Analysing US Output and the Great Moderation
by Simultaneous Unobserved Components

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Abstract

This paper seeks to determine the causal interaction between structural trend and cycle innovations in an unobserved components framework of aggregate output. For the purpose of identification, I propose allowing for shifts in volatility. This strategy provides good estimation precision when applied to US industrial production. In the early 1980s, predominance of cycle shocks gives way to strong negative spillovers of trend impulses, consistent with real business cycle theories. The coincident reduction of macroeconomic volatility was mainly caused by pronounced dampening of transitory disturbances. This is in accordance with an important role of macroeconomic policy in explaining the Great Moderation.

Keywords: Unobserved Components, Trend, Cycle, Identification, Great Moderation

JEL classification: C32, E32

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1 Introduction

What is the nature of business cycles? Are they a phenomenon simply triggered by transitory shocks to the economy, or is it permanent innovations that drive both long-run growth of output as well as its temporary fluctuations? Reversely, might the course of the business cycle leave a persistent imprint on a country’s development? These issues have initiated important progress in theoretical research, including prominent concepts like real business cycles (RBC) and New Keynesian economics, amongst others.

On the empirical side, unobserved components (UC) models, which specify the latent growth and cycle paths directly, naturally bear the potential to answer the introductory questions. However, the first UC models built to decompose GDP into trend and cycle assumed uncorrelated innovations (e.g. Harvey 1985, Clark 1987), thereby neglecting potential interactions a priori. More recently, Balke and Wohar (2002) as well as Morley et al. (2003) showed that correlation between permanent trend and transitory cycle shocks can be taken into account while maintaining identifiability of the structural model form. Indeed, the latter authors found a large negative correlation in their application to US GDP, just as the former for real dividend growth.

Economically, the prevalent interpretation sees the correlation as a causal effect from trend to cycle in the sense of partial GDP adjustment: When a positive permanent shock shifts up the long-run output path, we will see a negative transitory component, which vanishes over time while realigning real output with the elevated production potential. This view is consistent with Stock and Watson (1988) as well as stylised RBC theories, see Kydland and Prescott (1982). The latter suggest that transitory fluctuations represent dynamic reactions of output to real shocks, delayed by time-to-build effects. A further theoretical interpretation stresses the role of nominal rigidities triggering negative initial impacts of positive supply or technology shocks (e.g. Blanchard and Quah 1989, Galí 1999). Even though these particular explanations for the estimated correlation might appear plausible, in terms of statistics no case can be made to exclude alternative ones, even comprising totally reversed causality. That is to say, spillovers of cycle shocks to the trend can produce an observationally equivalent outcome.

Different prominent approaches bear the potential to rationalise such a reversed mechanism: For instance, Okun (1962) argued that transitory recessions might leave their mark on permanent output, when the average age of the nation’s capital stock rises (i.e., the vintage effect). The same effect on GDP is likely to occur in case unemployment does not regress to its starting point after a temporary increase, so-called hysteresis (e.g. Blan-
chard and Summers 1986). Clark (1989) gives the example of a surge in investment improving short-run demand along with long-run capacity. However, the previous considerations direct at a positive linkage of trend and cycle disturbances. As Proietti (2006) notes, negative correlation would go in line with adverse effects of temporary shocks on the permanent GDP component. For example, Clark (1987) argues that initially positive demand effects of fiscal policy shocks may be followed by rising tax and interest rates, lowering production potential hand in hand with output. The same may hold true for inflationary, e.g. monetary, shocks, if they provoke increased uncertainty, dampened trade development or inefficient product and labour substitution under price staggering of Calvo or Taylor type. Moreover, labour market policy actions like increases in unemp-loyment compensation (or disability benefits, following Clark 1989) might trigger short-run consumption-based upturns, but discourge productive work in the long run.

This list is surely extendable. Of course, a highly aggregate UC model is not designed to discover potentially different responses to all different types of underlying shocks. Nonetheless, regarding the dominant or average impact, the different arguments call for answering the much discussed question whether the dynamics of output are governed by permanent or transitory shocks; see King et al. (1991) and many others. Theoretically, it is common to achieve identification by assuming long-run neutrality of cycle (e.g., monetary policy) shocks, in line with the above interpretation of causality running from trend to cycle. Empirically however, a decision between the two potential directions of causality critically hinges on the ability to identify two simultaneous effects from the data. To this end, the present paper introduces the so-called simultaneous unobserved components (simultUC) model. Precisely, I seek to overidentify correlated UC models in order to reveal statistical evidence discriminating between the offered economic interpretations. For that purpose, I first consider to enhance the set of available information by extending the dynamic specification of the cyclical component and the reduced-form model version. However, since this strategy turns out to violate the sufficient rank condition for identification, a second innovative model is put forward exploiting the shift in shock variances going along with the so-called Great Moderation: As has been described by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), amongst others, the early 1980s saw a distinct decrease of variability in major macroeconomic indicators.

I show that implementing an additional variance regime allows identification from the output series alone of a random-walk trend, an autoregressive cycle, two according innovations as well as simultaneous cross-impacts between them. Furthermore, changes in the trend-cycle composition of output and even the structural origins of the Great
Moderation can be assessed. Empirical evidence is twofold: On the one hand, structural transitory disturbances are relatively dominant in the first post-war decades, possibly hinting at important policy influences on economic activity. This is in notable contrast to the above-mentioned mainstream interpretation given to the negative correlation phenomenon. However, structural permanent shocks survive as the only relevant source of both macroeconomic growth and fluctuations since the early 1980s. I.e., the correlation of reduced-form residuals can be traced back to spillovers of trend innovations to the cycle. This supports approaches ascribing a leading role to real shocks, such as RBC theories.

Concerning the discussion on the causes of the Great Moderation, the current results are consistent with a sizeable influence of changes in macroeconomic policies that might be associated to the tremendous reduction in genuine cyclical variability.

The reader can expect the following: Subsequently, the simultUC model is discussed along with key considerations on identification. Section 3 then presents the application to US industrial production (IP). The last section summarises and discusses the results and sets out implications for further research. Two appendices cover identification and estimation issues, respectively.

2 Model Specification and Identification

The classical UC model is built on the idea that (seasonally adjusted) log output $y_t$ can be represented as the sum of a stochastic trend $\tau_t$ and transitory deviations $c_t$, called the cycle. Formally, this is

$$y_t = \tau_t + c_t \quad (1)$$

$$\tau_t = \tau_{t-1} + \mu + \eta_t \quad , \quad \eta_t \sim N(0, \sigma_\eta^2) \quad (2)$$

$$c_t = b_1 c_{t-1} + \ldots + b_p c_{t-p} + \varepsilon_t \quad , \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (3)$$

where in modulus, all roots of the lag polynomial $B(L) = 1 - \sum_{i=1}^p b_i L^i$ lie outside the unit circle. Thus, the cycle is described by a stationary autoregressive process of order $p$, AR($p$). Periodic behaviour would only result for complex roots of $B(L)$, but in any case, I will stick to the name "cycle" for the transitory part of the output fluctuations. The trend component follows a random walk with a drift term $\mu$ that captures the steady-state growth rate of the economy. As explained later on in section 3.4, more sophisticated specifications for the drift did not prove essential for the underlying analysis.
While the original contributions assumed zero covariance between the permanent and transitory innovations, Balke and Wohar (2002) and Morley et al. (2003) relaxed this constraint. The latter specified $E(\eta_t \varepsilon_t) = r\sigma_\eta \sigma_\varepsilon$, with $r$ being the contemporaneous correlation. Decisively, $r$ becomes identifiable by setting the AR order $p = 2$, so that the structural UC model translates into a reduced-form autoregressive integrated moving average – ARIMA(2,1,2) – by virtue of Granger’s lemma (Granger and Newbold 1977). More precisely, substituting (2) and (3) into (1) leads to

$$B(L)\Delta y_t = B(1)\mu + B(L)\eta_t + \Delta \varepsilon_t .$$

(4)

The ARIMA representation in conventional form and notation is obtained as

$$B(L)\Delta y_t = c + A(L)u_t , \quad u_t \sim N(0, \sigma_u^2) ,$$

(5)

where $A(L)$ is a $p$-dimensional lag polynomial. Its coefficients $a_i, \ i = 1, \ldots, p$, are in general determined along with the variance $\sigma_u^2$ by matching autocovariances between the MA parts in (4) and (5). Evidently, the $b_i, \ i = 1, \ldots, p$, from the cycle equation (3) are directly identified by the autoregressive parameters in the reduced-form ARIMA process. Then, the drift term $\mu$ can be easily recovered from the constant $c$. Furthermore, the right-hand-side MA part delivers $p + 1$ non-zero autocovariances $\gamma(0), \ldots, \gamma(p)$, which are theoretically given as $\gamma(j) = E[(B(L)\eta_t + \Delta \varepsilon_t)(B(L)\eta_{t-j} + \Delta \varepsilon_{t-j})]$. For $p = 2$, the MA structure thus provides sufficient information to exactly identify three unknown parameters given as the correlation $r$ in addition to the variances $\sigma_\eta^2$ and $\sigma_\varepsilon^2$.

As has been set out in the Introduction, the present paper aims at incorporating further structure into the model in order to represent the causal mechanisms underlying the correlation of residuals. Thus, I split up the trend and cycle shocks from (2) and (3) according to the linear combinations

$$\eta_t = k_{11}\tilde{\eta}_t + k_{12}\tilde{\varepsilon}_t ,$$

(6)

$$\varepsilon_t = k_{21}\tilde{\eta}_t + k_{22}\tilde{\varepsilon}_t .$$

(7)

This simultaneous system is normalised by $E(\tilde{\eta}_t^2) = 1$ and $E(\tilde{\varepsilon}_t^2) = 1$ as well as $k_{11} \geq 0$ and $k_{22} \geq 0$. $\tilde{\eta}_t$ and $\tilde{\varepsilon}_t$ denote structural uncorrelated trend and cycle shocks, respectively. Therefore, $k_{12}$ and $k_{21}$ pick up the mutual spillover effects between both unobserved
components. In the following, I will refer to \( \eta_t \) and \( \varepsilon_t \) as the composite and to \( \tilde{\eta}_t \) and \( \tilde{\varepsilon}_t \) as the structural shocks.

Note that the equation system (6), (7) replaces the three parameters \( \sigma_{\eta}^2 \), \( \sigma_{\varepsilon}^2 \) and \( r \) by the four \( k_{ij} \), \( i, j = 1, 2 \). Naturally, the fully simultaneous UC specification with \( p = 2 \) then lacks one piece of information for identification. A straightforward solution to this problem seems to work through raising the AR order \( p \) of the cycle. That is, \( p = 3 \) implies an ARIMA(3,1,3) structure for the reduced form, which delivers one additional AR coefficient and a third non-zero autocovariance from the MA part (if \( p = 3 \) is justified by the data). Thereby, one seems to gain the required piece of information, because the number of unknowns in the structural form rises only by one (\( b_3 \)). However, this represents only a necessary condition for identification. As Appendix A explains, the AR(3) model does not fulfill the sufficient rank condition. I.e., the unknowns cannot be uniquely determined from the MA autocovariances. Therefore, I turn to an innovative and powerful solution to the fundamental identification problem in the following.

In essence, the task is to enlarge the set of information obtainable from the reduced form while extending the set of unknowns by as little as passable. For that purpose, imagine two regimes for the generating processes of the structural simultUC shocks \( \tilde{\eta}_t \) and \( \tilde{\varepsilon}_t \), one of high and one of low volatility. One retains the variance normalisations say for the first regime, that is \( \sigma_{\eta}^2 = \mathbb{E}(\tilde{\eta}_t^2 | t \in R_1) = 1 \) and \( \sigma_{\varepsilon}^2 = \mathbb{E}(\tilde{\varepsilon}_t^2 | t \in R_1) = 1 \), where \( R_i \) denotes the set of time points belonging to the \( i \)th regime. Accordingly, the variances for the second regime \( \sigma_{\eta}^2 = \mathbb{E}(\tilde{\eta}_t^2 | t \in R_2) \) and \( \sigma_{\varepsilon}^2 = \mathbb{E}(\tilde{\varepsilon}_t^2 | t \in R_2) \) are free parameters differing from unity in case breaks indeed occur.

Clearly, this specification introduces two additional unknown variance coefficients (\( \sigma_{\eta}^2 \) and \( \sigma_{\varepsilon}^2 \)) into the structural model. However, for the second variance regime, a completely new set of autocovariances from the reduced-form MA part can be calculated, providing \( p + 1 \) additional determining equations. It follows that for \( p \geq 2 \), the necessary summing-up condition for identifying the four \( k_{ij} \) in addition to \( \sigma_{\eta}^2 \) and \( \sigma_{\varepsilon}^2 \) is fulfilled. In detail, the number of unknowns (constant, AR parameters, shock loadings, variances) is given by \( 1 + p + 4 + 2(s - 1) \), where \( s \) is the number of regimes. The number of pieces of information from the reduced form (constant, AR parameters, MA autocovariances) amounts to \( 1 + p + s(1 + p) \). Comparing these terms, identification requires \( s(p - 1) \geq 2 \), what is met by AR orders of at least two in the presence of multiple regimes (and of at least three with

\footnote{“Completely new” holds as long as the variances do not break proportionally to each other. Otherwise, the new set of autocovariances would linearly depend on the existing one, delivering no additional identifying information. Again, see Appendix A for details.}
a single regime, however violating the sufficient condition, as discussed above).

Contrasting (4) with (5) reveals that heteroscedasticity of the structural shocks implies for the reduced form a break in the MA parameters $a_i$ and the residual variance $\sigma^2_u$, but not in the constant $c$ and the AR coefficients $b_i$. Importantly, this break represents a necessary identification condition that can (and will) be tested in an ARIMA model. Furthermore, shifts in the cycle propagation coefficients can be statistically tested since they would be directly identified from the polynomial $B(L)$ in (5). Breaks in the two contemporaneous spillover coefficients $k_{ij}, i \neq j$, would be more difficult to take into account. I.e., they could at best be identified if the inequality in $s(p - 1) \geq 2$ holds. Since we have seen that a third AR lag does not deliver identifying power, more than two regimes would be required, compare Rigobon (2003). In section 3.4 I will deliver some evidence on parameter stability based on a model with three variance regimes.

The idea of attaining identification by non-constant variances goes back as far as Wright (1928) and is comparable to Sentana and Fiorentini (2001) and Rigobon (2003), who treat factor models and simultaneous systems. However, while these authors rely on the contemporaneous residual covariance matrix as source of identifying information, the present approach involves the whole autocorrelation structure of the data. Furthermore, it identifies simultaneous impacts between unobserved components (i.e., simultUC) determined from a single observed series. In contrast, the existing approaches employ the conventional setup of left-hand-side observed variables depending on latent factors right hand side.

For estimation purposes, the structural model is cast in state-space form, see Appendix B. Maximum Likelihood is applied to estimate the model parameters. Thereby, the likelihood function is constructed using the prediction error decomposition from the Kalman filter, which delivers estimates of the states of the unobserved components.

3 Application to US Output

3.1 Data

Previous unobserved components studies have routinely employed quarterly US GDP. While applying the newly developed methodology to this series is feasible, I encountered no sufficient significance for effectively discriminating between competing theoretical explanations. Particularly, estimation uncertainty around the spillover coefficients $k_{12}$ and
proved to be too large. Instead, I use IP, what considerably raises the number of observations given that monthly data are available.\(^3\) Two well-known points should be addressed: First, IP development can differ quite substantially from GDP due to the limited share of industrial sectors. However, IP is quite common as an output measure in the macroeconomic literature. In particular, it usually counts among the more coincident series with the business cycle, what is underlined by its role in the NBER dating procedure. The mere point estimates in my model lead to similar conclusions for GDP and IP. That is, it is solely the precision of estimates that was improved by raising the frequency. Second, augmenting the number of observations through higher data frequency often does not provide the same quality of additional information as collecting time series of higher overall length. This argument holds especially when long-run variation is concerned. Nonetheless, the current focus lies additionally on volatility and short-/medium-run dynamics, where inference is more likely to benefit from higher frequency. Accordingly, given that starting the sample earlier than 1947 is not advisable, I choose IP as a second-best solution. In the present case, the gains in significance will be sufficient for clear-cut economic interpretation based on statistical evidence.

The monthly seasonally adjusted IP index of the United States for the sample 1947:1-2008:12 is obtained from the Federal Reserve. Slight changes in the start and end points would be uncritical, see section 3.4. Log IP (multiplied by 100) and its first differences are plotted in Figure 1.

\[\text{Figure 1: Log real US IP (×100) and first differences}\]

\(^3\)Quarterly GDP could be included along with monthly IP in a mixed-frequency state space setup. I leave this issue for future research, focusing the interest of the current paper on the fundamental identification problem in the case of a single observed variable.
3.2 Preliminary Steps

In a first step, I estimate a correlated UC model similar to Morley et al. (2003). For determining the lag length $p$ of the cycle, ARIMA($p,1,p$) models as in (5) are specified for the log IP series. Since IP is observed monthly, I set a large maximum lag length of $p = 12$. The Schwarz criterion (SC) prefers $p = 3$, clearly discarding higher lags. Unsurprisingly, the Akaike information criterion (AIC) chooses a higher lag length of $p = 10$. While I rely on the consistent SC estimate in the following, section 3.4 argues that AIC lags do not decisively alter results for the simultaneous model part. All coefficients (except the MA(1) parameter) are significant$^4$, and the residual autocorrelations are rather low. Therefore, it seems justified to let the unobserved cyclical component follow an AR(3) process.

The correlation $r$ is found to be close to $-1$. Statistical significance can be conveniently assessed by means of confidence sets based on inverted likelihood ratio (LR) statistics: In detail, LR tests reject the null hypothesis $r = r_0$ for all $r_0 \geq -0.59$ (i.e., closer to zero than $-0.59$ or positive) on the 5% level. Morley et al. (2003), using quarterly GDP data until 1998:2, nearly failed to reject even a zero correlation. Evidently, the additional ten years in my sample and the monthly frequency provide essential information for precise estimation. This fact shall be of avail for the analysis of the even more demanding simultUC model below.

The large negative estimate for the correlation leads Morley et al. (2003) to the interpretation that positive real trend shocks leave a lower transitory component, which gradually adjusts to the permanent output path with a lag. Balke and Wohar (2002) propose a similar explanation with regard to their real dividend growth model. I now head to reassess this assertion by identifying the causal structure underlying the inferred residual correlation. Particularly, this correlation could be either generated by two shocks with strong mutual spillovers or by a single relevant shock affecting trend and cycle alike. Here, one obviously faces a fundamental identification problem.

$^4$Admittedly, these coefficients might be subject to near cancellation, causing bias in t-tests (e.g. Nelson and Startz 2007). Indeed, the largest AR and MA roots of the ARIMA(3,1,3) model almost lie on the unit circle. However, for the moment I continue with the decision of the information criteria, which should be fairly reliable.
3.3 Identification and the Great Moderation

To begin with, reconsider the IP growth rates in Figure 1. The early 1980s witnessed a striking reduction in the volatility of macroeconomic fluctuations. This phenomenon, mainly for GDP, inflation and other aggregate series, found its way into the literature as the Great Moderation (see Kim and Nelson 1999, McConnell and Perez-Quiros 2000). Concerning its reasons, the debate goes on "good policies" (e.g. Clarida et al. 2000, Bernanke 2004) respectively changes in the structure of the economy (e.g. Dynan et al. 2006), versus "good luck", meaning a simple reduction in the size of shocks hitting the economy (e.g. Stock and Watson 2003).

Of course, I do not claim to be able to decide this discussion based on inference on a single time series, i.e. IP. What one can do on this study’s high level of abstraction is determining the contributions of the two types of shocks prior and subsequent to the breakpoint. Then, if the origin of the Great Moderation lies in better policies, and if one is willing to accept that policy shocks exert impacts on the real economy mainly on business cycle frequencies (e.g. Ahmed et al. 2004), the policy argument might be roughly associated with the cycle innovation in the present framework (e.g., "non-technology" shocks in Gali and Gambetti 2009). Accordingly, the trend disturbance is more prone to represent structural growth (or "technology") shocks not under the control of single political institutions. In so far, identifying the simultUC structure provides the means for discriminating between competing explanations of the Great Moderation. Of course, this comes in addition to the potential of assessing strength and nature of the trend-cycle interaction and the consequences for output dynamics.

Technically, I introduce shift dummies for the variances of both innovations, letting the data decide about the respective contributions. This shift in variability provides the statistical information required for identifying the simultaneous structure. As for the exact date of the change in regimes, I pick February 1984 based on visual inspection of the growth rates in Figure 1; that is, the last in a row of pronounced spikes might

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5Given the available sample, it is too early to assess the high fluctuations in the 2007/2008 financial and economic crisis. Notwithstanding, cutting the last few observations does not change the outcome of this paper. Moreover, it follows from Rigobon (2003) that underspecifying the number of regimes would not compromise identification, as long as there remains (disproportionate) heteroscedasticity. The same applies to a further break that might be presumed in the early 1960s; see section 3.4.

6Additionally, changes in the cycle propagation coefficients have already been excluded as sources of dampening IP volatility.

7Stochastic variance changes would be an extension of the current approach, even if the additional source of uncertainty might lower the sharpness of the estimation.
have immediately occurred in the preceding month. Indeed, for GDP, the literature (as cited above) has often identified the first quarter of 1984. However, one or two years more or less have been checked not make a decisive difference, neither to the parameter estimates nor to the likelihood. This is in line with Rigobon (2003), who states that slight misspecification of the volatility regimes does not impair identification and consistency.

Furthermore, recall that the break in the structural variances implies a change in the reduced-form MA dynamics and the residual variance in (5). In the ARIMA model for IP, the null of no break in these parameters is easily rejected by an LR test. The same holds true when only the MA coefficients (excluding the variance) are tested. In contrast, in an ARIMA model allowing for shifts in all parameters, the null of no break in the AR part cannot be rejected. Such a break would be present if the cycle coefficients $b_i$ were not stable.\footnote{Indeed, breaks in the $b_i$ are insignificant when estimating an according UC model.} Logically, this result is consistent with Ahmed et al. (2004) and Kim et al. (2008), who do not find important contributions of changes in propagation mechanisms to the Great Moderation. In sum, we have both good empirical and theoretical reasons assuring us that the current heteroscedastic specification is appropriate for the data and yields valid estimates.

The trend and cycle equations, including the second-regime variances are estimated as shown below. The third AR coefficient has been eliminated due to insignificance. Standard errors from the inverse Hessian are given in parentheses. However, due to potential distortion of Wald tests (e.g. Nelson and Startz 2007), I will additionally provide LR tests for the relevant coefficients.

\[
\tau_t = \tau_{t-1} + 0.201 + 1.430\hat{\eta}_t - 1.725\hat{\varepsilon}_t \\
(0.057) (0.199) (0.421)
\]

\[
c_t = 1.094c_{t-1} - 0.262c_{t-2} + 0.470\hat{\eta}_t + 2.122\hat{\varepsilon}_t \\
(0.168) (0.130) (0.252) (0.444)
\]

\[
\sigma^2_{\eta^2} = 0.589 \\
(0.140)
\]

\[
\sigma^2_{\varepsilon^2} = 0.039 \\
(0.091)
\]

The new identification strategy shows its merits in a tremendous reduction of the standard errors, leading to highly significant impact coefficients. In LR tests, both hypotheses $k_{12} = 0$ and $k_{21} = 0$ are clearly rejected on the 1% level. In the first regime, where both variances are normalised to 1, the simultUC system is strongly influenced by the structural cycle shock $\hat{\varepsilon}_t$. It hits the cyclical component three times stronger than the structural trend innovation does ($k_{22} = 2.122$), and even prevails in its effect on the trend
component itself \((k_{12} = -1.725)\). The latter can be explained by economic mechanisms as discussed in the Introduction, e.g. fiscal or monetary policy. The conventional view of real persistent shocks driving business cycle dynamics is reflected in the coefficient \(k_{21} = -0.740\). Nonetheless, until the Great Moderation, it is not this effect that mainly explains the negative UC correlation. This correlation of the composite residuals \(\eta_t\) and \(\varepsilon_t\), see (6) and (7), can be obtained from the covariance matrix (19) in Appendix B. As shown in Table 1, it takes a value that is common for GDP, e.g. as given in Morley et al. (2003). The variances of the composite disturbances are nearly identical in the first regime.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Var(Trend)</th>
<th>Var(Cycle)</th>
<th>Cor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.02</td>
<td>5.05</td>
<td>-93.7%</td>
</tr>
<tr>
<td>2</td>
<td>1.32</td>
<td>0.50</td>
<td>-94.5%</td>
</tr>
</tbody>
</table>

Table 1: Results for the composite residuals

The situation changes in the second regime: Here, variability of the structural cycle disturbance nearly vanishes, while it is reduced by only 41% for \(\tilde{\eta}_t\).\(^{10}\) The LR test statistic of \(H_0: \sigma_{z2}^2 = 0\) amounts to 0.089. Usual significance levels are unlikely to hold in this case, since under the null hypothesis, the variance is on the boundary of the admissible parameter space. Nevertheless, statistical (and economic) insignificance is too clear to be revised by any modification of the inference procedure. The correlation between the composite error terms increased very slightly in absolute value. Sinclair (2010), who finds an even larger increase in a correlated UC model for GDP, hypothesises that this might be explained either by more important adjustments of the transitory component to permanent movements or by less important "other transitory shocks". Now, the simultUC model shows that the latter point is most relevant due to the drop in the variance of \(\tilde{\eta}_t\). In particular, the strongly negative correlation can be traced back to transmission of structural trend shocks to the cyclical component, as for instance time-to-build effects imply. The phenomenon of the Great Moderation can thus be explained by (nearly) complete disappearance of genuine cyclical volatility, complemented by a much less important reduction of the size of trend innovations. This is in line with Gali and Gambetti (2009), who establish a sharp fall in the contribution of "non-technology" shocks to output volatility. A contrary view in a still unsettled debate can be found in Kim et al. (2008).

\(^{9}\)Note that since in the first regime both variances are normalised to one, variance decompositions yield identical conclusions.

\(^{10}\)These values provide the disproportionate break required for identification. This can be confirmed in the correlated UC model, see Appendix A.
The relation of standard deviations of the composite disturbances \( \eta_t \) and \( \varepsilon_t \) was near one in the first regime. After the break it rises to 1.63, close to the value found by Morley et al. (2003) for GDP in their full sample. The important point to notice is that the tremendous reduction of \( \sigma_{z2}^2 \) concerns the genuine (structural) cycle shock \( \tilde{\varepsilon}_t \). Of course, the cycle itself becomes clearly less volatile, too, but the relation 1.63 is moderate. This is explained by the facts that first, the reduction of \( \sigma_{z2}^2 \) similarly reduced its contribution to the overall trend variance, and second, the cycle continues to be driven by spillovers of the structural trend innovation (e.g. time-to-build effects). Thereby, the first point clarifies that the dampening of structural cycle shocks reduced volatility of both composite disturbances. Here, one might see an interesting parallel to Clarida et al. (2000), who argue that monetary policy contributed to the Great Moderation by adopting a policy rule that eliminates ”sunspot” equilibria, i.e. prevents shifts in expectations unrelated to fundamentals from influencing the economy. The effect would be a general fall in the aggregate volatility of shocks. In the current setup, the decline of structural cyclical volatility would mirror the underlying cause.

The latter point is of particular importance when comparing the current model outcome to further results from the existing literature. For example, Ahmed et al. (2004) cannot reject the hypothesis that the reduction in volatility of US GDP is proportional across all frequencies. In (10), the simultUC model shows that the structural cycle shock nearly loses all of its importance as a source of fluctuations. Notwithstanding, the cyclical component (i.e., the composite cycle shock) survives. As a matter of fact, it is the importance of the composite innovations that roughly compares to the importance of the different frequencies in output. It is right that the cycle component loses more of its volatility than the trend component (recall the relation of standard deviations of 1:1.63), but this is much less in contradiction to the empirical results from Ahmed et al. (2004) than the structural decomposition developed in the present paper might have suggested at first glance.

Finally, to gain a graphical impression, Figure 2 plots the filtered unobserved components. As it has usually been discovered in Beveridge-Nelson-type decompositions and correlated UC models, the trend is relatively variable and the cycle quite noisy.\(^{11}\) Since the lag polynomial in (9) has no complex roots, one cannot observe any pronounced periodicity. Nevertheless, section 3.4 shows that the AR(10) specification (as preferred by AIC)

\(^{11}\)In this context, note that the mere identification of the sources of the UC correlation does not change the trend and cycle components themselves. It is not the aim of this paper to provide a decomposition that is different from correlated UC or Beveridge-Nelson. Of course, what has a certain impact on the components is the break in the variances.
produces a more "intuitive" cycle measure while leaving the outcome for the spillovers unaffected. At the time of the Great Moderation, the amplitude of the cyclical UC clearly drops. However, as explained above, this reduction in composite cycle volatility is far less extreme than the decline in $\sigma_{\epsilon_2}^2$. In a word, business cycles do not disappear — but the main source has changed.

![Figure 2: Filtered IP trend and cycle](image)

### 3.4 Model Extensions

This section discusses some model extensions and robustness issues. The tests support the reported empirical results, notably including the identified causalities.

**Trend Specification**

The deterministic drift term $\mu$ was allowed to follow a random walk. However, I encountered no relevant differences to the current investigation. This is in line with Oh and Zivot (2006), who showed that the "double-drift" specification yields results similar to Morley et al. (2003).

Furthermore, I examined the role of a trend break as proposed by Perron and Wada (2009). While these authors proposed 1973 as the break date for GDP, Figure 1 suggests that reasonable candidates might be found in the 1970s and early 1980s. Therefore, I conducted an endogenous break date search over the twelve years following 1973 by determining the specification with the largest likelihood. The according break in 1973:3 improved the likelihood of the no-break model by not more than 1.02. This is insignificant in a usual LR test, even if estimation uncertainty concerning the break point is taken into account. The parameter estimates are very close to equations (8)-(10).
Lippi and Reichlin (1994) specify the permanent output component as an ARIMA process reflecting the diffusion of technological change. Ma and Wohar (2010) add AR dynamics to the trend in a correlated UC model taking into account adjustment of equilibrium output to its steady-state level. The coefficients from the AR($q$) are identified since they enter the AR lag polynomial in the reduced-form ARIMA($q+p$, 1, $max(p, q+1)$) process. Evidently, the case where this model can be superior to the random-walk trend / ARIMA($p$, 1, $p$) is given when the AR order is larger than the MA order. For example, Ma and Wohar (2010) choose $q = p = 2$, what results in ARIMA(4,1,3) in a univariate setting. I checked such flexible AR orders within my sample, but SC still prefers ARIMA(3,1,3) and AIC, as discussed below, ARIMA(10,1,10). Additionally, I extended the simultUC model to take into account higher lags in the trend equation. However, an AR($q$) part in the trend made no significant contribution to IP dynamics for various $q$. The autoregressive cycle dynamics were largely unchanged.

**Cycle Specification**

While in section 3.2, the SC had preferred an ARIMA(3,1,3) process for IP, the third lag in the cycle was omitted in section 3.3 due to insignificance. Keeping it in the model did not change the conclusions.

Likewise, AIC chooses a lag length of $p = 10$. Naturally, more lags leave less volatility to the shocks, implying lower estimates for the $k_{ij}$. However, the relations of these coefficients hardly change. It follows that none of the structural interpretations is affected by the choice of the lag length. Moreover, the richer cycle specification absorbs a larger share of the IP variation, as shown in Figure 3 together with NBER recession periods.

The cycle follows NBER recessions quite accurately. Only in 1991 and 2001 the trough lags behind the turning point (in 2008 the cycle just follows the erratic IP fluctuations that can as well be found in Figure 1). It is important to see that Figure 2 does correctly pick up NBER recessions, too; but here, they are mostly allocated to the trend (see as well Morley at al. 2003). In sum, results concerning the simultaneous structure do not depend on whether the transitory component is measured as a relatively noisy component or as a more "intuitive" cycle.

**Three Regimes**

Figure 1 might suggest the presence of a second volatility break in the early 1960s. This issue is examined for two reasons: First, I seek to collect further evidence of robustness of the previous results. Second, a third regime would deliver additional identifying in-
formation in line with the discussion in section 2. This can then be exploited for testing stability of the spillover coefficients.

A potential break in the early 1960s is not as well discussed in the literature as the Great Moderation. Therefore, the additional break date is determined endogenously. The best likelihood within six years from 1959 onwards is found for 1960:1 and is clearly higher than in the two-regime model. The simultUC equations are estimated as follows:

\[
\begin{align*}
\tau_t &= \tau_{t-1} + 0.210 + 1.911 \tilde{\eta}_t - 2.618 \tilde{\xi}_t \\
(0.059) & \quad (0.407) \quad (0.837) \\
\end{align*}
\]

\[
\begin{align*}
c_t &= 1.136 c_{t-1} - 0.277 c_{t-2} - 1.095 \tilde{\eta}_t + 3.178 \tilde{\xi}_t \\
(0.155) & \quad (0.123) \quad (0.508) \quad (0.798) \\
\end{align*}
\]

\[
\begin{align*}
\sigma_{\eta 2}^2 &= 0.427 & \sigma_{e 2}^2 &= 0.237 \\
(0.107) & \quad (0.064) \\
\sigma_{\eta 3}^2 &= 0.367 & \sigma_{e 3}^2 &= 0.032 \\
(0.100) & \quad (0.039) \\
\end{align*}
\]

The relations of the shock loadings are largely unaffected by the additional regime. This mirrors Rigobon’s (2003) conclusion that specifying too few regimes does not lead to inconsistency as long as heteroscedasticity remains. As expected, the variances are first reduced in 1960 and a second time in the Great Moderation. Thereby, the latter mainly affects structural cycle volatility, what is in line with previous discussion in the present paper.
The additional shift delivers three additional determining equations from the covariance matrix but only two additional unknowns. Logically, the model is overidentified with one degree of freedom. This makes it possible to test for breaks in the propagation mechanism in addition to breaks in the variances of the shocks. When allowing for shifts in $k_{12}$ or $k_{21}$ at the beginning of the second or third regime, LR statistics lie below one. This supports the hypothesis of parameter stability.

**Further Robustness Checks**

Jarque-Bera tests for the ARIMA residuals indicated deviations from normality both in the full sample and within the regimes. Quasi Maximum Likelihood was adopted in order to make statistical inference robust. Naturally, this increases the standard errors, but all hitherto significant coefficients remained significant. Of course, the already preferred LR tests are not affected.

The sample could be shortened without notable effects either at the beginning to leave out the Korean War, or at the end to cut off the observations affected by the subprime crisis.

4 **Summary and Discussion**

The underlying paper has presented the novel simultUC approach to assess the importance of structural trend and cycle shocks in a macroeconomy. Building on the conventional UC model and its correlated extension, the focus was on determining causal structure in the interaction of permanent and transitory output components. This task makes high demand on the extraction of identifying information from the data. An effective strategy was developed by specifying a shift in the volatility of the structural disturbances. By providing more statistical information than introducing additional unknowns, such a type of heteroscedasticity bears the potential to identify simultaneity among unobserved components.

The application to US IP revealed strikingly different patterns in the periods prior and subsequent to the Great Moderation. The first post-war decades were characterised by strong structural cycle shocks driving the transitory component and even leaving a sustained mark on the long-run growth path. As for the first regime, this stands in contrast to the popular view of prevailing real or permanent shocks. Recurring to the Introduction, one might instead locate an important source of both transitory and persistent output
variability in the conduct of interventionist and discretionary monetary, fiscal and labour market policy, or "sunspot" equilibria (Clarida et al. 2000), which might have characterised the decades until the 1980s.

Coinciding with the Great Moderation, the cyclical influence has disappeared, leaving a permanent-transitory composition of IP largely governed by structural trend impulses. Logically, in the second regime the spillovers underlying the residual correlation found in UC models mainly originate from the trend innovations. The Great Moderation phenomenon seems to be a product of sustained reduction of the size of structural cycle shocks. In other words, until the early 1980s, the cycle was predominantly triggered by transitory shocks with independent variation. Since then, however, it represents temporary adjustments of actual IP to the development of the production potential. The latter fact is in line with RBC-type theories emphasising the role of real innovations in driving business cycles in addition to long-run economic growth.

Furthermore, the results are compatible with the "good policies" position in the discussion on the origins of the Great Moderation. So, it might appear plausible that the reduction of excessive policy interventions, most likely in place until the 1980s, at least in part stands behind the decline of the size of transitory shocks. Nevertheless, it is clear that policy impacts cannot be uniquely identified for instance from other demand shocks within the underlying highly parsimonious framework (i.e., it cannot be ruled out that the dampening of structural cycle shocks was based on "luck"). In sum, the statistical evidence should not be unduly stressed, since inference on a single time series cannot be expected to deliver imperatively compelling arguments on a topic as complex as the functioning of a whole macroeconomy. Including policy variables in an augmented model and explicitly identifying further shocks might dissolve some of the remaining ambiguity. Moreover, it should not be overlooked that the general volatility decrease also includes the structural trend disturbances, to a much lesser extent though. As far as those can be associated to fundamental growth shocks exogenous to macroeconomic policy, the "good luck" hypothesis additionally helps in explaining the Great Moderation.

There is considerable potential for future research drawing on this paper’s accomplishments. Particularly, it seems promising to extend multivariate approaches as in Cochrane (1994), Morley (2007), Basistha (2009) or Sinclair (2009). Based on the current methodology, further time series containing valuable information on macroeconomic trends and cycles should be employed. Those might put the permanent-transitory identification on an empirically even more firm footing, allow more precise economic interpretation and facilitate comparison to other multivariate (e.g., VAR) approaches. By the same token, richer
causal structures implied by economic theory might be assessed econometrically. Finally, interesting studies may gauge how both the simultaneous setup and heteroscedasticity relate to identifiability of further UC specifications, for example incorporating cyclical growth, hysteresis or ARMA cycles as discussed in Proietti (2006).

5 Appendix

A Sufficient Identification Conditions

Applying the simultaneous specification (6), (7) to the ARIMA model (4) leads to

\[ B(L)\Delta y_t = B(1)\mu + B(L)(k_{11}\tilde{\eta}_t + k_{12}\tilde{\varepsilon}_t) + k_{21}\Delta \tilde{\eta}_t + k_{22}\Delta \tilde{\varepsilon}_t . \]  

(15)

The autocovariances \( \gamma(j) \) of the MA part are calculated according to

\[
\gamma(j) = E[(B(L)(k_{11}\tilde{\eta}_t + k_{12}\tilde{\varepsilon}_t) + k_{21}\Delta \tilde{\eta}_t + k_{22}\Delta \tilde{\varepsilon}_t) \\
(B(L)(k_{11}\tilde{\eta}_{t-j} + k_{12}\tilde{\varepsilon}_{t-j}) + k_{21}\Delta \tilde{\eta}_{t-j} + k_{22}\Delta \tilde{\varepsilon}_{t-j})].
\]  

(16)

For \( j = 0, \ldots, p \), one gets \( p + 1 \) equations. This nonlinear equation system has a locally unique solution, if the first derivatives matrix \( \partial \gamma / \partial k' \) has full column rank 4 (the number of unknowns). Therein, the column vectors \( \gamma \) and \( k \) stack all \( \gamma(j), j = 0, \ldots, p \), respectively \( k_{ij}, i, j = 1, 2 \). In presence of nonlinear terms, the rank naturally depends on unknown parameters \( k_{ij} \). Nonetheless, it can be shown that for \( p = 3 \), the first derivatives matrix is irregular, leaving the model unidentified. In other words, the third autocovariance delivers no additional information on the \( k_{ij} \). (Of course, for \( p < 3 \), a column rank of 4 cannot be reached due to the lack of rows – the necessary identification condition is violated.)

Above, \( E(\tilde{\eta}_t^2) = 1 \) and \( E(\tilde{\varepsilon}_t^2) = 1 \) applied. In the two-regime case, these normalisations are retained for the first regime, while the variances \( \sigma^{2}_{\eta_1} \) and \( \sigma^{2}_{\varepsilon_2} \) in the second regime are freely estimated. The autocovariances \( \gamma_1(0), \ldots, \gamma_1(p) \) and \( \gamma_2(0), \ldots, \gamma_2(p) \) can thus be calculated separately for both regimes, providing extra equations available for identifying the simultaneity. In this, a further (sufficient) condition must be taken into account: A proportional break would occur for \( \sigma^{2}_{\eta_2} = \sigma^{2}_{\varepsilon_2} \) (since the variances are identical in the first regime by definition). Then, the \( \gamma_2(j), j = 0, \ldots, p, \) would simply result as multiples of
their first-regime counterparts, a special case of linear dependence. Formally, let vector $\gamma_R$ ($R$ for “regime”) stack all $\gamma_1(j)$ and $\gamma_2(j)$ and vector $k_R$ stack all $k_{ij}$, as well as $\sigma^2_{\gamma_1}$ and $\sigma^2_{\gamma_2}$. Then, even though the dimension of the first derivatives matrix $\partial\gamma_R/\partial k'_R$ would rise compared to the constant variance case, its rank would not be augmented by the introduction of the shift in volatility. Note that the condition of linear independence does not require breaks in both of the structural variances.

One can collect some evidence on disproportionate breaks in the correlated UC model. Particularly, the variances and the covariance of the composite residuals are linear combinations of the structural variances. If the shifts in the latter are identical, the former must break proportionately. In the correlated UC model this implies constant correlation and a constant relation of the variances. These two restrictions are tested by LR and clearly rejected with a $p$-value of 2.1%. This confirms the impression from (10) that the structural variances break differently.

### B State-Space Model

Setting up a state-space model, both trend and cycle are treated as state variables. According to (1), the observation equation is the simple identity

$$y_t = \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tau_t \\ c_t \\ c_{t-1} \\ c_{t-2} \end{pmatrix}.$$  \hspace{1cm} (17)

Combining (2) and (3) gives the transition equation

$$\begin{pmatrix} \tau_t \\ c_t \\ c_{t-1} \\ c_{t-2} \end{pmatrix} = \begin{pmatrix} \mu \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b_1 & b_2 & b_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ c_{t-3} \end{pmatrix} + \begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix}.$$  \hspace{1cm} (18)

Based on the simultaneous extensions (6) and (7), the covariance matrix of the vector of transition errors can be written as
Note that the covariance matrix of $\eta_t$ and $\varepsilon_t$ (the upper left block) as a quadratic form in the matrix of the $k_{ij}$s is guaranteed to be positive definite. In the two-regime case, for the observations after the shift a second covariance matrix applies. It differs from (19) only in that each $k_{i1}$ is to be multiplied by $\sigma_{\eta 2}$ and each $k_{i2}$ by $\sigma_{\varepsilon 2}$, $i = 1, 2$.

Initial values for the AR parameters and the constant are obtained from estimating the appropriate ARIMA process. The trend starts at the first observation of the series $y_t$ and is assigned an extremely large variance. The cycle is initialised at zero with the variance of the IP growth rates. Then, the log-likelihood function can be constructed and numerically maximised passing through the standard Kalman filter equations.

References


