and

$$-\, m(\sigma_1, \sigma_2)\,(3-2\psi(\sigma_1, \sigma_2)) \tag{2}$$

in the second period. Thus the interest rate implicit in a loan as characterized by Definition 1 is $(1/\kappa(\sigma_1, \sigma_2)) - 1$.

Since we are interested in settings where financial intermediaries are absent, we consider a decentralized economy where all individuals trade with each other symmetrically.

<u>*Definition 2:*</u> *In a n-person trading economy* $\Omega_n(\Lambda_n)$, *each individual enters a credit contract with each of the other n-1 individuals according to the mechanism* Λ_n.

Of course, the announcement an individual makes in each contract is a strategic variable. In general, an individual can make a different announcement in each of her n - 1 bilateral trades.

<u>Definition 3:</u> *Consider a* $\Omega_n(\Lambda_n)$*-economy. For all* $i \in S$*, a strategy* $\Sigma_i(\Lambda_n)$*:* $y_i \in I \rightarrow (\sigma_1,\ldots,\sigma_{i-1},\sigma_{i+1},\ldots,\sigma_n) \in \mathbb{R}^{n-1}$ *assigns a collection of announcements to i's first-period income* y_i*. For a uniform strategy* $\Sigma_i^u(\Lambda_n)$*, the map* $\Sigma_i^u(\Lambda_n)$*:* $y_i \rightarrow (\sigma_1,\ldots,\sigma_{i-1},\sigma_{i+1},\ldots,\sigma_n)$ *is a function and satisfies* $\sigma_1=\ldots=\sigma_{i-1}=\sigma_{i+1}=\ldots=\sigma_n$*. A* $\Omega_n(\Lambda_n)$*-economy has a symmetric, uniform strategy Nash equilibrium if there exists a uniform strategy* $\breve{\Sigma}^u(\Lambda_n)$ *such that it is a Nash equilibrium if each individual in the economy plays* $\breve{\Sigma}^u(\Lambda_n)$*. The announcement function corresponding to* $\breve{\Sigma}^u(\Lambda_n)$ *is denoted by* $\breve{\sigma}_{\Lambda_n}(y)$*.*

Since, without doubt, making $n-1$ different announcements is intellectually more demanding than making the same announcement in all $n-1$ contracts, and we are concerned with environments of low levels of human capital, for the rest of the paper, we restrict our attention exclusively to uniform strategies and uniform strategy equilibria.

The revelation principle says that, for each mechanism Λ_n which induces a symmetric, uniform strategy Nash equilibrium in $\Omega_n(\Lambda_n)$, there exists an incentive-compatible mechanism, $\hat{\Lambda}_n$ say, which, in the corresponding symmetric, uniform strategy Nash equilibrium, generates identical payoffs. In such an equilibrium, however, each individual announces her first-period income truthfully. Further, since risk sharing typically involves a transfer from the higher to the lower income, we restrict our attention to mechanisms which, in each bilateral trade, make the individual with the lower first-period income the borrower.

<u>Definition 4:</u> *An incentive-compatible risk sharing mechanism, denoted by* $\hat{\Lambda}_n$, *satisfies the following two conditions:*

(i) $\psi(\sigma_i, \sigma_j) = i$ *if, and only if,* $\sigma_i < \sigma_j$;

(ii) there exists a symmetric, uniform strategy Nash equilibrium where each individual announces her first-period income truthfully, i.e. $\exists\ \breve{\Sigma}^u(\Lambda_n)$ *with* $\breve{\sigma}_{\hat{\Lambda}_n}(y) = y$.

Now consider $\Omega_\infty(\hat{\Lambda}_\infty)$, a trading economy with infinitely many individuals trading according to the incentive-compatible risk-sharing mechanism $\hat{\Lambda}_\infty$. Using the definitions given above and applying the law of large numbers to (1) and (2), we obtain the payoffs in a symmetric, uniform strategy Nash equilibrium to an individual with first-period income y as

$$\begin{aligned} &E[\kappa(y, Y)\, m(y, Y)\,(2\,^1\{y < Y\} -1)] \\ &\quad = E[\kappa(y, Y)\, m(y, Y) \mid y < Y](1-F(y)) - E[\kappa(y, Y)\, m(y, Y) \mid Y < y]\, F(y) \equiv \beta_1(y) \end{aligned} \tag{3}$$

in the first period and

$$\begin{aligned} &- E[m(y, Y)\,(2\,^1\{y < Y\} -1)] \\ &\quad = E[m(y, Y) \mid Y < y]\, F(y) - E[m(y, Y) \mid y < Y](1-F(y)) \equiv \beta_2(y) \end{aligned} \tag{4}$$

in the second period, where $^1\{\cdot\}$ denotes the indicator function. In this notation, incentive-compatibility requires that

$$y = \arg\max_{\sigma} \tilde{u}\left(y + \beta_1(\sigma), Y + \beta_2(\sigma)\right) \text{ for all } y \in I. \tag{5}$$

With this in hand, we can turn to the payoffs of an ex-ante efficient, incentive-compatible risk-sharing mechanism. By 'ex ante' we refer to the stage before first-period incomes are observed. Thus, at this stage, first-period and second-period incomes have to be treated as random variables and are denoted by Y_1 and Y_2, respectively. In a trading economy with infinitely many individuals, such a mechanism is characterized by the following problem:

$$(6) \quad \max_{\{\beta_1(\cdot),\beta_2(\cdot)\}} E_{Y_1}[\tilde{u}\left(Y_1+\beta_1(Y_1), Y_2+\beta_2(Y_1)\right)] \text{ s.t.}$$

$$(7) \quad \tilde{u}_1\left(y_1+\beta_1(y_1), Y_2+\beta_2(y_1)\right)\beta_1'(y_1)+\tilde{u}_2\left(y_1+\beta_1(y_1), Y_2+\beta_2(y_1)\right)\beta_2'(y_1)=0 \text{ for all } y_1 \in I,$$

$$(8) \quad E[\beta_1(Y_1)] = 0,$$

$$(9) \quad E[\beta_2(Y_1)] = 0.$$

Note that in deriving the incentive-compatibility constraint (7), we have followed the first-order approach, since, in general, (7) is only a necessary condition for the general incentive-compatibility constraint given by (5). Although the first-order approach has some limitations (see e.g. Rogerson, 1985, for the case of principal-agent problems), we have chosen this representation because it is most suited for the analysis of the next sections. Equations (8) and (9) represent budget-balancing constraints for the economy, formalizing the fact that transfers in both periods sum to zero. In all these derivations, as for the rest of the paper, we have assumed that the location of I and the shape of $\beta_1(\cdot)$ and $\beta_2(\cdot)$ are such that, for each pair of realised incomes (y_1, y_2), consumption in both periods is non-negative.

3 Classifying the Complexity of Risk-Sharing Mechanisms

As mentioned in the introduction, this paper is concerned with environments where large-scale financial intermediaries are absent. If there was an omniscient benevolent dictator in Ω_∞, he could compute the solution to the problem (6) to (9), $\beta_1^*(\cdot)$ and $\beta_2^*(\cdot)$ say, and, after first-period incomes are observed, accept deposits and hand out loans at conditions corresponding to $\beta_1*(\cdot)$ and $\beta_2*(\cdot)$. As can be seen from (3) and (4), however, the implementation of $\beta_1*(\cdot)$ and $\beta_2*(\cdot)$ in decentralized trading in general involves bilateral credit contracts in which both the principal and the interest rate (represented by κ) depend on observed first-period incomes. In an environment where written contracts do not exist and human capital is at a low level, mechanisms of this kind can most likely not be implemented because they are too complex. For example, two individuals are faced with an entire menu that assigns an interest rate and a principal to each pair of announcements, most likely in a non-linear fashion. Our mechanism framework facilitates a formal classification of this notion of complexity. Recall Definition 1, where a credit contract is defined by three elements, the discount factor κ, the principal m, and an indicator determining which of the two parties is the borrower, ψ. A natural classification partitions the set of mechanisms Λ in three subsets. First, mechanisms where κ, m and ψ respond to the participants' announcements. Second, mechanisms where either the principal or the discount factor is fixed in advance. Third, mechanisms where the principal and the discount factor are fixed and the trading parties' announcements only matter for determining ψ. These ideas are summarized in Table 1.

Complexity Category	Mechanism Label	ψ	m	κ
1	Λ	free	free	free
2	Λ^{κ}	free	free	fixed
	Λ^{m}	free	fixed	free
3		free	fixed	fixed

Table 1. A classification scheme for credit contracts

Following Definition 4, we denote the incentive-compatible mechanisms corresponding to Λ^{κ} and Λ^{m} by $\hat{\Lambda}^{\kappa}$ and $\hat{\Lambda}^{m}$, respectively.

An example for $\Lambda^{\kappa}_{\infty}$ is a competitive credit market with a state-independent interest rate, or 'credit market' for short, where each individual is a price (i.e. interest rate) taker and announces her demand for credit. Such mechanisms correspond to the notion of a 'going interest rate', which is reported by many empirical studies of informal financial markets.

In a symmetric, uniform strategy Nash equilibrium of a $\Omega_{\infty}(\hat{\Lambda}^{\kappa}_{\infty})$-economy, we obtain the payoffs to an individual with first-period income y by replacing the function $\kappa(\cdot,\cdot)$ in (3) by the constant κ, namely $E[\kappa m(y, Y)(2^{1}\{y < Y\} -1)] = \kappa(E[m(y, Y) \mid y < Y](1-F(y)) - E[m(y, Y) \mid Y < y]\, F(y)) \equiv \kappa\, \xi(y)$ in the first period and $- E[m(y, Y)(2^{1}\{y < Y\} -1)] = - \xi(y)$ in the second period. Incentive compatibility now requires that

(10) $\quad y = \arg\max_{\sigma} \tilde{u}\big(y + \kappa\xi(\sigma), Y - \xi(\sigma)\big)$ for all $y \in I$,

and, analogous to equations (6) to (9), the payoffs of an ex-ante efficient $\hat{\Lambda}^{\kappa}_{\infty}$-mechanism are characterized by

(11) $\quad \max_{\{\kappa,\xi(\cdot)\}} E_{Y_1}[\tilde{u}\big(Y_1 + \kappa\xi(Y_1), Y_2 - \xi(Y_1)\big)]$ s.t.

(12) $\quad \kappa\, \tilde{u}_1\big(y_1 + \kappa\xi(y_1), Y_2 - \xi(y_1)\big) - \tilde{u}_2\big(y_1 + \kappa\xi(y_1), Y_2 - \xi(y_1)\big) = 0$ for all $y_1 \in I$,

(13) $\quad E[\beta(Y_1)] = 0.$

Note that in obtaining (12) from (10), we have again adopted the first-order approach. Any ex-ante efficient $\hat{\Lambda}^{\kappa}_{\infty}$-mechanism will be denoted by $\hat{\Lambda}^{\kappa}_{\infty}*$.

4 The Bidding Rosca as a Fixed-principal Mechanism

Λ^m-mechanisms are less familiar when we think of competitive equilibria because, in this latter category, the price in each transaction has to be determined while the quantity is fixed. Before we turn to bidding Roscas as a prominent real-world example belonging to this category, we determine the equilibrium payoffs in a $\Omega_\infty(\hat{\Lambda}^m_\infty)$-economy in general. This is achieved by substituting the constant m for the function $m(\cdot,\cdot)$ in (3) and (4). For an individual with first-period income y we obtain

$$\begin{aligned} &E[\kappa(y, Y)\, m\,(2\,^1\{y < Y\} -1)] \\ &= (E[\kappa(y, Y) \mid y < Y](1-F(y)) - E[\kappa(y, Y) \mid Y < y]\, F(y))\, m \equiv \phi(y)\, m \end{aligned} \tag{14}$$

in the first and

$$- E[m\,(2\,^1\{y < Y\} -1)] = (2F(y) - 1)\, m \tag{15}$$

in the second period. Incentive compatibility requires that

$$y = \arg\max_{\sigma} \tilde{u}\left(y + \phi(\sigma)m, Y + (2F(\sigma) - 1)m\right) \text{ for all } y \in I, \tag{16}$$

and, again adopting the first-order approach, the payoffs of an ex-ante efficient $\hat{\Lambda}^m_\infty$-mechanism are consequently characterized by

$$\max_{\{m,\phi(\cdot)\}} E_{Y_1}[\tilde{u}\left(Y_1 + \phi(Y_1)m, Y_2 + (2F(Y_1) - 1\right)m] \text{ s.t.} \tag{17}$$

$$\tilde{u}_1\left(y_1 + \phi(y_1)m, Y_2 + (2F(y_1) - 1)m\right)\phi'(y_1) + \tilde{u}_2\left(y_1 + \phi(y_1)m, Y_2 + (2F(y_1) - 1)m\right)2f(y_1) = 0 \tag{18}$$

for all $y \in I$,

$$E[\phi(Y_1)] = 0. \tag{19}$$

If a parameter m and a function $\phi(\cdot)$ satisfy (18) and (19), we denote the corresponding incentive-compatible mechanism by $\hat{\Lambda}_{\infty}^{m}(m, \phi(\cdot))$. Any efficient $\hat{\Lambda}_{\infty}^{m}$-mechanism will be denoted by $\hat{\Lambda}_{\infty}^{m}*$. If for a particular fixed-principal mechanism, $\Lambda_{\infty}^{\#}$ say, the equilibrium payoffs to all individuals in a $\Omega_{\infty}(\Lambda_{\infty}^{\#})$-economy and a $\Omega_{\infty}\left(\hat{\Lambda}_{\infty}^{m}\left(m,\phi(\cdot)\right)\right)$-economy are identical, we say that $\Lambda_{\infty}^{\#}$ implements $\hat{\Lambda}_{\infty}^{m}(m, \phi(\cdot))$.

Let us now turn to the bidding Rosca as an example of a fixed-principal mechanism. In keeping with Definition 1, we only deal with two-person Roscas. As in Klonner (2000), consider two individuals, i.e. $n = 2$, who, before first-period incomes are observed, agree to contribute an amount of m^R to a pot in both the first and the second period. After each individual has observed her first-period income privately and the first-period pot (or 'pot one' for short) of $2m^R$ has been established, an auction for pot one is staged. The winner of this auction receives the pot at a price of b, say, which is equally distributed between the two participants. Thus, in the first period, the payoffs resulting from the Rosca participation, are $-m^R + 2m^R - b + b/2 = m^R - b/2$ for the winner and $- m^R + b/2$ for the loser. In the second period, again a pot of $2m^R$ is established, but this time the loser of the first-period auction receives the pot without a discount. Consequently, in the second period, the payoffs resulting from the Rosca, are $-m^R$ for the first-period winner and m^R for the first-period loser. If the auction takes the form of an oral ascending bid auction, then b is equal to the standing bid at which one of the two bidders quits the bidding process and thereby ends the auction. Let σ_i, $i = 1, 2$, denote the standing bid at which bidder i intends to quit the bidding process. Then b is equal to $\min[\sigma_1, \sigma_2]$ and, in the terminology of Definition 1, such a Rosca is a Λ_2^m-mechanism with $m = m^R$, $\kappa(\sigma_1, \sigma_2) = 1 - (b/2m^R) = 1 - (\min_i [\sigma_i]/2m)$ and $\psi(\sigma_1, \sigma_2) = \arg\max_i [\sigma_i]$. More generally, in a n-person trading economy, each individual contributes $m^R/(n\text{-}1)$ to each of the n - 1 Roscas in which she participates. We denote such a mechanism by Λ_n^R or by $\Lambda_n^R(m^R)$ when we refer to a particular value of m^R. The limiting case Λ_∞^R shall be called a 'Rosca network'.

5 The Bidding Equilibrium of a Rosca Network

In the theoretical literature on Roscas, the issue of bidding equilibria when individuals participate in several Roscas simultaneously has so far not been addressed. In connection with the present analysis, two issues need to be clarified. First, whether a Rosca network has a symmetric, uniform strategy equilibrium. Second, whether a Rosca network can implement any, in particular any efficient, $\hat{\Lambda}_\infty^m$-mechanism or whether the set of $\hat{\Lambda}_\infty^m$-mechanisms which can be implemented by a Rosca network is smaller then the set of all $\hat{\Lambda}_\infty^m$-mechanisms. To answer these questions, we have to take a closer look at the bidding in a Rosca network.

When an individual participates in several Roscas simultaneously, she has to decide at which level of the standing bid she wants to quit the bidding process in each Rosca. If she participates in $n - 1$ Roscas, a uniform strategy implies that she quits each auction at the same level of the standing bid. Define b_n as the sum of all bids that an individual submits to the n - 1 Roscas in which she participates, or 'aggregate bid' for short. In a uniform strategy, b_n is a function of first-period income y and, following the terminology of Definition 3, we denote the corresponding uniform strategy, symmetric Nash equilibrium in a Rosca Network, provided it exists, by $\breve{\Sigma}^u(\Lambda_\infty^R)$ and the corresponding aggregate bidding function by $b(y)$. Since we are interested in risk-sharing mechanisms, we require that each auction generates a transfer from the bidder with the higher to the bidder with the lower first-period income, that is the latter one shall always be the winner of the first-period pot, which requires that $b(y_i) < b(y_j)$ for all $y_i > y_j$. If $b(\cdot)$ is continuously differentiable, this latter condition is equivalent to $b'(y) < 0$ for almost all y.

Further, for the sake of plausibility, we require that aggregate bids are always non-negative and finite. By the law of large numbers, the first-period payoff to an individual, i say, with first period income y is $-m^R + F(y)\, b(y)/2 + (1 - F(y))\,(2m^R - E[b(Y)|Y > y]/2)$, where $-m^R$ is i's aggregate contribution to the Roscas and $F(y)\, b(y)/2$ is her payoff from that fraction of Roscas where her respective trading partner, j say, observes a lower income than herself. In all those Roscas, i ends the auction by quitting each of these auctions at a standing bid corresponding to the aggregate bid $b(y)$. Since, in all Roscas, each of the two participants receives half of the price the winner pays, i receives $b(y)/2$ times the fraction $F(y)$ from those Roscas. In the remaining fraction of Roscas, $1 - F(y)$, i observes a lower income than her respective trading partner. In those cases, i receives the pot minus half the price that is determined by her trading partner, on average $E[b(Y)|Y > y]$. In the second period, as in (15), i's payoff is $-m^R + F(y)2m^R$, where, again, $-m^R$ is i's aggregate contribution and $F(y)$ is the fraction of Roscas where i does not win the first-period auction and consequently receives the second pot.

<u>Proposition 1:</u> *Define*

(20) $$b^*(y) \equiv 2m\Big(1 + \big(\phi(y) - E[\phi(Y) \mid Y < y]\big)/F(y)\Big).$$

(i) *Any $\hat{\Lambda}_\infty^m(\phi(\cdot), m)$-mechanism can be implemented by the Rosca network $\Lambda_\infty^R(m)$ if, and only if, $b^{*\prime}(y) < 0$ for almost all y. If $b^{*\prime}(y) < 0$ for almost all y, then, in the symmetric, uniform strategy equilibrium, each individual bids according to the aggregate bidding function $b^*(y)$.*

(ii) $b^{*\prime}(y) < 0$ *if, and only if,*

(21) $$\int_{y_l}^{y} F(\rho)\big[f(\rho)\phi'(y) - f(y)\phi'(\rho)\big]\,d\rho < 0.$$

Proof: see the Appendix.

Proposition 1 says that whether a $\hat{\Lambda}_{\infty}^{m}(\phi(\cdot), m)$-mechanism can be implemented by a Rosca network, depends on the properties of $\phi(\cdot)$ and $F(\cdot)$. Note that a sufficient condition for (21) to hold for all y is found to be $(f'(y)/ f(y)) < (\phi''(y)/\phi'(y))$ for all y, which relates the curvature properties of $\phi(y)$ and $F(y)$. It can be shown, in contrast, that (21) is violated if, for small values of y, $\phi''(y) \geq 0$ and if $f(y)$ is increasing at the left tail of the distribution. Proposition 1 does not state whether aggregate bids are always positive as we require for the sake of plausibility. What can be seen, however, is that aggregate bids are always finite. For the highest possible income, we obtain $b^*(y_u) = 2m^R(1 + \phi(y_u))$ because, by (19), $\int_{y_l}^{y_u} \phi(\rho) f(\rho) d\rho = 0$, while for the lowest possible income, by L'Hôpital's rule, we have $b^*(y_l) = 2m^R\left(1+\frac{\phi'(y_l)}{2f(y_l)}\right)$.

A further theoretical analysis of the questions raised in the previous paragraph is limited by the complexity of the problem. The next paragraph, therefore, illustrates all these issues for a specific example.

6 The Risk-sharing Performance of Rosca Networks and Credit Markets: An Example

In this section, we compare the risk-sharing performance of a Rosca network and a credit market with a fixed interest rate in an economy with infinitely many individuals. Since, as has been argued, it is sufficient to look at the respective incentive-compatible mechanisms when the payoffs are concerned, the question is whether ex-ante expected utility with an efficient $\hat{\Lambda}_{\infty}^{\kappa}$-mechanism (obtained from (11) to (13)) or an efficient $\hat{\Lambda}_{\infty}^{m}$-mechanism (obtained from (17) to (19)) is higher and whether a Rosca network can implement the payoffs of such an efficient

$\hat{\Lambda}_{\infty}^{m}$-mechanism. As to the former of these two questions, since there are two optimization problems involved which are not nested, it is impossible to determine in general which of the two attains a higher value of the objective function. Both problems, however, have in common that there is one parameter and one function to be determined, where, in each of the two problems, the expected value of the said function has to equal zero. Provided the first-order approach is valid, the major difference between the two problems is that (12), which belongs to the fixed-interest rate mechanism, is an algebraic equation, whereas (18) is an ordinary first-order differential equation. Since the solution to an ordinary first-order differential equation involves one degree of freedom while, under mild conditions, the algebraic equation (12) assigns exactly one β to each y, the fixed-principal mechanism has potentially one more degree of freedom. In general, however, it is impossible to make this point rigorous because, due to its integral form, (13) does not necessarily define a unique κ, while, similarly, (19) is not a proper boundary condition.

It is on these grounds that, for the rest of this section, we turn to specific families of utility functions and income-shock distributions, which facilitate a comparison of $\hat{\Lambda}_{\infty}^{\kappa}$ and $\hat{\Lambda}_{\infty}^{m}$-mechanisms. Suppose that utility takes the CARA form

(22) $u(c_1, c_2) = v(c_1) + \delta v(c_2)$, $v(c_t) \equiv -\exp[-ac_t]$ with $a > 0$ and $0 < \delta \leq 1$,

and that the income random variable is uniformly distributed on the unit interval, i.e. $F(y) = y$, $I = [0, 1]$. Since, for the CARA specification (22), decisions are independent of the level of income, the following analysis extends to any uniform distribution of the form $F_{\eta}(y) \equiv F(y) - \eta$, where η is an arbitrary constant. To exclude any problems of inability to pay, η has to be bigger than some threshold value, η^* say, which will be specified shortly.

Under these assumptions, the solution to the fixed-interest rate problem (11) to (13) is easily found to be

$$\kappa^* = \delta \frac{E[v(Y)]}{v(E[Y])} = (\delta/a)\left(Exp[a/2] - Exp[-a/2]\right),$$

$$\xi^*(y) = \frac{1/2 - y}{1+\kappa^*}.$$

From the shape of $\xi^*(\cdot)$ it follows that the minimum income ensuring every individual's ability to pay is $\eta^* = \xi^*(0)$. Further note that κ^* and $\xi^*(\cdot)$ are uniquely determined by the two constraints (12) and (13) in this case. Thus, by the revelation principle, any Λ_∞^κ-mechanism with a symmetric, uniform strategy equilibrium generates the payoffs defined by κ^* and $\xi^*(\cdot)$.

Turning to fixed-principal mechanisms, the solution to the differential equation (18) is

(23) $$\phi(y) = -1 - \frac{1}{am}\left(Log\left[\delta E[v(Y)]\right] + Log\left[\frac{2m}{2m-1} v\left((2m-1)y\right) + \chi\right]\right)$$

with some constant χ. Note that $\frac{\partial \phi(y)}{\partial \chi}$ is negative for all y and so, given a particular m, (19) determines χ uniquely. Let us consider the case $\chi = 0$. Then $\phi(\cdot)$ as defined by (23) is a linear function and the constraint (19) implies that

(24) $$m^* = \frac{1}{2}\left(\delta \frac{E[v(Y)]}{v(E[y])} + 1\right)^{-1} = \frac{1}{2(\kappa^*+1)}$$

and, by substituting (24) into (23),

(25) $\phi^*(y) = \kappa^*(1 - 2y)$.

Thus, for $\chi = 0$, the payoffs of the fixed-principal arrangement are equivalent to the payoffs of the fixed-interest rate arrangement, namely $\kappa^*(1/2 - y)/(\kappa^* + 1)$ in the first and $(1/2 - y)/(\kappa^* + 1)$ in the second period. Since χ is a free parameter in general, we immediately obtain

<u>Proposition 2:</u> *For CARA utility and income distributed uniformly according to F_η, there exists a $\hat{\Lambda}_\infty^m$-mechanism which is at least as efficient as $\hat{\Lambda}_\infty^\kappa *$.*

Can a Rosca Network implement $\hat{\Lambda}_\infty^m(m^*, \phi^*(\cdot))$, which is the incentive-compatible fixed-principal mechanism that generates the same payoffs as $\hat{\Lambda}_\infty^\kappa *$? Plugging (24) and (25) into (20) gives

(26) $$b^*(y) = \frac{1-\kappa^*}{1+\kappa^*} \equiv b^*.$$

Thus, in a symmetric, uniform strategy bidding equilibrium, the same price is paid in every auction. Whether the said price in form of b^* is positive, depends on whether $\kappa^* < 1$, which, in turn, requires a sufficiently high degree of impatience. In particular we find that κ^* is smaller than unity, if, and only if,

$$\delta < \frac{v(E[Y])}{E[v(Y)]} = \frac{a}{\left(Exp[a/2] - Exp[-a/2]\right)} = \frac{a}{2}\text{csch}\left[\frac{a}{2}\right],$$

which is a strictly decreasing function of a and equal to one for $a = 0$. Thus, loosely speaking, a positive interest rate, and thus a positive b^*, only obtains if intertemporal discounting is stronger than risk aversion. Since with (26), in each auction both bidders quit the bidding process at the same level of the standing bid, the auction does not fulfil its task, namely to identify the individual with the lower first-period income. Thus a Rosca network is not able to implement $\hat{\Lambda}_{\infty}^{m}(m^*, \phi^*(\cdot))$.

In general, rational individuals will not choose a $\hat{\Lambda}_{\infty}^{m}$-mechanism with m equal to m^* because $\hat{\Lambda}_{\infty}^{m}(m^*, \phi^*(\cdot))$ is not the result of the optimisation (17) to (19). Since, however, the said optimization cannot be treated analytically and our prime task is to compare the efficiency of a credit market and a Rosca network, we focus on a small variation of m with $m = m^*$ as the reference point. In particular, we seek conditions determining whether a level of m bigger or smaller than m^* is advantageous. For the CARA-uniform case, consider the expected utility of an individual at the interim stage, that is after observing her first-period income y, and let $E[U(\hat{\Lambda}_{\infty}^{m}) | y]$ denote interim expected utility of an individual with first-period income y when a $\hat{\Lambda}_{\infty}^{m}(m, \phi(\cdot))$-mechanism is in effect. If an increase in m increases $E[U(\hat{\Lambda}_{\infty}^{m}) | y]$ for all y, then the resulting mechanism will be more efficient at the interim stage and thus also at the ex-ante stage. This gives rise to

<u>Proposition 3:</u> *For CARA utility and income distributed uniformly according to F_η, consider the fixed-principal mechanism $\hat{\Lambda}_{\infty}^{m}(m^*, \phi^*(\cdot))$. At the interim stage, individuals with sufficiently big and sufficiently small incomes prefer an increase in m, while there exists an individual with an intermediate income, y* say, who prefers a decrease in m. Formally,*

(27) $$\frac{dE[U(\hat{\Lambda}_{\infty}^{m}) | y_l]}{dm}_{|m=m^*} > 0 \text{ and } \frac{dE[U(\hat{\Lambda}_{\infty}^{m}) | y_u]}{dm}_{|m=m^*} > 0.$$

Further, for some y,*

$$\frac{dE[U(\hat{\Lambda}_{\infty}^{m}) \mid y^*]}{dm}_{|m=m^*} < 0. \tag{28}$$

Proof: see the Appendix.

Thus, after observing first-period incomes, the economy's individuals cannot decide on an increase in m unanimously. For this reason, we have to turn to the ex-ante stage, that is before first-period incomes are observed. Specifically, we want to determine whether ex-ante expected utility is increasing or decreasing in m with m^* as the reference point. In this connection, the following result can be proven analytically.

Proposition 4: *Define ex ante expected utility* $E[U(\hat{\Lambda}_{\infty}^{m})] \equiv E_Y[U(\hat{\Lambda}_{\infty}^{m}) \mid Y]$. *For CARA utility and income distributed uniformly according to* F_η,

(i) $$\frac{dE[U(\hat{\Lambda}_{\infty}^{m})]}{dm}_{|m=m^*} = 0 \text{ if } \kappa^* = 1.$$

Further, there exists an $\varepsilon > 0$ *such that*

(ii) $$\frac{dE[U(\hat{\Lambda}_{\infty}^{m})]}{dm}_{|m=m^*} > 0 \text{ for all } 0 < \kappa^* < \varepsilon,$$

(iii) $$\frac{dE[U(\hat{\Lambda}_{\infty}^{2b})]}{dm}_{|m=m^*} > 0 \text{ for all } 1 - \varepsilon < \kappa^* < 1,$$

(iv) $$\frac{dE[U(\hat{\Lambda}_{\infty}^{m})]}{dm}_{|m=m^*} < 0 \text{ for all } 1 < \kappa^* < 1 + \varepsilon.$$

Proof: see the Appendix.

Proposition 4 says that if the equilibrium interest rate in the credit market, $(1/\kappa^*)$-1, is equal to zero, then the payoff-equivalent fixed-principal mechanism cannot be improved on by an infinitesimal change in *m*. Further, for a very large or slightly positive equilibrium interest rate, the payoff-equivalent fixed-principal mechanism can be improved on by increasing *m*. If, on the other hand, there is a slightly negative equilibrium interest rate, then the payoff-equivalent fixed-principal mechanism can be improved on by decreasing *m*. Numerical computations, moreover, suggest

Result 1: For CARA utility, income distributed uniformly according to F_η, and

$0.01 < a < 200$,

$$\frac{dE[U(\hat{\Lambda}_{\infty}^{m})]}{dm}_{|m=m^*} > 0 \textit{ for all } \kappa^* < 1,$$

$$\frac{dE[U(\hat{\Lambda}_{\infty}^{m})]}{dm}_{|m=m^*} < 0 \textit{ for all } \kappa^* > 1.$$

The findings of Proposition 4 are generalized by the previous result, which states that, whenever there is a positive (negative) equilibrium interest rate in the credit market, the payoff-equivalent fixed-principal mechanism can be improved on by increasing (decreasing) *m*.

We now return to Rosca networks and ask whether the bidding allocation mechanism can implement a $\hat{\Lambda}_{\infty}^{m}$-mechanism with *m* slightly bigger than *m** because, as has been argued, increasing *m* is advantageous whenever a positive equilibrium interest rate occurs in the credit market. Recall from (26) that, in the symmetric equilibrium, $\Lambda_{\infty}^{R}(m^*)$ involves a flat aggregate bidding function, i.e. $b^{*\prime}(y) = 0$. As m changes, the question is how the slope of the said bidding function, $b^*(y)$, changes. Recalling that only a strictly decreasing aggregate bidding function facilitates the implementation of a $\hat{\Lambda}_\infty^m$-mechanism by a Rosca network, it needs to be determined whether $\frac{\partial b^{*\prime}(y)}{\partial m}_{|m=m^*}$ is negative for all y. The answer is provided by

<u>Proposition 5:</u> *For CARA utility and income distributed uniformly according to F_η,*

$$\frac{\partial b^{*\prime}(y)}{\partial m}_{|m=m^*} > 0 \textit{ for all of } \kappa^*,$$

i.e. a Rosca network with m slightly bigger than m has no symmetric, uniform strategy bidding equilibrium.*

<u>Proof:</u> see the Appendix.

Thus $b^*(y)$ will be downward-sloping if, and only if, m is chosen slightly smaller than m^*. Recall from Result 1, however, that a decrease in m from m^* is only advantageous if the equilibrium interest rate in the credit market is negative and that on the other hand, by virtue of (18), a negative equilibrium interest rate in the credit market implies a negative b^*, which is empirically implausible. These considerations give rise to

<u>Result 2:</u> *For CARA utility, income distributed uniformly according to F_η, and* $0.01 < a < 200$, *there exists an* $\varepsilon > 0$ *such that for all* $m \in [m^*-\varepsilon, m^*+\varepsilon]$ *at least one of the following conditions is violated:*

(i) $E[U(\Lambda_\infty^R(m))] \geq E[U(\hat{\Lambda}_\infty^{\kappa}{}^*)]$,

(ii) *$b^*(\cdot)$ is strictly decreasing,*

(iii) *$b^*(\cdot)$ is non-negative.*

That is, when we focus on small variations of m around m^*, there does not exist a Rosca network with a symmetric, uniform strategy bidding equilibrium and non-negative bids, which is more ex-ante efficient than any fixed-interest-rate mechanism with a symmetric, uniform strategy equilibrium. Thus, although there exists an incentive-compatible fixed-principal mechanism which is more efficient than any fixed-interest-rate mechanism, a Rosca network is not able to implement the said fixed-principal mechanism. The only scenario under which a Rosca network functions with non-negative bids involves sufficient impatience and a principal smaller than m^*. Result 1 suggests, however, that, in this latter case, a Rosca network is less efficient than any fixed-interest-rate mechanism with a symmetric, uniform strategy equilibrium.

7 Concluding Remarks

Recent theoretical literature has explained the prevalence of Roscas exclusively with the desire to finance an indivisible good, be it a consumption good as in Besley et al. (1993, 1995) or an investment good as in Kovsted and Lyk-Jensen (1999). Moreover, all of these authors have assumed that the incomes of Rosca participants are deterministic. Guided by a body of empirical evidence, this paper and its companion (Klonner, 2000) analyze Roscas among ex-ante identical, risk-averse individuals, who are exposed to privately observed income shocks and use Rosca funds for consumption. In such an environment, bidding Roscas are a mechanism to mitigate the problem of asymmetric information and generate gains from intertemporal trade.

We have shown that, within a two period, private-information environment, first-best risk-sharing mechanisms involve credit contracts where both the interest rate and the principal are state-dependent. We have argued that in environments where informal financial institutions like Roscas prevail, such first-best mechanisms may be too complex to implement. We classified credit contracts with a predetermined interest rate and bidding Roscas with a predetermined contribution as simple intertemporal trading mechanisms because for the former only the principal, and for the latter only the interest rate is determined by the trading parties. For the family of CARA utility functions and uniformly distributed incomes, we have shown that an incentive-compatible fixed-principal mechanism can be designed such that it is more efficient than an incentive-compatible fixed-interest-rate mechanism. Under plausible conditions, however, a bidding Rosca, which is a fixed-principal mechanism, cannot be designed such that it is more efficient than a fixed-interest-rate mechanism because the auction allocation mechanism limits the set of incentive-compatible fixed-principal mechanisms which can be implemented by bidding Roscas.

In some ways, the approach of this paper is related to that of Besley et al. (1995), where it is shown that neither a Rosca nor a credit market is efficient for financing a lumpy good when incomes are deterministic. Analogous to the findings of the present paper, Besley et al. find that a Rosca and a credit market generate constrained-efficient allocations when first-best mechanisms are not feasible. In their approach, however, the element of chance inherent in a random Rosca can generate allocations that are more ex-ante efficient than those generated by a credit market, while a bidding Rosca is always less efficient than a credit market. In contrast, we find that, when incomes are stochastic and Rosca funds are used for consumption, there is no room for random Roscas, whereas a network of bidding Roscas helps individuals to insure against income shocks, although less efficiently than a credit market.

Together with its companion, this paper formalizes a view on Roscas which has long been advocated in the informal literature and which is complementary to the lumpy-good story elaborated by several theoretical authors. In practice, bidding Roscas often appear to serve both purposes, risk-sharing as well as accumulation of funds for a lumpy good. Future research has to show whether and how the two complementary approaches can be reconciled and unified.

Appendix

Proof of Proposition 1

(i) According to (14) and (15), the equilibrium payoffs from a $\hat{\Lambda}^m_\infty$-mechanism to an individual with first-period income y can be written as $m\phi(y)$ in the first and $(2F(y) - 1)\,m$ in the second period. As derived in Section 5, the equilibrium payoffs from participation in a Rosca network to an individual with first-period income y are given by

$$\begin{aligned} &-m + F(y)\, b(y)/2 + (1 - F(y))\,(2\,m - E[b(Y)|Y > y]/2) \\ &= F(y)\, b(y)/2 + (1 - 2F(y))\, m - \frac{1}{2}\int_y^{y_u} b(\rho) f(\rho) d\rho, \end{aligned} \tag{29}$$

in the first and $(2F(y) - 1)\,m$ in the second period. Equating $m\phi(y)$ and the RHS of (29) gives the integral equation

$$m\phi(y) = F(y)\, b(y)/2 + (1 - 2F(y))\, m - \frac{1}{2}\int_y^{y_u} b(\rho) f(\rho) d\rho. \tag{30}$$

Differentiating (30) w.r.t. y gives

$$m\phi'(y) = F(y)\, b'(y)/2 + b(y)\, f(y) - 2f(y)\, m,$$

which is an ordinary first order differential equation in $b(y)$ with solution

$$\begin{aligned} b(y) &= 2\frac{m}{\left(F(y)\right)^2}\left(\int_{y_l}^{y}\left(\phi'(\rho) + f(\rho)\right)F(\rho)d\rho + c\right) \\ &= 2\frac{m}{\left(F(y)\right)^2}\left(\left(F(y)\right)^2 + \phi(y)F(y) - \int_{y_l}^{y}\phi(\rho)f(\rho)d\rho + c\right), \end{aligned} \tag{31}$$

where c is a constant. To determine c, we substitute (31) into (30) to obtain

$$m\phi(y_u) = - m + m\left(1+\phi(y_u)-\int_{y_l}^{y_u}\phi(\rho)f(\rho)d\rho+c\right).$$

Noting that, by virtue of (19), $\int_{y_l}^{y_u}\phi(\rho)f(\rho)d\rho = E[\phi(Y)] = 0$, c is found to equal zero. We thus obtain

$$(32)\quad b(y) = 2\frac{m}{\left(F(y)\right)^2}\left(\left(F(y)\right)^2+\phi(y)F(y)-\int_{y_l}^{y}\phi(\rho)f(\rho)d\rho\right),$$

which is the unique solution to (30). Using the fact that $\int_{y_l}^{y}\phi(\rho)f(\rho)d\rho$ $= E[\phi(Y)\,|\,Y<y]F(y)$, $b(y)$ as defined by (32) is immediately seen to equal $b^*(y)$ as defined by (20).

To see that $\Lambda_\infty^R(m)$ implements $\hat{\Lambda}_\infty^m(\phi(\cdot), m)$ if, and only if, $b^{*\prime}(y) < 0$ for almost all y, suppose that there exists a y which lies in the interior of I, and an $\varepsilon > 0$ such that $b(x_1) = b(x_2)$ for all $x_1, x_2 \in [y - \varepsilon, y + \varepsilon]$. Then all auctions where both bidders observed incomes lying in $[y - \varepsilon, y + \varepsilon]$, will end in a tie. Thus, in all such cases, the auction does not necessarily generate a transfer from the higher to the lower first period income, which contradicts the assumption that $\Lambda_\infty^R(m)$ is a risk-sharing mechanism in the sense of Definition 4 (i).

(ii) Taking the derivative of $b^{*\prime}(y)$ as defined by (32) gives

$$b^{*\prime}(y) = 2\frac{m}{\left(F(y)\right)^3}\left(\phi'(y)\left(F(y)\right)^2-2\phi(y)F(y)f(y)+2f(y)\int_{y_l}^{y}\phi(\rho)f(\rho)d\rho\right).$$

Integrating the integral term by parts and writing $(F(y))^2$ as $2\int_{y_l}^{y}F(\rho)f(\rho)d\rho$ gives

$$b^{*\prime}(y) = 2\frac{m}{\left(F(y)\right)^3}\left(2\phi'(y)\int_{y_l}^{y}F(\rho)f(\rho)d\rho-2f(y)\int_{y_l}^{y}\phi'(\rho)F(\rho)d\rho\right),$$

$$= 2\frac{m}{(F(y))^3}\int_{y_l}^{y} F(\rho)\left[f(\rho)\phi'(y) - f(y)\phi'(\rho)\right]d\rho \,.$$

Clearly, the sign of $b^{*\prime}(y)$ is only determined by the integral term which appears in (21).

Proof of Proposition 3:

For expositional convenience, define $\gamma(y,m) \equiv m\phi(y)$, where $\phi(y)$ is as defined by (23). Then

$$(33)\quad \frac{dE[U(\hat{\Lambda}_\infty^m)\,|\,y]}{dm}_{|m=m^*} = v'\big(y+\gamma(y,m^*)\big)\frac{\partial\gamma(y,m)}{\partial m}_{|m=m^*} + E\Big[v'\big(Y+(2F(y)-1)m^*\big)\Big](2F(y)-1)\,.$$

From (23) we further obtain

$$(34)\quad \frac{\partial\gamma(y,m)}{\partial m} = -1-\frac{1}{a}\left(\frac{\dfrac{2}{2m-1}v\big((2m-1)y\big)\left(\dfrac{1}{1-2m}-2amy\right)-\dfrac{d\chi(m)}{dm}}{\dfrac{2m}{2m-1}v\big((2m-1)y\big)-\chi(m)}\right),$$

where we have written χ as a function of m. $\chi(m)$ is implicitly defined by the equation

$$(35)\quad E[\gamma(Y,m)] = E\left[-m-\frac{1}{a}\left(Log\big[\delta E[v(Y)]\big]+Log\left[\frac{2m}{2m-1}v\big((2m-1)y\big)+\chi\right]\right)\right] = 0,$$

which, in turn, follows from (19), (23) and the definition of $\gamma(y,m)$. Evaluating the total derivative of (35) at $m = m^*$ and substituting the RHS of (24) for m^* gives

$$(36)\quad \frac{d\chi}{dm}_{|m=m^*} = -2a\frac{1+\kappa^*}{\kappa^*\big(1+v\big(\kappa^*/(1+\kappa^*)\big)\big)}\,.$$

Substituting (36) into (34) gives

$$(37)\quad \frac{\partial\gamma(y,m^*)}{\partial m} = -1-2\frac{\kappa^*+1}{\psi}+2y+2(\kappa^*+1)\frac{\exp(-\psi y)}{1-\exp(-\psi)},$$

where $\psi \equiv \dfrac{a\kappa^*}{1+\kappa^*}$. Substituting (37) into (33) and simplifying gives

$$(38)\quad \frac{dE[U(\hat{\Lambda}_\infty^m)\,|\,y]}{dm}_{|m=m^*} = (1+\kappa^*)\exp\left(-a\frac{y+(\kappa^*/2)}{1+\kappa^*}\right)\pi(y),$$

where $\pi(y) \equiv 2y - 1 - \dfrac{2}{\psi} + 2\dfrac{\exp(-\psi y)}{1-\exp(-\psi)}$.

Note that the term $(1+\kappa^*)\exp\left(-a\dfrac{y+(\kappa^*/2)}{1+\kappa^*}\right)$ is clearly positive and that $\pi(0) = \pi(1) = \left[\psi\left(\exp(\psi)-1\right)\right]^{-1}\left(2+\psi+\exp(\psi)(\psi-2)\right) > 0$ for all $\psi > 0$. This proves (27).

To prove (28), we define $y^* = \arg\min_y[\pi(y)] = -\left(\log\left[\left(1-\exp(-\psi)\right)/\psi\right]\right)/\psi$ which can be shown to lie in the interior of the unit interval when ψ is positive. Hence we obtain $\pi(y^*) = -1-\left(2\log\left[\left(1-\exp(-\psi)\right)/\psi\right]\right)/\psi$ which can be shown to be strictly negative for all $\psi > 0$. Applying these findings to (38), we obtain $\dfrac{dE[U(\hat{\Lambda}_\infty^m)\,|\,y^*]}{dm}_{|m=m^*} < 0$, which completes the proof of (28).

<u>Proof of Proposition 4:</u>

We obtain $\dfrac{dE[U(\hat{\Lambda}_\infty^m)]}{dm}_{|m=m^*} = \displaystyle\int_0^1 \frac{dE_Y[U(\hat{\Lambda}_\infty^m)\,|\,y]}{dm}_{|m=m^*}\,dy$ from (38). After simplifying we obtain

$$(39)\quad \frac{dE[U(\hat{\Lambda}_\infty^m)]}{dm}_{|m=m^*} = 2\left[\alpha(1-\rho)(1+\rho)^3\right]^{-1}\exp\left(-\alpha(1+\rho)/2\right)\operatorname{csch}\left(\alpha(1-\rho)\right)\,\psi(\rho),$$

where $\psi(\rho) \equiv (1-\rho^2)\left[\rho\sinh(\alpha)+\sinh(\alpha\rho)\right]+4\rho\left[\cosh(\alpha\rho)-\cosh(\alpha)\right]/\alpha$, $\rho \equiv \dfrac{1-\kappa^*}{1+\kappa^*}$ and $\alpha \equiv a/2$. Note that $-1 < \rho < 1$, $\rho = 0$ when $\kappa^* = 1$, and $\rho'(\kappa^*) < 0$. Obviously, when $-1 < \rho < 1$, the sign of the RHS of (39) depends only on $\psi(\rho)$, since all other terms are positive. The key properties of $\psi(\rho)$ are summarized in the following

Lemma 1: a) $\psi(0) = 0$ for all α,

b) $\psi'(0) > 0$ for all α,

c) $\psi(1) = \psi'(1) = \psi''(1) = 0$ and $\psi'''(1) < 0$ for all α.

Part a) of Lemma 1 proves Proposition 4 (i), b) proves (iii) and (iv), and c) proves (ii).

Proof of Lemma 1:

a) Evaluating $\psi(\rho)$ at $\rho = 0$ immediately yields the desired result.

b) $\psi'(0) = \left[\alpha \sinh(\alpha) + \alpha^2 - 4\left(\cosh(\alpha) - 1\right)\right] / \alpha$. To determine the sign of $\psi'(0)$, we develop the Taylor series of the term in square brackets at $\alpha = 0$. At zero, all derivatives of order smaller than six are found to equal zero, while, as is usual with hyperbolic functions, the higher order derivatives can be obtained by an inductive argument. We thus obtain $\psi'(0) = \dfrac{1}{\alpha}\sum\limits_{n=3}^{\infty}\left[(2n-4)\dfrac{\alpha^{2n}}{2n!}\right]$, which is clearly positive for all $\alpha > 0$.

c) $\psi'(1) = 2\left[\alpha \cosh(\alpha) - (\alpha^2 + 3)\sinh(\alpha)\right]$. As in b), we develop the Taylor series of the term in square brackets at $\alpha = 0$. Again, the derivatives can be obtained by induction. We thus obtain $\psi'(1) = -4\left(1 + 2\sum\limits_{n=1}^{\infty}\left[(n-1)^2\dfrac{\alpha^{2n-1}}{(2n-1)!}\right]\right)$, which is clearly negative for all $\alpha > 0$.

Proof of Proposition 5:

For the general case, from (20), we obtain

$$(40)\quad \frac{\partial b^*(y)}{\partial m}_{m=m^*} = \frac{2}{\left(F(y)\right)^2}\left(\left(F(y)\right)^2 + F(y)\frac{\partial\gamma(y,m^*)}{\partial m} - \int_{y_l}^{y}\frac{\partial\gamma(\rho,m^*)}{\partial m}f(\rho)d\rho\right),$$

where, as above $\gamma(y,m) \equiv m\phi(y)$. For the CARA-uniform case, plugging $\frac{\partial\gamma(y,m^*)}{\partial m}$ from (37) into (40) and solving the integral gives

$$(41)\quad \frac{\partial b^*(y)}{\partial m}_{|m=m^*} = 4\left(1 + \frac{(\kappa^*+1)\left[(\psi y+1)\exp(-\psi y)-1\right]}{\psi\left(1-\exp(-\psi)\right)y^2}\right).$$

Taking the derivative of (41) w.r.t. y gives

$$(42)\quad \frac{\partial b^{*\prime}(y)}{\partial m}_{|m=m^*} = 4\frac{(\kappa^*+1)\exp(-\psi y)}{\psi\left(1-\exp(-\psi)\right)y^3}\left[2\exp(\psi y)-(\psi y+1)^2-1\right].$$

Clearly, the sign of (42) depends only on the term in square brackets, which can be shown to be strictly positive for all $\psi, y > 0$.

References

Atkeson, A., Lucas, R. E. Jr., 1992. On efficient distribution with private information. Review of Economic Studies, 59, 427-453.

Besley, T., 1995. Nonmarket institutions for credit and risk sharing in low-income countries. Journal of Economic Perspectives, 9 (3), 115-127.

Besley, T., Coate, S., Loury, G., 1993. The economics of rotating savings and credit associations. American Economic Review, 83 (4), 792-810.

Besley, T., Coate, S., Loury, G., 1994. Rotating savings and credit associations, credit markets and efficiency. Review of Economic Studies, 61, 701-720.

Cheung, S., 1968. Private property rights and share-cropping. Journal of Political Economy, 76, 107-122.

Coate, S., Ravallion, M., 1993. Reciprocity without commitment: Characterization and performance of informal insurance arrangements. Journal of Development Economics, 40 (1), 1-24.

Eswaran, M., Kotwal, A, 1989. Credit as insurance in agrarian economies. Journal of Development Economics, 31 (1), 37-54.

Geertz, C., 1962. The rotating credit association: a middle rung in development. Economic Development and Cultural Change, 10 (2), 241-263.

Green, E., 1987. Lending and smoothing of uninsurable income, in: E. Prescott and N. Wallace (eds.), "Contractual arrangements for intertemporal trade". Minneapolis: University of Minnesota Press.

Klonner, S., 2000. Rotating savings and credit associations when participants are risk-averse. Manuscript, Heidelberg University.

Kovsted, J., Lyk-Jensen, P., 1999. Rotating savings and credit associations: the choice between random and bidding allocation of funds. Journal of Development Economics, 60 (1), 143-17.

Phelan, C., Townsend, R., 1990. Computing multi-period, information constrained optima. Review of Economic Studies, 58, 853-881.

Rogerson, W. P., 1985. The first-order approach to principal-agent problems. Econometrica, 53 (6), 1357-1367.

Udry, C., 1990. Rural credit in northern Nigeria: credit as insurance in a rural economy. World Bank Economic Review, 4, 251-269.

Udry, C., 1994. Risk and insurance in a rural credit market: an empirical investigation in northern Nigeria. Review of Economic Studies, 61, 495-526.

Wang, C., 1995. Dynamic insurance with private information and balanced budgets. Review of Economic Studies, 62, 577-595.