Auctions with Anticipated Emotions: Overbidding, Underbidding, and Optimal Reserve Prices*

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Abstract

The experimental literature has documented that there is overbidding in second-price auctions, regardless of bidders' valuations. In contrast, in first-price auctions there tends to be overbidding for large valuations, but underbidding for small valuations. We show that the experimental evidence can be rationalized by a simple extension of the standard auction model, where bidders anticipate (constant) positive or negative emotions caused by the mere fact of winning or losing. Even if the "emotional" (dis-)utilities are very small, the revenue-maximizing reserve price r^* may be significantly different from the standard model. Moreover, r^* is decreasing in the number of bidders.

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I Introduction

In standard auction theory, it is assumed that a bidder cares only about whether or not he gets the object to be auctioned off and the payment he has to make. If a bidder wins, his utility is given by his intrinsic valuation of the object minus his payment. If a bidder does not win, his utility is zero, just as if he had not participated in the auction.

While this may be a good approximation in many circumstances, it cannot be denied that bidders in auctions are just human beings. As such, they experience emotions.[1] A bidder may well enjoy the thrill and excitement of being a winner. He may thus derive some utility from the mere fact of winning, over and above his value from receiving the object. Similarly, a bidder might be disappointed when he loses, because in contrast to someone who did not participate, he had a chance to win.

The aims of this paper are threefold. First, we want to study the robustness of the standard symmetric sealed-bid auctions model with risk-neutral players and private independent values. Specifically, we are interested in the implications for the resulting bidding behavior when bidders anticipate (small) positive emotions of winning and (small) negative emotions of losing. Second, we investigate whether introducing anticipated emotions in a particularly simple way (where the direct utility of winning and the direct disutility of losing are given by constant terms) may shed light on various experimental findings on the bidding behavior in auctions with independent private values.

Specifically, in laboratory experiments on second-price auctions there tends to be overbidding, the extent of which is (relatively) constant across all valuations bidders might have (see, e.g., Kagel, Harstad, and Levin, 1987; Kagel and Levin, 1993; Kirchkamp

[1] The role of anticipated emotions in decision making has been pointed out in the literature on experimental psychology (see, e.g., Mellers, Schwartz, and Ritov, 1999). For a recent experiment on the role of emotions in auctions, see e.g., Cooper and Fang (2008).

and Reiss, 2004; Cooper and Fang, 2008). In contrast, in first-price auctions there tends to be overbidding when bidders have large valuations, but underbidding when bidders have small valuations (see, e.g., Cox, Smith, and Walker, 1985, 1988; Ivanova-Stenzel and Sonsino, 2004; Kirchkamp and Reiss, 2004, 2006). In particular, Cox, Smith, and Walker (1988) document overbidding in first-price auctions, but also emphasize that, when bids are approximated by linear bidding functions, intercepts tend to be negative. Cox, Smith, and Walker (1988, p. 77f.) point out that some "subjects bid a much smaller proportion (sometimes zero) of their value" when their realized valuation was sufficiently small. They call the observation of bids equal to zero the "throw away" bid phenomenon and argue that these bidders did not "seriously" participate in the auction.[2] More recently, to investigate this issue further, Ivanova-Stenzel and Sonsino (2004) and Kirchkamp and Reiss (2004, 2006) facilitate underbidding at low valuations by considering experimental settings where the lowest possible valuation of buyers is strictly positive (which is in contrast to most of the experimental literature where the lower bound of valuations is usually set equal to zero). They find that the average amount of overbidding in first-price auctions is increasing in bidders' valuations (see also Goeree, Holt, and Palfrey, 2002), and that, whenever valuations are sufficiently small, there is indeed substantial underbidding (i.e., bids are strictly positive, but below their risk-neutral equilibrium value).[3] Moreover, Kirchkamp and Reiss (2006) document that subjects indeed expect underbidding at low valuations by other players.

To summarize, with respect to first-price auctions and second-price auctions the following stylized facts emerge from the experimental literature. On the one hand, in

[2] Note that actual non-participation was not a feasible option in these laboratory experiments.

[3] For graphical illustrations, cf. Figure 6 and Table 2 in Kirchkamp and Reiss (2004) and Figure 8 in Cox, Smith, and Walker (1988). In Ivanova-Stenzel and Sonsino (2004), 7.4 percent of bids are even below the lowest possible valuation.

first-price auctions, the amount of overbidding seems to be increasing in the underlying valuation, and it is negative (i.e., there is underbidding) for low valuations. On the other hand, in second-price auctions, there seems to be overbidding, the extent of which is relatively constant across valuations.

We show that our simple extension of the standard symmetric auctions model (where bidders anticipate some (constant) positive emotions of winning and some (constant) negative emotions of losing) is able to simultaneously rationalize the experimental findings on both the first-price auction and the second-price auction. Intuitively, if bidders anticipate joy of winning, bids will be larger than in the standard model, both in first-price and second-price auctions. However, if bidders anticipate a disutility of losing, the implications depend on the auction format. In a second-price auction, bidders who still participate bid more when they anticipate negative emotions, because they are more eager to avoid losing. To summarize, in a second-price auction, while bidders with very low valuations will not participate, due to anticipated emotions all participating bidders overbid by the same amount. While it turns out that, in second-price auctions, joy of winning and disutility of losing affect bids in the same way, this is markedly different in first-price auctions. In a first-price auction, bidders with relatively small valuations might again not participate (where, in both auction formats, participation is increasing in the joy of winning and decreasing in the disutility of losing). Those bidders with small valuations who do participate, when anticipating that losing is painful, bid less than in the standard model (i.e., for participating bidders with small valuations a disutility of losing actually *reduces* bids). In contrast, bidders with high valuations bid more. The reason that in a first-price auction there may be underbidding is that anticipated disutility of losing reduces the number of participating bidders. In a first-price auction, the winner has to pay his own bid, so that bids are smaller when there are fewer bidders (and hence the probability of winning is larger). Provided that a bidder's valuation is sufficiently small, this effect dominates the willingness to bid more in order to avoid negative emotions.

Third, we derive novel predictions regarding the optimal reserve price, i.e., the lower bound on acceptable bids that maximizes the seller's expected revenue. Reserve prices have received considerable attention in the theoretical literature (see, e.g., the seminal work of Myerson, 1981, and Riley and Samuelson, 1981) and are widely used in practice. It turns out that the seller's optimal reserve price goes up when winners enjoy positive emotions, while it goes down when losers experience negative emotions. Perhaps more surprisingly, we can show that the optimal reserve price is decreasing in the number of potential buyers. This result is in stark contrast to the standard model, where the optimal reserve price is independent of the number of potential buyers (cf. Myerson, 1981; Riley and Samuelson, 1981). Intuitively, in a second-price auction, a reserve price increases the seller's revenue when it is binding, but it also reduces participation.[4] When there are more buyers, a given reserve price will be binding with a smaller probability. It thus becomes relatively more important to compensate the fact that the anticipation of negative emotions already reduces participation. Hence, the optimal reserve price will go down. Since revenue-equivalence will hold, this result will also hold true in a first-price auction. Finally, we also demonstrate that if there are many bidders, the optimal reserve price can be significantly smaller than the one in the standard model, even when the extent of the emotions seems to be almost negligible compared to the expected value of the object.

The remainder of the paper is organized as follows. In Section II, we discuss the

[4] Note that in a second-price auction (but not in a first-price auction), the bidding behavior of the participating bidders is not changed by the presence of a reserve price.

related literature, and in Section III, the model is introduced. In Section IV, bidding behavior in second-price auctions and first-price auctions is analyzed. In Section V, the implications of positive and negative emotions for the seller's optimal reserve price are studied. In Section VI, we briefly examine the robustness of our results on over- and underbidding in a setting where the emotional (dis-)utilities are not given by constants. Concluding remarks follow in Section VII. All proofs are relegated to an Appendix.

II Related Literature

Our paper is related to three strands of the literature. First, at a general level, the paper is part of the behavioral economics literature that tries to enrich economic theory by taking into consideration psychological insights. Papers that are thus related in spirit include recent work on the implications of fairness, ethics, and anticipated emotions in mechanism design theory.[5]

Second, and more specifically, our paper is related to a growing theoretical literature that aims at explaining the bidding behavior observed in the laboratory experiments discussed above. This literature has enriched the standard auction model by a variety of (potentially complex) behavioral phenomena that are each consistent with *some* of the experimental findings. We contribute to this literature by showing that a rather simple extension of the standard auction model (that incorporates constant positive and negative emotions of winning respectively losing) can simultaneously rationalize the patterns of overbidding and underbidding in first-price auctions and second-price auctions that have been observed in the experimental literature.

For example, various authors have shown that risk aversion may lead to overbidding

[5]For a survey of the role of emotions in economic theory, see e.g., Elster (1998).

in first-price auctions (see, e.g., Riley and Samuelson, 1981; Cox, Smith, and Walker, 1988). However, in contrast to the experimental evidence discussed above, risk aversion cannot account for overbidding in second-price auctions (where bidding one's valuation remains a weakly dominant strategy). Also, risk aversion would predict overbidding for *all* valuations in first-price auctions. Evidence casting doubt on the (exclusive) role of risk aversion is also provided by Cox, Smith, and Walker (1985), who employ the binary lottery procedure to induce risk neutral behavior.

Other proposed explanations for the experimental findings are based on bounded rationality of participants. For example, focusing on second-price auctions, Kagel, Harstad, and Levin (1987) argue that overbidding in this auction format might be driven by the illusion that overbidding increases the probability of winning with little cost (because only the second highest bid has to be paid). Moreover, limited negative feedback from overbidding in this auction format might also play a role.[6] Kirchkamp and Reiss (2004) investigate the possibility that experimental subjects might follow simple rules-of-thumb when making their bids. In first-price auctions, such rules-of-thumb can indeed lead to underbidding at low valuations and overbidding at high valuations. However, in many of the second-price auction experiments studied in the literature applying such heuristics would predict the same behavior as the standard auction model. More recently, Crawford and Iriberri (2007) study a non-equilibrium model of level-k strategic thinking. In second-price auctions as well as in first-price auctions with a uniform distribution of valuations (which has frequently been used in experiments), Crawford and Iriberri's (2007) approach predicts the same behavior as the standard model. However, given a non-uniform distribution, Crawford and Iriberri (2007) can explain observed behavior in

[6] However, see e.g., Kagel and Levin (1993) and Cooper and Fang (2008) for evidence that overbidding in second-price auctions does not seem to vanish with experience.

first-price auctions.

The present paper belongs to a further strand of attempts to explain the experimental findings that considers the role of emotions in auctions. With respect to negative emotions, Filiz-Ozbay and Ozbay (2007) study the implications of anticipated loser regret that may be felt by a losing bidder if he learns that the winning bid was smaller than his valuation, where the resulting disutility depends on the difference between these two (see also Engelbrecht-Wiggans and Katok, 2007). They show that anticipated loser regret may lead to overbidding in first-price auctions, but does not alter the prediction with respect to second-price auctions. Morgan, Steiglitz, and Reis (2003) introduce a spite motive, i.e., the utility of bidders is negatively affected by the surplus of the winning rival. Such a spite motive predicts overbidding in second-price auctions (which, however, is decreasing in valuations) as well as overbidding in first-price auctions (which is increasing in valuations), but, in contrast to the experimental evidence, never results in underbidding. More closely related, we build on Cox, Smith, and Walker (1988) who introduce a (constant) joy of winning into standard auction models (see also Goeree, Holt, and Palfrey, 2002).[7] In a second-price auction, joy of winning implies overbidding by the same amount independent of the underlying valuation, which is in line with the experimental evidence. However, in a first-price auction without a reserve price (which is the setting considered in the first-price auction experiments discussed above), a pure joy of winning model would predict an amount of overbidding that is independent of the underlying valuation, i.e., which is constant. Hence, a pure joy of winning model would fail to replicate two stylized facts with respect to first-price auctions: namely, that the

[7]Note that one could also imagine that a winning bidder feels regret (i.e., a negative emotion) if he learns that he could have won with a smaller bid, which potentially might reduce the (positive) joy of winning. Yet, Filiz-Ozbay and Ozbay (2007) argue that bidders do not seem to *anticipate* such winner regret. In a similar vein, in principle, losing the auction might trigger positive emotions in bidders who have overbid. The relative importance of such countervailing effects awaits more scrutiny in the future.

extent of overbidding seems to be increasing in the underlying valuation and that there seems to be underbidding for small valuations. We contribute to the joy of winning literature by showing that our model that combines a constant joy of winning and a constant disappointment from losing is able to rationalize the above described stylized facts of *both* second-price auctions and first-price auctions. Hence, while sofar joy of winning has received considerable attention in the literature, it seems that anticipated disappointment from losing the auction might also play an important role in explaining the experimental findings.[8]

Third, as discussed above, our paper is related to the literature on optimal reserve prices (i.e., minimum acceptable bids) in auctions. To the best of our knowledge, the present paper is the first to investigate how bidders' positive emotions from winning and negative emotions from losing affect optimal reserve prices, and we show that the optimal reserve price may be decreasing in the number of potential buyers. Thereby, we contribute to a literature that has investigated reasons why the standard model's prediction of an optimal reserve price that is independent of the number of bidders (see Myerson, 1981; Riley and Samuelson, 1981) might fail. For example, like the present paper, Levin and Smith (1996) find that optimal reserve prices may decline if, instead of independent private values, correlated information is assumed. Intuitively, with correlated values the cost of lost trade due to a reserve price that is too high may be increasing in the number of bidders. In contrast, Rosenkranz and Schmitz (2007) and Chen and Greenleaf (2008) provide reasons why an optimal reserve price may be increasing in the number of bidders. Rosenkranz and Schmitz (2007) assume that reserve prices may play the role of reference

[8]There is also a growing body of neuroeconomic evidence on the role of negative emotions in decision-making (see, e.g., Sanfey et al., 2003; Delgado et al., 2008). Note that the term "disappointment" has been used previously (e.g., by Gul, 1991), however in contexts different from ours. For example, in Gul's (1991) non-expected utility approach to rationalize the Allais Paradox, the term "disappointment" relates to unfavorable outcomes in lotteries.

points, so that a larger reserve price can have a positive impact on the bidders' willingness-to-pay. Chen and Greenleaf (2008) consider a seller who experiences regret if the reserve price he set is too low relative to the winning bid, which is more likely if the number of bidders is large. However, in contrast to the present paper, Chen and Greenleaf (2008) assume that bidders have standard preferences.

III The Model

A monopolistic seller possesses a single, indivisible object. There are $n \geq 2$ potential buyers. The seller conducts a (second-price or first-price) sealed-bid auction with a publicly announced reserve price $r \geq 0$. Thus, a buyer who wants to participate in the auction must at least bid r. If bidder i wins the object and pays the price t_i according to the rules of the auction, then his utility is given by $v_i - t_i + \varepsilon$, where $v_i \in [\underline{v}, \overline{v}]$ denotes his intrinsic valuation of the object to be auctioned off ($\overline{v} > \underline{v} \geq 0$). If buyer i participates (i.e., if he bids at least the reserve price), but does not win the auction, his utility is given by $-\gamma$. The case $\varepsilon = \gamma = 0$ is the standard case analyzed in the literature on auctions. A strictly positive ε captures the joy that a bidder feels when he wins the auction. Similarly, a strictly positive γ captures the disappointment that a bidder feels when he loses. Importantly, as will become clear below, even if ε and γ are relatively small, the presence of anticipated emotions may affect equilibrium behavior substantially.

Apart from the (possibly very small) modification with regard to $\varepsilon \geq 0$ and $\gamma \geq 0$, our model is identical to the standard model of the independent private values environment with symmetric bidders that completely ignores positive and negative emotions (see, e.g., Riley and Samuelson, 1981). Buyer i's type v_i is the realization of a random variable $\check{v}_i$. Each $\check{v}_i$ is independently and identically distributed on $[\underline{v}, \overline{v}]$. The cumulative distribution function F is strictly increasing and the density function is denoted by f. We make the usual monotone hazard rate assumption, according to which $[1-F(v)]/f(v)$ is decreasing in v. As a consequence, we are in Myerson's (1981) "regular case," i.e., the "virtual valuation" $v - [1 - F(v)]/f(v)$ is increasing in v. Only buyer i knows his realized value v_i, while the other components of the model are common knowledge. The seller sets the reserve price r that maximizes her expected revenue, and each buyer is interested in maximizing his expected utility. Throughout, we will focus attention on symmetric equilibria.

IV Bidding Behavior

We now study participation and bidding behavior in both second-price auctions and first-price auctions when anticipated emotions of the present simple form are taken into account.

Second-Price Auction

If at least two bidders participate, in a second-price auction the buyer submitting the highest bid wins the object, but he has to pay only the second-highest bid.[9] If there is a tie, so that there is more than one bidder with the highest bid, let the object go to each of them with equal probability. A similar assumption is made in the first-price auction. However, in equilibrium, the tie-breaking rule is not important, because the probability of a tie will be zero. If only one bidder participates (i.e., bids at least r), he wins and has to pay the reserve price r. It is well known that in the standard model (where $\varepsilon = \gamma = 0$), each buyer i with $v_i \geq r$ will participate in the auction and bid his type v_i. In the present

[9]Recall that (following Riley and Samuelson, 1981), participating buyers must make a bid (weakly) above the announced reserve price.

framework with $\varepsilon \geq 0$ and $\gamma \geq 0$, this result can be generalized as follows.[10]

Proposition 1 *In a second-price auction, only buyers of types $v \geq \tilde{v}$ will participate, and their symmetric equilibrium bidding strategies are given by $b^S(v) = v + \varepsilon + \gamma$. The threshold valuation $\tilde{v}$ is increasing in r. If $\gamma > 0$, then $\tilde{v}$ is implicitly defined by $[\tilde{v} - r + \varepsilon + \gamma]F(\tilde{v})^{n-1} = \gamma$. In this case, $\tilde{v} \in (\underline{v}, \overline{v})$, and $\tilde{v} > r$ if and only if $r < \bar{r}$, where $F(\bar{r})^{n-1} = \gamma/(\varepsilon + \gamma)$ and $\bar{r} \in (\underline{v}, \overline{v})$. If $\gamma = 0$, then $\tilde{v} \equiv r - \varepsilon \leq r$.*

If there were no negative emotions of losing ($\gamma = 0$), then every buyer with $v \geq r - \varepsilon$ would participate and bid $b^S(v) = v+\varepsilon$, which is more than the usual second-price auction bid v, because a bidder now anticipates that he will feel happy when he wins. Yet, if a buyer also anticipates that it is painful to lose ($\gamma > 0$),[11] his valuation must be larger than $r - \varepsilon$ in order to make participation individually rational for him. Specifically, the larger is n, the larger will be the threshold valuation $\tilde{v}$ below which a buyer will not participate, because the probability of losing increases when there are more bidders (formally, this follows immediately from the implicit definition of $\tilde{v}$ in Proposition 1). Hence, even if the reserve price is set equal to zero, in the presence of emotions, bidders with sufficiently low valuations might not necessarily participate.

Corollary 1 *In a second-price auction with a given reserve price $r \geq 0$, the following results hold. (i) Increasing ε weakly increases participation, while increasing γ reduces participation. (ii) The bids are increasing in ε and γ. (iii) The amount of overbidding of a participating bidder, $b^S(v) - v = \varepsilon + \gamma$, is independent of v.*

[10]Notice that positive and negative emotions have similar effects on the bidding behavior in second-price auctions. In contrast, it will turn out that this is not the case in first-price auctions.

[11]Notice that anticipated negative emotions are somewhat related to entry fees or participation costs (however, the emotional costs are incurred by losers only and they are not payments that accrue to the seller). On auctions with costly participation, see e.g., Menezes and Monteiro (2000).

Note that increasing ε has no effect on participation when $\gamma = 0$ and $r - \varepsilon < \underline{v}$. Now, consider a buyer of type $v \geq \tilde{v}$. The expected payment of the buyer to the seller is given by

$$T^S(v) = rF(\tilde{v})^{n-1} + \int_{\tilde{v}}^{v} (w + \varepsilon + \gamma) dF(w)^{n-1}, \tag{1}$$

because the buyer will win only if he has the highest value. He then must pay r if all other buyers have types smaller than $\tilde{v}$ (which happens with probability $F(\tilde{v})^{n-1}$), and he must pay $b^S(w) = w + \varepsilon + \gamma$ if $w \in (\tilde{v}, v)$ is the highest value of the other $n-1$ buyers.

First-Price Auction

If the seller conducts a first-price auction, the bidder with the highest bid wins and has to pay what he has bid. In the standard framework (where $\varepsilon = \gamma = 0$), there is a symmetric equilibrium in which each bidder bids less than his true type. In the present model, this result can be generalized, so that a bidder who participates in a first-price auction bids $b^F(v)$, which is less than $b^S(v)$. Specifically, the following result can be obtained.

Proposition 2 *In a first-price auction, only buyers of types $v \geq \tilde{v}$ will participate, where $\tilde{v}$ is defined in Proposition 1 above. Their symmetric equilibrium bidding strategies are given by*

$$b^F(v) = v + \varepsilon - \frac{1 - F(v)^{n-1}}{F(v)^{n-1}}\gamma - \int_{\tilde{v}}^{v} \frac{F(w)^{n-1}}{F(v)^{n-1}} dw. \tag{2}$$

Just as in the second-price auction, anticipated joy of winning increases the bids. Yet, while anticipated disappointment increases a buyer's bid in the second-price auction (provided the buyer still participates), it may decrease his bid in the first-price auction. When a buyer wants to avoid the pain of losing, in a second-price auction he either does not participate or he bids more aggressively (recall that the bid determines the winner, but not his payment). In contrast, in a first-price auction, a winning bidder must pay his own bid. When the buyers anticipate disappointment, fewer bidders will participate. Thus, on the one hand, a participating bidder will win with a larger probability, so he may reduce his bid. On the other hand, anticipated disappointment increases a bidder's desire to win, so he might be willing to bid more. The first effect is stronger than the second effect if the bidder's valuation is sufficiently small. These findings can be summarized as follows.

Corollary 2 *In a first-price auction with a given reserve price $r \geq 0$, the following results hold. (i) Increasing ε weakly increases participation, while increasing γ reduces participation. (ii) The bids are increasing in ε. They are increasing in γ if v is sufficiently large and decreasing in γ if v is sufficiently small. (iii) If $\gamma > 0$, the amount of overbidding of a participating bidder, $b^F(v) - \left(v - \int_r^v \frac{F(w)^{n-1}}{F(v)^{n-1}} dw\right)$, is increasing in v. Moreover, there is always overbidding for large valuations, and there is underbidding for small valuations whenever $r < \bar{r} \in (\underline{v}, \overline{v})$. (iv) If $\gamma = 0$, there is never underbidding and, if additionally $r \leq \underline{v}$ holds, the amount of overbidding is given by ε, and hence independent of v.*

Importantly, Corollary 2 shows that relying on joy of winning alone cannot explain the experimental findings discussed in the Introduction. In the absence of a reserve price (which is exactly the setting considered in these experiments), Corollary 2(iv) shows that a model with $\varepsilon > \gamma = 0$ would predict constant overbidding, which is in contradiction to the experimental evidence. On the other hand, a model that allows for potential disappointment (i.e., $\gamma > 0$) can rationalize the experimental observations of underbidding at low valuations as well as overbidding that is increasing in the underlying valuation. Consequently, in addition to joy of winning (which has received substantial attention in the earlier auction literature), anticipated disappointment might have a non-negligible impact on the behavior of bidders in auctions.

As an illustration, consider Figure 1, where $F(v) = v$ for $v \in [0,1]$, $r = 0$, and $n = 3$. The dotted line is the usual equilibrium bidding strategy when $\varepsilon = \gamma = 0$, while the solid curve is the equilibrium bidding strategy $b^F(v)$ when $\varepsilon = \gamma = 1/20$. Note that in the latter case, bidders with a valuation smaller than $\tilde{v} \approx 0.338$ do not participate, bidders with a valuation between $\tilde{v}$ and $\hat{v} \approx 0.609$ bid less than in the standard model, and bidders with a valuation larger than $\hat{v}$ bid more.

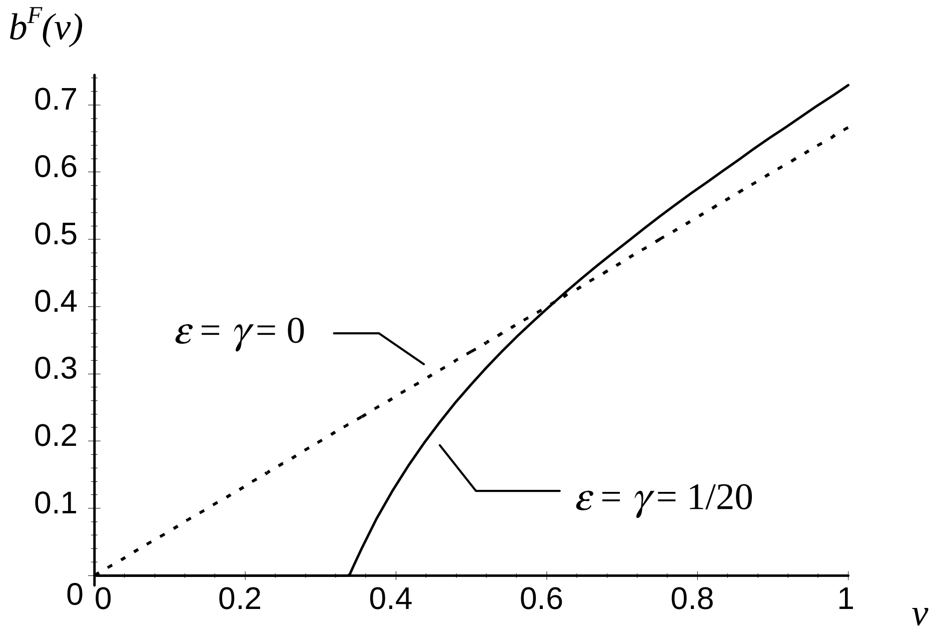

Figure 1. Bidding strategy in the first-price auction.

To summarize, our model can explain experimental findings according to which in second-price auctions there is always overbidding that does not vary in the underlying valuation, while in first-price auctions there is underbidding for small valuations and overbidding for large valuations, the extent of which is increasing in the underlying valuation.

To conclude the analysis of the first-price auction, consider again a buyer of type $v \geq \tilde{v}$. He pays $b^F(v)$ if all other buyers have types smaller than v, so his expected payment to the seller is

$$T^F(v) = b^F(v)F(v)^{n-1} = (v+\varepsilon+\gamma)\,F(v)^{n-1} - \gamma - \int_{\tilde{v}}^{v} F(w)^{n-1}dw. \qquad (3)$$

Using the definition of $\tilde{v}$, integration by parts immediately shows that $T^F(v) = T^S(v)$, which is in accordance with the well-known revenue equivalence principle.

V The Optimal Reserve Price

A revenue-maximizing seller will set a reserve price. As we have indicated above, if the seller takes the bidders' joy of winning respectively disappointment from losing into account, the implications for the optimal reserve price might be markedly different from the standard model. This will be illustrated in the present Section.

The seller does not know the buyers' types. Hence, her expected revenue Π is given by n times the expected value of the payment that a buyer makes to the seller (which is $T^F(v) = T^S(v)$ if $v \geq \tilde{v}$, and 0 otherwise). With integration by parts, it follows that

$$\begin{aligned}
\Pi &= n\int_{\tilde{v}}^{\overline{v}} \left((v+\varepsilon+\gamma)\,F(v)^{n-1} - \gamma - \int_{\tilde{v}}^{v} F(w)^{n-1}dw \right) dF(v) \qquad (4) \\
&= n\int_{\tilde{v}}^{\overline{v}} \left[\left((v+\varepsilon+\gamma)\,F(v)^{n-1} - \gamma \right) f(v) - F(v)^{n-1} + F(v)^{n-1}F(v) \right] dv \\
&= n\int_{\tilde{v}}^{\overline{v}} \left(v - \frac{1-F(v)}{f(v)} + \varepsilon + \gamma \right) F(v)^{n-1}dF(v) - n\gamma[1-F(\tilde{v})].
\end{aligned}$$

In the present Section, where we characterize the optimal reserve price and its properties, we maintain the assumption $\varepsilon > 0$ and $\gamma > 0$, i.e., we assume that there are positive emotions of winning and negative emotions of losing.[12]

[12] Note that Proposition 3 continues to hold in the cases (i) $\gamma > 0$ and $\varepsilon = 0$ and (ii) $\gamma = 0$ and $\varepsilon < 1/f(\underline{v}) - \underline{v}$. If, however, $\gamma = 0$ and the joy of winning is sufficiently large (i.e., $\varepsilon \geq 1/f(\underline{v}) - \underline{v}$),

Proposition 3 *The optimal reserve price r^* is given by $r^* = \frac{1-F(\tilde{v}^*)}{f(\tilde{v}^*)}$, where $\tilde{v}^*$ is uniquely defined by $\tilde{v}^* - \frac{1-F(\tilde{v}^*)}{f(\tilde{v}^*)} = \frac{1-F(\tilde{v}^*)^{n-1}}{F(\tilde{v}^*)^{n-1}}\gamma - \varepsilon$ and $\tilde{v}^* \in (\underline{v}, \overline{v})$ holds.*

We can now analyze what implications anticipated joy and disappointment have for the optimal reserve price. As might have been expected, the joy of winning increases the optimal reserve price. Even though the disappointment from losing increases the (remaining) bids in a second-price auction (and the bids of buyers with high valuations in a first-price auction), the optimal reserve price goes down when disappointment is anticipated, because negative emotions already reduce the buyers' willingness to participate in the auction.

Proposition 4 *The optimal reserve price r^* is increasing in ε and decreasing in γ.*

Let us now turn to the impact of the number n of potential buyers on the optimal reserve price. Interestingly, it turns out that the optimal reserve price r^* is decreasing in the number of buyers. This result is in stark contrast to the standard result, according to which the optimal reserve price r_0 is independent of the number of buyers. Intuitively, when buyers anticipate negative emotions, they are less inclined to participate. In order to increase participation, it makes sense to reduce the reserve price. This effect is the more important, the larger is the number n of potential buyers.

Proposition 5 *The optimal reserve price r^* is decreasing in n.*

Remark 1 *Even if ε and γ are very small, the optimal reserve price r^* can be significantly smaller than the reserve price r_0 that is optimal in the absence of emotions, provided the number of potential buyers is sufficiently large.*

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then the seller optimally induces all types to participate and any reserve price r^* satisfying $r^* \leq \underline{v} + \varepsilon$ is optimal. Note that in the standard model (where $\varepsilon = \gamma = 0$), it is, in general, assumed that $0 < 1/f(\underline{v}) - \underline{v}$ holds (in particular, this is satisfied in the case of strictly positive density and $\underline{v} = 0$); implying an interior optimal reserve price $r_0 \in (\underline{v}, \overline{v})$ that is implicitly defined by $r_0 = (1 - F(r_0))/f(r_0)$ (see case (ii) and Proposition 3).

For example, consider the uniform distribution on the unit interval, so that $r_0 = 0.5$. Assume that $\varepsilon = \gamma = 0.01$. If we measured payoffs in 100 dollar units, this would mean that while the expected valuation is 50 dollars, the monetary equivalent of a bidder's joy of winning or his disappointment when he loses is only 1 dollar. If there are only two potential buyers, the effects of joy and disappointment just cancel out, so that the optimal reserve price is still $r^* = r_0 = 0.5$. However, if $n = 25$, then the optimal reserve price is significantly reduced to $r^* \approx 0.162$.

VI A Remark on Robustness

In this Section, we briefly illustrate how the main insights of our analysis with respect to over- and underbidding can be generalized. So far, we have assumed that the emotional (dis-)utilities are simply given by constants. Let us now consider a slightly extended model.[13] For simplicity, in this Section, we assume that there is no reserve price (as is typically the case in the experimental literature on over- and underbidding).

Suppose that a winning bidder i who pays the price t_i has utility $\omega(v_i) - t_i$ and a losing buyer i has utility $l(v_i)$, where $\omega(v) > v$, $l(v) < 0$, and $\omega'(v) > l'(v) \geq 0$. In analogy to Proposition 1, it follows that in a second-price auction, only buyers of types $v \geq \tilde{v}$ participate, where $\tilde{v} \in (\underline{v}, \overline{v})$ is implicitly defined by $[\omega(\tilde{v}) - l(\tilde{v})]F(\tilde{v})^{n-1} = -l(\tilde{v})$. Their symmetric equilibrium bidding strategies are given by $b^S(v) = \omega(v) - l(v)$. If the extended model is to explain the stylized facts that motivated our analysis, there must be constant overbidding in the second-price auction, i.e., we have to impose the restriction $\omega(v) - l(v) = v + const$ on the admissible functions $\omega(v)$ and $l(v)$, where $const > 0$.

[13] We are grateful to an anonymous referee for suggesting this extension. The proofs of the subsequent claims with respect to participation and bidding strategies in the second-price auction and the first-price auction are analogous to the proofs of Propositions 1 and 2, and complete proofs are therefore omitted (for an outline of the main arguments, see the Appendix).

In analogy to Proposition 2, in a first-price auction, only buyers of types $v \geq \tilde{v}$ will participate. Their symmetric equilibrium bidding strategies are now given by

$$b^F(v) = v + const + \frac{l(\tilde{v})}{F(v)^{n-1}} - \int_{\tilde{v}}^{v} \frac{F(x)^{n-1}}{F(v)^{n-1}} dx. \tag{5}$$

Thus, the amount of overbidding in a first-price auction is $const + \frac{l(\tilde{v})}{F(v)^{n-1}} + \int_0^{\tilde{v}} \frac{F(x)^{n-1}}{F(v)^{n-1}} dx$. The derivative of this expression with respect to v is $(n-1)f(v)F(v)^{-n} \left(-l(\tilde{v}) - \int_0^{\tilde{v}} F(x)^{n-1} dx \right)$, which is larger than $(n-1)f(v)F(v)^{-n} \left([\tilde{v} + const]F(\tilde{v})^{n-1} - \tilde{v}F(\tilde{v})^{n-1} \right) > 0$, i.e., the amount of overbidding in the first-price auction is increasing in v. Note that there is underbidding at $v = \tilde{v}$, because $const - \frac{[\tilde{v}+const]F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1}} + \int_0^{\tilde{v}} \frac{F(x)^{n-1}}{F(\tilde{v})^{n-1}} dx < -\tilde{v} + \frac{1}{F(\tilde{v})^{n-1}} \tilde{v}F(\tilde{v})^{n-1} = 0$. Moreover, observe that there is overbidding at $v = \overline{v}$, because $const + l(\tilde{v}) + \int_0^{\tilde{v}} F(x)^{n-1} dx > \omega(\tilde{v}) - \tilde{v} > 0$.

Hence, while our simple way of modeling emotions as positive and negative constants leads to a particularly tractable model that can explain the patterns of over- and underbidding observed in the experimental literature, qualitatively similar conclusions can also be obtained in more general version of our model.

VII Conclusion

It seems to be fair to say that most human beings are excited when they win and they are disappointed when they lose. We simultaneously include such positive and negative emotions in the standard auction model. We certainly do not claim that the simple way in which we have tried to capture joy and disappointment in our model is the ultimate way to introduce emotions into auction theory. Clearly (but also at the cost of additional complexity), there might be private information about emotions or emotions might dependent on various variables such as the winning bid, the second-highest bid, or the number of bidders. In principle, the extent of emotions might also vary across auction formats, in which case revenue equivalence between the first-price auction and the second-price auction would in general fail. In particular, note that the seller's expected profit given an optimal reserve price is increasing in ε and decreasing in γ.[14] Hence, if one of the auction formats were to generate both larger joy of winning and smaller disappointment, the seller would strictly prefer this auction format.

However, leaving aside these potentially interesting issues, we think it is instructive to see that the present simple extension of the standard auction model can simultaneously rationalize patterns of overbidding and underbidding in both first-price auctions and second-price auctions observed in the experimental literature, while earlier (potentially more complex) approaches have failed to do so. It would be an interesting next step to conduct an experiment that aims to assess the relative importance of joy of winning and disappointment in an auction context.

We also study optimal reserve prices and show that, in contrast to the standard result, in the presence of emotions the optimal reserve price might be decreasing in the number of bidders. As we are not aware of any conclusive experimental evidence regarding how the optimal reserve price varies with the number of bidders, this question might also be an interesting topic for future experimental research.

[14]This follows from a straightforward application of the envelope theorem (see (4))

VIII Appendix

Proof of Proposition 1

Consider a buyer i and assume that all other buyers follow the strategy given in the proposition. If buyer i participates and bids $b^S(v_i)$, he wins and pays t_i if $b^S(v_i) > t_i$, where t_i is the maximum of the other bids if there are any, and $t_i = r$ otherwise. Consider a downward deviation to some $\tilde{b} < b^S(v_i)$. If $t_i < \tilde{b} < b^S(v_i)$, he still wins and pays t_i, so his utility is not changed. If $\tilde{b} < b^S(v_i) < t_i$, his utility $-\gamma$ also remains unchanged. If $\tilde{b} < t_i < b^S(v_i)$, he now loses and his utility is $-\gamma$, while he could have attained the utility $v_i - t_i + \varepsilon$ by bidding $b^S(v_i)$. The latter utility level would have been larger, because $v_i - t_i + \varepsilon > v_i - b^S(v_i) + \varepsilon = -\gamma$. A similar argument shows that an upward deviation $\tilde{b} > b^S(v_i)$ cannot be profitable for the buyer. Finally, note that the expected payoff of a participating buyer is increasing in his valuation. A buyer with the smallest valuation for which it is not yet unprofitable to participate in the auction will win (and pay r) only if all other buyers have smaller valuations. Hence, a buyer will participate whenever $v \geq \tilde{v}$, where $\tilde{v}$ is implicitly defined by $[\tilde{v} - r + \varepsilon]F(\tilde{v})^{n-1} - \gamma[1 - F(\tilde{v})^{n-1}] = 0$. Consequently, if $\gamma = 0$, a buyer will participate whenever $v \geq max\{\underline{v}, r - \varepsilon\}$ holds. If $\gamma > 0$, we have $\tilde{v} \in (\underline{v}, \overline{v})$ and $\tilde{v}$ is uniquely defined: when v moves from $\underline{v}$ to $\overline{v}$, then $\gamma[1 - F(v)^{n-1}]$ decreases from γ to 0. If $r \in [0, \varepsilon]$, then $[v - r + \varepsilon]F(v)^{n-1}$ increases from 0 to $\overline{v} - r + \varepsilon$, and hence an intermediate value argument can be applied. Similarly, if $r \in (\varepsilon, \overline{v} + \varepsilon]$, then $[v - r + \varepsilon]F(v)^{n-1}$ increases from 0 to $\overline{v} - r + \varepsilon$ when v moves from $max\{\underline{v}, r - \varepsilon\}$ to $\overline{v}$. The seller will never set the reserve price r larger than $\overline{v} + \varepsilon$, because then no one would participate (even though a buyer's bid may well be larger than $\overline{v} + \varepsilon$).

Proof of Corollary 1

Part (i) follows from the implicit definition of $\tilde{v}$. Specifically, the threshold valuation $\tilde{v}$ below which buyers do not participate satisfies

$$\frac{d\tilde{v}}{d\varepsilon} = \frac{-F(\tilde{v})^n}{F(\tilde{v})^n + \gamma(n-1)f(\tilde{v})} < 0 \text{ and } \frac{d\tilde{v}}{d\gamma} = \frac{1 - F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1} + \gamma(n-1)f(\tilde{v})/F(\tilde{v})} > 0. \quad \text{(A1)}$$

Parts (ii) and (iii) of the corollary are obvious.

Proof of Proposition 2

A buyer's strategy (as a function of his type) specifies both his participation decision and, conditionally on participation, his bidding function. Hence, consider buyer i and suppose that all other buyers follow the strategy given in the proposition (i.e., abstain if $v < \tilde{v}$, and participate and bid $b^F(v)$ if $v \geq \tilde{v}$). For the moment, suppose that $\tilde{v} \leq \overline{v}$ (i.e., at least some types participate in the auction).

Given this assumption, in a preliminary step, one can show that $r \leq b^F(\overline{v})$ holds. To see this, consider the case $\tilde{v} \in (\underline{v}, \overline{v})$, and note that the implicit definition of $\tilde{v}$ in Proposition 1 (namely, $[\tilde{v} - r + \varepsilon]F(\tilde{v})^{n-1} - \gamma[1 - F(\tilde{v})^{n-1}] = 0$) implies that $r \leq b^F(\overline{v})$ is equivalent to

$$\tilde{v} + \varepsilon - \frac{1 - F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1}}\gamma \leq \overline{v} + \varepsilon - \frac{1 - F(\overline{v})^{n-1}}{F(\overline{v})^{n-1}}\gamma - \int_{\tilde{v}}^{\overline{v}} \frac{F(w)^{n-1}}{F(\overline{v})^{n-1}} dw \quad \text{(A2)}$$

$$\Leftrightarrow \tilde{v} - \frac{1 - F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1}}\gamma \leq \overline{v} - \int_{\tilde{v}}^{\overline{v}} F(w)^{n-1} dw. \quad \text{(A3)}$$

Moreover, note that $\overline{v} - \int_{\tilde{v}}^{\overline{v}} F(w)^{n-1}dw \geq \overline{v} - (\overline{v} - \tilde{v})F(\overline{v})^{n-1} = \tilde{v}$, where the inequality follows from the fact that $F(w)^{n-1}$ is an increasing function. Hence, for $\tilde{v} \in (\underline{v}, \overline{v})$,

$r \leq b^F(\overline{v})$ holds because

$$\tilde{v} - \underbrace{\frac{1 - F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1}}\gamma}_{\geq 0} \leq \tilde{v} \leq \overline{v} - \int_{\tilde{v}}^{\overline{v}} F(w)^{n-1} dw. \tag{A4}$$

In the case $\tilde{v} \leq \underline{v}$ (which according to Proposition 1 can only emerge if $\gamma = 0$; implying $r = \tilde{v} + \varepsilon$), we have that $r \leq b^F(\overline{v})$ is equivalent to

$$\tilde{v} + \varepsilon \leq \overline{v} + \varepsilon - \frac{1 - F(\overline{v})^{n-1}}{F(\overline{v})^{n-1}}\gamma - \int_{\tilde{v}}^{\overline{v}} \frac{F(w)^{n-1}}{F(\overline{v})^{n-1}} dw \Leftrightarrow \tilde{v} \leq \overline{v} - \int_{\tilde{v}}^{\overline{v}} F(w)^{n-1} dw, \tag{A5}$$

where $\overline{v} - \int_{\tilde{v}}^{\overline{v}} F(w)^{n-1} dw \geq \overline{v} - (\overline{v} - \tilde{v})F(\overline{v})^{n-1} = \tilde{v}$, and hence $r \leq b^F(\overline{v})$ for all $\tilde{v} \leq \overline{v}$.

After this preliminary step, we now derive player i's best response. Suppose that, for a given v_i, player i's strategy prescribes participation. In this case, it cannot be optimal for player i to place a bid b larger than $b^F(\overline{v})$ because otherwise (as $b^F(v)$ is increasing) he would win for sure and could increase his payoff by slightly reducing his bid. Moreover, in the model we have assumed that a buyer who wants to participate in the auction must at least bid r;[15] implying that bidder i's optimal bid (conditional on participation) must satisfy $b \in [r, b^F(\overline{v})]$. There exists a value $z \in [\tilde{v}, \overline{v}]$ such that $b^F(z) = b$. Consequently, if bidder i participates his expected payoff from bidding b can be written as

$$\begin{aligned} & \left(v_i - b^F(z) + \varepsilon\right) F(z)^{n-1} - \gamma[1 - F(z)^{n-1}] && \text{(A6)} \\ = \; & \left(v_i - z\right) F(z)^{n-1} + \int_{\tilde{v}}^{z} F(w)^{n-1} dw. && \text{(A7)} \end{aligned}$$

Note that, for $v_i = z = \tilde{v}$, (A7) is equal to zero. Moreover, the derivative of (A7) with

[15] Note that this assumption is not restrictive, because a strategy that, for given v_i, would prescribe participation in combination with a bid smaller than r would not be optimal: in this case, bidder i would lose for sure and obtain a payoff of $-\gamma$, while by not participating he could secure himself a payoff of 0.

respect to v_i is (weakly) positive and given by $F(z)^{n-1}$, and the derivative of (A7) with respect to z is given by

$$\underbrace{(n-1)F(z)^{n-2}f(z)}_{\geq 0}\cdot(v_i - z), \tag{A8}$$

which is (weakly) negative for all $v_i \leq \tilde{v}$. These observations imply that, for given v_i, bidder i will optimally participate only if $v_i \geq \tilde{v}$.

Hence, if, conditional on v_i, buyer i participates and bids $b^F(v_i)$ (i.e., sets $z = v_i \in [\tilde{v}, \overline{v}]$), his expected payoff is $\int_{\tilde{v}}^{v_i} F(w)^{n-1}dw$ (which follows from (A7)). Moreover, as

$$\begin{aligned} & (v_i - z)\,F(z)^{n-1} + \int_{\tilde{v}}^{z} F(w)^{n-1}dw - \int_{\tilde{v}}^{v_i} F(w)^{n-1}dw \\ = & \int_{v_i}^{z} \left[F(w)^{n-1} - F(z)^{n-1}\right] dw \leq 0, \end{aligned} \tag{A9}$$

it cannot be profitable for buyer i to deviate from $b^F(v_i)$, which completes the proof of the Proposition for the case $\tilde{v} \leq \overline{v}$.

Finally, suppose that $\tilde{v} > \overline{v}$; implying that none of the other buyers participates in the auction. Proposition 1 implies that $\tilde{v} > \overline{v}$ can only emerge if $\gamma = 0$ (in which case we have $\tilde{v} = r - \varepsilon$); implying $r - \varepsilon > \overline{v}$. If, conditional on v_i, bidder i decides not to participate his payoff is zero. If bidder i decides to participate, he has to bid at least r, and his payoff is weakly smaller than $v_i - r + \varepsilon < 0$. Hence, it is optimal for bidder i to follow the candidate equilibrium strategy of not participating for any v_i. This concludes the proof.

Proof of Corollary 2

Ad (i): Part (i) is identical to Corollary 1(i). Ad (ii): From Proposition 2 and Corollary

1 it follows that

$$\frac{db^F(v)}{d\varepsilon} = 1 + \frac{F(\tilde{v})^{n-1}}{F(v)^{n-1}}\frac{d\tilde{v}}{d\varepsilon} = 1 - \frac{F(\tilde{v})^{n-1}}{F(v)^{n-1}}\frac{F(\tilde{v})^n}{F(\tilde{v})^n + \gamma(n-1)f(\tilde{v})}, \tag{A10}$$

which is positive for $v \geq \tilde{v}$. Similarly, it follows that

$$\begin{aligned} \frac{db^F(v)}{d\gamma} &= -\frac{1-F(v)^{n-1}}{F(v)^{n-1}} + \frac{F(\tilde{v})^{n-1}}{F(v)^{n-1}}\frac{d\tilde{v}}{d\gamma} \\ &= -\frac{1-F(v)^{n-1}}{F(v)^{n-1}} + \frac{1}{F(v)^{n-1}}\frac{1-F(\tilde{v})^{n-1}}{1+\gamma(n-1)F(\tilde{v})^{-n}f(\tilde{v})}, \end{aligned} \tag{A11}$$

so that

$$\frac{db^F(v)}{d\gamma} < 0 \Leftrightarrow \frac{1-F(\tilde{v})^{n-1}}{1+\gamma(n-1)F(\tilde{v})^{-n}f(\tilde{v})} < 1 - F(v)^{n-1}. \tag{A12}$$

Hence, when γ grows, a bidder will reduce his bid if v is sufficiently close to $\tilde{v}$, while he will increase his bid if v is sufficiently close to $\overline{v}$.

Ad (iii): Note that in this case, we suppose $\gamma > 0$. Recall that $\tilde{v} = r$ in the standard model where $\varepsilon = \gamma = 0$, so that the amount of overbidding is

$$\begin{aligned} \Delta(v,\varepsilon,\gamma,r) &= b^F(v) - \left(v - \int_r^v \frac{F(w)^{n-1}}{F(v)^{n-1}}dw\right) \\ &= \varepsilon - \frac{1-F(v)^{n-1}}{F(v)^{n-1}}\gamma + \int_r^{\tilde{v}} \frac{F(w)^{n-1}}{F(v)^{n-1}}dw. \end{aligned} \tag{A13}$$

To see that the amount of overbidding is increasing, note that

$$\frac{d}{dv}\Delta(v,\varepsilon,\gamma,r) = (n-1)f(v)F(v)^{-n}\left(\gamma - \int_r^{\tilde{v}} F(w)^{n-1}dw\right), \tag{A14}$$

which is positive, because (using the fact that F is an increasing function)

$$\gamma - \int_r^{\tilde{v}} F(w)^{n-1} dw = [\tilde{v} - r + \varepsilon + \gamma] F(\tilde{v})^{n-1} - \int_r^{\tilde{v}} F(w)^{n-1} dw \tag{A15}$$
$$> [\tilde{v} - r + \varepsilon + \gamma] F(\tilde{v})^{n-1} - (\tilde{v} - r) F(\tilde{v})^{n-1} > 0.$$

Consider the case $r < \bar{r}$ and hence $r < \tilde{v}$, so that the smallest valuation of a bidder who participates both in the standard model (where $\varepsilon = \gamma = 0$) and in our model is $v = \tilde{v}$. Notice that in this case $b^F(\tilde{v})$ can be even smaller than $\underline{v}$. Specifically, $b^F(\tilde{v}) < \underline{v}$ holds whenever $r \leq \underline{v}$. There is underbidding at $v = \tilde{v}$, because (again using the definition of $\tilde{v}$)

$$\Delta(\tilde{v}, \varepsilon, \gamma, r) = \frac{1}{F(\tilde{v})^{n-1}} \left[(\varepsilon + \gamma) F(\tilde{v})^{n-1} - \gamma + \int_r^{\tilde{v}} F(w)^{n-1} dw \right] \tag{A16}$$
$$< \frac{1}{F(\tilde{v})^{n-1}} \left[(\varepsilon + \gamma) F(\tilde{v})^{n-1} - \gamma + (\tilde{v} - r) F(\tilde{v})^{n-1} \right] = 0.$$

Consider now the case $r \geq \tilde{v}$, so that the smallest valuation of a bidder who participates in the standard model as well as in our model is $v = r$. There is overbidding at $v = r$, because

$$\Delta(r, \varepsilon, \gamma, r) = \frac{1}{F(r)^{n-1}} \left[(\varepsilon + \gamma) F(r)^{n-1} - \gamma - \int_{\tilde{v}}^{r} F(w)^{n-1} dw \right] \tag{A17}$$
$$> \frac{1}{F(r)^{n-1}} \left[(\varepsilon + \gamma) F(r)^{n-1} - (r - \tilde{v}) F(r)^{n-1} - \gamma \right]$$
$$= \frac{1}{F(r)^{n-1}} (\tilde{v} - r + \varepsilon + \gamma) \left[F(r)^{n-1} - F(\tilde{v})^{n-1} \right]$$
$$= \frac{1}{F(r)^{n-1}} \frac{\gamma}{F(\tilde{v})^{n-1}} \left[F(r)^{n-1} - F(\tilde{v})^{n-1} \right] > 0.$$

Finally, at $v = \overline{v}$, there is overbidding, because

$$\Delta(\overline{v}, \varepsilon, \gamma, r) = \varepsilon + \int_r^{\tilde{v}} F(w)^{n-1} dw, \tag{A18}$$

which is obviously positive if $r < \tilde{v}$. Otherwise, if $r \geq \tilde{v}$, then there is overbidding at $v = r$ already.

Ad (iv): Note that in this case, we suppose $\gamma = 0$. Analogous to the proof of part (iii) and due to $\tilde{v} = r - \varepsilon$, in the present case the amount of overbidding is

$$\Delta(v, \varepsilon, \gamma, r) = \varepsilon - F(v)^{1-n} \cdot \int_{r-\varepsilon}^{r} F(w)^{n-1} dw. \tag{A19}$$

A similar reasoning as in part (iii) above implies that $\Delta(v, \varepsilon, \gamma, r)$ is weakly increasing in v (it is strictly increasing if $\int_{r-\varepsilon}^{r} F(w)^{n-1} dw > 0$). Consequently, $\Delta(v, \varepsilon, \gamma, r) = \varepsilon$ (and hence, independent of v) if $r \leq \underline{v}$. With respect to overbidding respectively underbidding, recall that in the present case, we have $r \geq \tilde{v} = r - \varepsilon$. Hence, the smallest valuation of a bidder who participates in the standard model as well as in our model is $v = r$. There is no underbidding at $v = r$, because

$$\begin{aligned} \Delta(r, \varepsilon, \gamma, r) &= \varepsilon - F(r)^{1-n} \cdot \int_{r-\varepsilon}^{r} F(w)^{n-1} dw \\ &\geq \varepsilon - F(r)^{1-n} \cdot [r - (r - \varepsilon)] \cdot F(r)^{n-1} = 0 \end{aligned} \tag{A20}$$

In combination with the above observations this implies that there in never underbidding.

Proof of Proposition 3

The first derivative of the seller's expected profit with respect to the threshold valuation

$\tilde{v}$ is given by

$$\frac{d\Pi}{d\tilde{v}} = -n\left(\tilde{v} - \frac{1-F(\tilde{v})}{f(\tilde{v})} + \varepsilon + \gamma\right) F(\tilde{v})^{n-1} f(\tilde{v}) + n\gamma f(\tilde{v}). \tag{A21}$$

The first-order condition of the seller's profit-maximization problem can thus be written as $\lambda(\tilde{v}^*) = \mu(\tilde{v}^*, \varepsilon, \gamma, n)$, where

$$\lambda(\tilde{v}) = \tilde{v} - \frac{1-F(\tilde{v})}{f(\tilde{v})} \text{ and } \mu(\tilde{v}, \varepsilon, \gamma, n) = \frac{1-F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1}}\gamma - \varepsilon. \tag{A22}$$

Note that given the monotone hazard rate assumption, there always exists a unique $\tilde{v}^* \in (\underline{v}, \overline{v})$ that satisfies the first-order condition. When $\tilde{v}$ moves from $\underline{v}$ to $\overline{v}$, then $\lambda(\tilde{v})$ strictly increases from $\underline{v} - 1/f(\underline{v})$ to $\overline{v}$, while $\mu(\tilde{v}, \varepsilon, \gamma, n)$ strictly decreases from $+\infty$ to $-\varepsilon$. Due to continuity, an intermediate value argument can thus be applied. The optimal reserve price can then be derived from $[\tilde{v}^* - r^* + \varepsilon + \gamma]F(\tilde{v}^*)^{n-1} = \gamma$, which can be rewritten as $\tilde{v}^* - r^* = \mu(\tilde{v}^*, \varepsilon, \gamma, n)$, so that $r^* = \tilde{v}^* - \lambda(\tilde{v}^*)$ must hold.

Proof of Proposition 4

From the implicit definition of the optimal threshold valuation $\tilde{v}^*$ in Proposition 3, it follows that

$$\frac{d\tilde{v}^*}{d\varepsilon} = -\frac{1}{\lambda_{\tilde{v}}(\tilde{v}^*) - \mu_{\tilde{v}}(\tilde{v}^*)} < 0, \tag{A23}$$

where subscripts denote partial derivatives. The denominator is positive, because the monotone hazard rate condition implies that $\lambda_{\tilde{v}}(\tilde{v}^*)$ is positive and $\mu_{\tilde{v}}(\tilde{v}^*) = -\gamma(n-1)F(\tilde{v}^*)^{-n}f(\tilde{v}^*) < 0$. Since $[1-F(\tilde{v}^*)]/f(\tilde{v}^*)$ is decreasing, this implies that $dr^*/d\varepsilon > 0$.

Similarly, it follows that

$$\frac{d\tilde{v}^*}{d\gamma} = \frac{1 - F(\tilde{v}^*)^{n-1}}{[\lambda_{\tilde{v}}(\tilde{v}^*) - \mu_{\tilde{v}}(\tilde{v}^*)]F(\tilde{v}^*)^{n-1}} > 0 \tag{A24}$$

and thus $dr^*/d\gamma < 0$.

Proof of Proposition 5

In the proof, we treat n as if it were a continuous variable. From the implicit definition of $\tilde{v}^*$ in Proposition 3 it follows that

$$\frac{d\tilde{v}^*}{dn} = \frac{\mu_n(\tilde{v}^*)}{\lambda_{\tilde{v}}(\tilde{v}^*) - \mu_{\tilde{v}}(\tilde{v}^*)} > 0, \tag{A25}$$

where the denominator is positive (see the proof of Proposition 4) and

$$\mu_n(\tilde{v}^*) = -\gamma F(\tilde{v}^*)^{1-n} \ln F(\tilde{v}^*) > 0. \tag{A26}$$

As a consequence, it follows that $dr^*/dn < 0$ must hold.

Participation and Bidding Strategies in Section VI

The bidding strategies described in Section VI follow from arguments analogous to those in the proofs of Propositions 1 and 2. Specifically, the central step to prove that $b^S(v) = \omega(v) - l(v)$ is as follows. Consider bidder i, who contemplates bidding $\tilde{b}$, where $\tilde{b} < t_i < b^s(v_i)$ and t_i is the largest amount bid by another buyer. Bidder i is better off bidding $b^s(v_i)$, because bidding $\tilde{b}$ yields a utility $l(v_i)$, while bidding $b^s(v_i)$ leads to a utility $\omega(v_i) - t_i > \omega(v_i) - b^s(v_i) = l(v_i)$. Regarding the definition of $\tilde{v}$, note that the expected payoff of a participating buyer is increasing, and the buyer with the smallest valuation for which it is not yet unprofitable to participate will win only if all other buyers have smaller valuations, so that $\omega(\tilde{v})F(\tilde{v})^{n-1} + l(\tilde{v})[1 - F(\tilde{v})^{n-1}] = 0$. Note that $\tilde{v} \in (\underline{v}, \overline{v})$ is uniquely defined, since $\omega(v)F(v)^{n-1}$ increases from 0 to $\omega(\overline{v}) > \overline{v}$ and $-(1-F(\tilde{v})^{n-1})l(v)$ decreases from $-l(\underline{v}) > 0$ to 0 when v moves from $\underline{v}$ to $\overline{v}$. Regarding the bidding strategies in the first-price auction, the central step is as follows. Let z be such that $b^F(z) = b$, consider buyer i and suppose all other buyers follow the postulated strategy. Buyer i's expected payoff from bidding b is $[\omega(v_i) - b^F(z)]F(z)^{n-1} + l(v_i)[1 - F(z)^{n-1}] = [v_i - z]F(z)^{n-1} + l(v_i) - l(\tilde{v}) + \int_{\tilde{v}}^{z} F(x)^{n-1}dx$, where the equality follows from the definition of $b^F(z)$ and the restriction $\omega(v) - l(v) = v + const$ that is imposed on the admissible functions $\omega(v)$ and $l(v)$. When bidder i bids $b^F(v_i)$, his expected payoff is $l(v_i) - l(\tilde{v}) + \int_{\tilde{v}}^{v_i} F(x)^{n-1}dx$. A deviation cannot be profitable, since

$$[v_i - z]F(z)^{n-1} + l(v_i) - l(\tilde{v}) + \int_{\tilde{v}}^{z} F(x)^{n-1}dx - [l(v_i) - l(\tilde{v}) + \int_{\tilde{v}}^{v_i} F(x)^{n-1}dx]$$

$$= \int_{v_i}^{z} [F(x)^{n-1} - F(z)^{n-1}]dx < 0. \quad \text{(A27)}$$

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