A letter by Leibniz to L'Hôpital on measurement. [originally *Leibnizens Mathematische Schriften*, (ed. C.I. Gerhardt) Berlin and Halle, vol. 2 (1849-1863):

Translated from the French by Peter McLaughlin

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5.2 Documents to Chapter 2: Conservation and Contrariety

5.2.1 Leibniz on Measurement by Congruence and Equipollence: From a Letter to de l'Hospital (Jan. 15, 1696)

I remain in agreement with you that a body acts through its mass and its speed; also it is only through these things that I determine the motive force. But it does not at all follow from this that forces are in a proportion composed of the masses and the speeds. Right cones are determined by the height and the base of the triangle that generates them, but they are not in a compound proportion of these two quantities. However, just as two of these cones are equal in size when the generating triangles have the same base and the same height, it is also true that two bodies are equal in force when their masses and their speeds are equal. From which I infer that, given a body AB [Fig. 5.11], having speed H, and a body BCD, double body AB, having speed M equal to speed H, the force of the double body BCD will be double that of the simple body AB when their speeds M and H are equal. For BCD, having two parts BC and CD, each equal to AB, and each part of BCD having its speed equal to that of the whole, that of BC, namely L, will be equal to M and consequently to H, and likewise that of CD, namely N, will be equal to M or to H. Thus, the case of BC with speed L is precisely congruent to the case of AB with speed H and consequently equipollent; likewise the case CD with speed N. Thus, the case BCD with the speed M contains precisely two times the case AB with speed H and consequently it also contains double its force; or a double body is double in force to a simple body of the same speed. This is only all too clear, you say, Sir. However, this is the foundation of my Dynamics, and even of all mathematical estimation [*estime*] and measurement – on condition that one adds here the single principle that the entire effect is equipollent to its cause. For it is the relation between the two that is being dealt with here, since the force is known by the action. And as measurement [*l'estime*] is made by the repetition of the measure, there are two [kinds of] repetitions, a formal repetition which I call *congruence*, when the same subject in which the force is located is repeated; the other [repetition is] virtual, which I call *equipollence*, when this formal repetition or congruence is not located in the subjects themselves which one compares but rather in their full causes or in their entire effects. But neither by the principle of congruence nor by that of equipollence can one demonstrate that the simple body DE with the double speed P is exactly double in force to the simple body AB with the simple speed H, or rather that the double body BCD with the simple speed M is equal in force to the simple body DE with the double

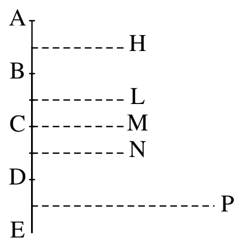

Fig. 5.11 (GM II, Fig. 59)

speed P. Congruence there is none at all, and equipollence shows the opposite, for, taking ED with [speed] P, it is true that the speed H is comprehended twice in P but the body AB is not comprehended twice in body DE. Thus there is no repeated congruence at all. And to say that the speed compensates virtually the body, taking the rectangle [i.e. product] of the mass and the speed as the measure of force is to take something not at all demonstrated and in fact just the contrary is demonstrated by the principle of equipollence. Thus, since the case of the two bodies of different speeds cannot be compared by simple congruence or the exact repetition of the same or a congruent [unit], it is necessary to have recourse to the equipollence of cause and effect; that is to say it is necessary to inquire whether there is not a means of producing by a body of double speed an effect which precisely repeats that of a body of simple speed. Now, this can be obtained in a number of ways. For example, if a body of simple speed can raise a pound one foot, a body of double speed can raise precisely four times one pound one foot, either by raising four pounds one foot or by raising one pound four feet; for each, the one and the other, is precisely the fourfold repetition of the elevation of one pound to one foot. So that (to say it in passing) the equality of the raising of one pound four feet and four pounds one foot is also demonstrated by the principle of congruence. This proves thus that a body of double speed has four times the force of a similar body with simple speed. And if body A [Fig. 5.12] with a simple speed AQ can tighten a spring Q (which it meets in its path) to a certain degree of tension – without being able to do more – the similar body E with a double speed ET would be able to tighten precisely to a similar degree four such springs T, S, R, Q. And what is more: a body of double speed can give the simple speed not only to two but to four bodies which are similar to it in size, as is easy to demonstrate. Thus (according to the principle of the equipollence of cause and effect) a body with a double speed is equipollent to four similar bodies with simple speeds; but (according to the principle of congruence) four equal bodies which have the simple speed have the fourfold force of a single one of them whose speed is simple; finally, a simple body of double speed has the fourfold force of a simple body with a simple speed.

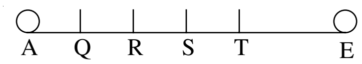

Fig. 5.12 (GM II, Fig. 60)

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