Unintended Hedging in Ambiguity Experiments*

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Abstract

In this note, we point out a potential hedging problem in standard Ellsberg-type experiments. This hedging problem may yield an incorrect classification of ambiguity averse subjects as expected utility-maximizers (or ambiguity loving) or vice versa. We demonstrate that subjects in these experiments can choose combinations of ambiguous options in a way that eliminates all ambiguity in expected payoffs, and we propose a new classification strategy for asymmetric Ellsberg urn that allows the identification of ambiguity averse subjects who employ hedging.

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1 Introduction

The number of ambiguity experiments using an Ellsberg (1961) urn has exploded in recent years.[1] In order to get clear evidence – within subject – for a violation of subjective expected utility (SEU), the experimenter must ask the subject *two* questions regarding the *same* Ellsberg urn. In this note, we point out a potential hedging problem when both questions are incentivised. In particular, we show that the standard questions asked in the Ellsberg 3-color experiment allow for hedging that would lead to an incorrect classification of ambiguity averse subjects as SEU-maximizers (or ambiguity loving) or vice versa. We show that the same problem exists for the 2-color Ellsberg urn and propose a new classification strategy for asymmetric Ellsberg urn that allows the identification of ambiguity averse subjects who employ hedging.

2 Vanishing Ambiguity in the 3-color Ellsberg urn

Consider a typical three-color Ellsberg urn experiment with 30 balls. Subjects are told that there are 10 green balls and that the remaining 20 balls can be either blue or yellow. Typically, subjects are now asked to make a decision between betting on green (G) and betting on, say, blue (B), where the "act" G means that a subject receives 1$ if a green ball is drawn and 0 otherwise (see Table 1). Then subjects are asked to make a second decision, namely betting on "not green" ($\neg G$) or on "not blue" ($\neg B$). The standard choice of ambiguity averse subjects would be $(G, \neg G)$, i.e. betting on "green" in the first decision and on "not green" in the second.

Table 1: Ellsberg 3-color experiment

	ball drawn		
bet (act)	green	blue	yellow
G	\$1	0	0
B	0	\$1	0
$\neg G$	0	\$1	\$1
$\neg B$	\$1	0	\$1

Indeed, viewed separately, betting on green is the only unambiguous option for the first decision, and betting on "not green" is the only unambiguous option for the second decision. However, viewed together, betting for and against the same color, no matter which of the colors, produces an unambiguous expected value. As an extreme case suppose first that subjects believe that only one ball is drawn, the color of this ball is relevant for both bets, and both bets are being paid out. Then, the combination of bets $(G, \neg G)$ yields a guaranteed fixed payment of 1$. But so does the bet $(B, \neg B)$.

[1] See Trautmann and van de Kuilen, 2013, and Oechssler and Roomets, 2013, for recent surveys.

A well designed ambiguity experiment would of course avoid this mistake. So let us suppose that two independent draws are made from the urn and that one of the two decisions is selected randomly (e.g. by a fair coin) for payment.[2] However, this procedure still does not solve the problem. In the following we show formally that a decision maker, who follows the α-MEU approach and who considers the two draws from the urn as independent from the coin toss, should be indifferent between the combination of bets $(G, \neg G)$ and $(B, \neg B)$ (or even $(Y, \neg Y)$).

Consider now combination bets and suppose that two independent draws are made from the urn and that one of the two decisions is selected randomly (e.g. by a fair coin) for payment.

A state in the experiment is described by a triplet listing the colors (g, b, or y) of the two balls drawn from the urn and the result of the coin toss determining whether the first or the second decisions are being paid out (1 or 2). Thus, in total the state space S contains 18 states,

$$S = \{g, b, y\} \times \{g, b, y\} \times \{1, 2\} = \{s_1, ..., s_{18}\},$$

listed in Table 2. For example, we denote state s_2 by $by1$ because in this state, ball b was drawn for the first decision, ball y was drawn for the second decision, and the coin decided that the first decision was paid out.

We only compare three bets (or acts) here although, of course, there are 9 different acts f. The three acts are $(G, \neg G)$, $(B, \neg B)$, and $(B, \neg G)$. The consequences (or payoffs) associated with these bets are also shown in Table 2. We assume that there is a utility function $u(\cdot)$ over consequences. Without loss of generality, we set $u(1) = 1$ and $u(0) = 0$.

As a benchmark, consider first a subjective expected utility (SEU) maximizer who must have a unique prior $\pi := (\pi_s)_{s=1,...,18}$ over S. In principle any probability distribution can be a prior but given the physical setup of the experiment we make the following plausible assumptions.

Assumption 1 *The draws from the two urns and the coin flip are considered as independent events.*

Assumption 2 *The probabilities for the coin flip are 50:50 and the probability of a green ball drawn from the urn is $p(G) = \frac{1}{3}$ because this composition was announced.*

The next assumption may appear innocuous but when applied to ambiguity this is not so clear.[3]

Assumption 3 *If a decision maker draws several times with replacement from what he believes to be the same probability distribution, then his beliefs from each drawing are the same and independent.*

[2] Azriely et al. (2012) discuss the conditions under which this procedure (RPS) is incentive compatible in experiments.

[3] We thank a referee for pointing this out.

Table 2: States, bets, and probabilities

S		$(B,\neg B)$	$(G,\neg G)$	$(B,\neg G)$	probabilities
s_1	$bb1$	1	0	1	$\pi_1 = p(B)^2\frac{1}{2}$
s_2	$by1$	1	0	1	$\pi_2 = p(B)\left(2/3 - p(B)\right)\frac{1}{2}$
s_3	$bg1$	1	0	1	$\pi_3 = p(B)\frac{1}{3}\frac{1}{2}$
s_4	$gg1$	0	1	0	$\pi_4 = \frac{1}{3}\frac{1}{3}\frac{1}{2}$
s_5	$gb1$	0	1	0	$\pi_5 = \frac{1}{3}p(B)\frac{1}{2}$
s_6	$gy1$	0	1	0	$\pi_6 = \frac{1}{3}\left(2/3 - p(B)\right)\frac{1}{2}$
s_7	$yb1$	0	0	0	$\pi_7 = \left(2/3 - p(B)\right)p(B)\frac{1}{2}$
s_8	$yy1$	0	0	0	$\pi_8 = \left(2/3 - p(B)\right)^2\frac{1}{2}$
s_9	$yg1$	0	0	0	$\pi_9 = \left(2/3 - p(B)\right)\frac{1}{3}\frac{1}{2}$
s_{10}	$bb2$	0	1	1	$\pi_{10} = p(B)^2\frac{1}{2}$
s_{11}	$by2$	1	1	1	$\pi_{11} = p(B)\left(2/3 - p(B)\right)\frac{1}{2}$
s_{12}	$bg2$	1	0	0	$\pi_{12} = p(B)\frac{1}{3}\frac{1}{2}$
s_{13}	$gg2$	1	0	0	$\pi_{13} = \frac{1}{3}\frac{1}{3}\frac{1}{2}$
s_{14}	$gb2$	0	1	1	$\pi_{14} = \frac{1}{3}p(B)\frac{1}{2}$
s_{15}	$gy2$	1	1	1	$\pi_{15} = \frac{1}{3}\left(2/3 - p(B)\right)\frac{1}{2}$
s_{16}	$yb2$	0	1	1	$\pi_{16} = \left(2/3 - p(B)\right)p(B)\frac{1}{2}$
s_{17}	$yg2$	1	0	0	$\pi_{17} = \left(2/3 - p(B)\right)\frac{1}{3}\frac{1}{2}$
s_{18}	$yy2$	1	1	1	$\pi_{18} = \left(2/3 - p(B)\right)^2\frac{1}{2}$

This seems particularly compelling if the draws are made from the same physical randomization device (e.g. when it is obvious to a subject in an experiment that the same urn is used twice). In the current context this implies that a decision maker who believes that the probability of drawing a blue ball once is $p(B)$, would believe that the probability of drawing (with replacement) a blue ball twice from the same urn is $p(B)^2$.

Given these assumptions, and given any probability distribution $p \in P$ with

$$P := \left\{ p = \left(\frac{1}{3}, p(B), \frac{2}{3} - p(B) \right) : 0 \leq p(B) \leq \frac{2}{3} \right\}$$

on the outcome of a single draw from the urn, a SEU must therefore have a prior of the form $\pi = \frac{1}{2}(p \otimes p)$. The last column of Table 2 lists the resulting probabilities for each of the 18 possible states.

A SEU maximizer with a unique prior π would evaluate a bet f in the usual way

$$SEU(f) = \sum_{s \in S} \pi_s u(f(s)).$$

Consider for the moment what would happen if a SEU maximizer violated Assumption 3. When evaluating bet $(B,\neg B)$, he could then have one prior for the first draw (in the extreme, this could be $p(B) = 0$) and another prior for the second draw (e.g. $p'(B) = 2/3$), which would make the combination bet $(B, \neg B)$ very unattractive, $SEU(B, \neg B) = 1/6$. However, given that the two draws come from the same urn with replacement, clearly this does not make much sense. In fact, we find it very reasonable to impose Assumption 3 not just for SEU maximizers but for every rational decision maker, including ambiguity averse ones.[4]

When we turn to ambiguity, modelling the situation becomes slightly more intricate. Intuitively, our Assumption 3, when applied to an ambiguity averse subject in an Ellsberg setting who mistrusts the experimenter, can be thought of as follows. The subject believes that the experimenter, having perhaps anticipated his decision, will have filled the urn in whatever way makes the subject worst off, but cannot affect the outcome beyond that. Without Assumption 3, the subject would believe that the experimenter could not only fill the urn to the subject's detriment, but also either tamper with the urn between draws, or tamper with the draws themselves (such that $p(B)$ was no longer directly tied to the number of blue balls in the urn). Clearly the second subject displays not merely ambiguity aversion but an additional irrationality (or severe mistrust) that the first subject does not. We will differentiate between ambiguity aversion (loving) that does and does not satisfy Assumption 3 as "rational" and "superstitious" ambiguity aversion (loving), respectively.[5]

For non-SEU maximizing subjects, we need to make the following additional assumptions.

Assumption 4 *The order in which the balls are drawn and the coin is tossed does not matter.*

This assumption is implied by the Reversal of Order axiom (see e.g. Seo, 2009).[6] For SEU maximizers the latter is implied by the independence axiom.

It seems pretty intuitive to assume that the two balls and the coin toss are independent from each other.[7] Thus we would like to retain Assumption 1. However, in the presence of ambiguity, the notion of independence is no longer clear. Depending on how ambiguity is modeled, different concepts of independence arise. Here, we adopt the notion of independence suggested in Gilboa and Schmeidler (1989, p. 150).[8]

Assumption 5 *For rational ambiguity attitudes, the set of priors $\mathcal{P}$ is the set of Gilboa-Schmeidler-independent product measures,*

$$\mathcal{P} := \text{co}\left\{\frac{1}{2}(p \otimes p), p \in C\right\},$$

[4]Accoding to the multiple prior approach, decison makers can, of course, have different priors for different acts. However, if one considers a combination bet like $(B, \neg B)$ as one act, as we do here, then one cannot pick different probabilities $p(B)$ for the two sub-acts B and $\neg B$.

[5]We do not want to suggest Assumption 3 as a normative assumption for all ambiguity averse subjects. However, we would submit that there are rational subjects who react to the "basic acts" B and Y in an Ellsberg urn with ambiguity aversion but nevertheless satisfy Assumption 3.

[6]In fact, if the coin toss comes last in the experiment, the reversal of order axiom is not required.

[7]Whether subjects in an experiment actually consider these events to be independent is another matter.

[8]Bade (2008) provides a discussion of alternative ways for defining independence of sets of priors.

for some closed and convex set $C \subseteq P$.

The set of priors is thus the closed convex hull of (a subset of) the priors a SEU maximizer could have. Alternatively, for superstitious ambiguity attitudes the priors would instead be

$$\mathcal{P} := \mathrm{co}\left\{\frac{1}{2}(p \otimes p'); p, p' \in C\right\}.$$

We model ambiguity–averse subjects (rational as well as superstitious) using the α-MEU approach (for an axiomatization see Ghirardato et al. 2004). A decision maker whose preferences are described by α-MEU evaluates a bet f by

$$\alpha MEU(f) = \alpha \min_{\pi \in \mathcal{P}} \sum_{s \in S} \pi_s u(f(s)) + (1 - \alpha) \max_{\pi \in \mathcal{P}} \sum_{s \in S} \pi_s u(f(s)), \tag{1}$$

for $\alpha \in [0, 1]$. The maxmin expected utility (MEU) model of Gilboa and Schmeidler corresponds to the case of $\alpha = 1$, and is thought to model the behavior of an ambiguity averse decision maker. The optimistic case of $\alpha = 0$ would correspond to ambiguity loving behavior.

Proposition 1 *Rational-α-MEU–maximizers and SEU-maximizers are indifferent between the combinations of bets $(G, \neg G)$ and $(B, \neg B)$ (or $(Y, \neg Y)$).*[9]

Proof. Using equation 1 and if Assumption 3 holds, then by the probabilities in Table 2 we obtain

$$\alpha MEU(G, \neg G) = \alpha MEU(B, \neg B) = \alpha MEU(Y, \neg Y) = \frac{1}{2}$$

for any $\alpha \in [0, 1]$.

Furthermore,

$$SEU(G, \neg G) = SEU(B, \neg B) = SEU(Y, \neg Y) = \frac{1}{2}.$$

□

At least two kinds of misclassification can happen as a result of this.

- Suppose a subject chose $(G, \neg G)$. By the usual interpretation he would be considered ambiguity averse. Yet he could just as well be a SEU-maxizer who subscribes to the principle of insufficient reason and therefore believes that $p(B) = p(Y) = \frac{1}{3}$.

[9] We point out that superstitious-α-MEU-maximizers do not share this indifference.

- Suppose a subject chose $(B, \neg B)$. By the usual interpretation he would be considered ambiguity loving (absent our additional specification). Yet by the above proposition, a rational ambiguity averse person could optimally choose $(B, \neg B)$. Meanwhile, for plausible $\mathcal{P}$, a rational ambiguity loving person would get a higher utility by betting on $(B, \neg G)$.

Is there thus no way of distinguishing a rational ambiguity averse subject from a SEU-maximizer? We would argue there is, although Kuzmics (2013) would disagree (at least under two additional assumptions). The next section presents a suggestion.

3 An asymmetric 3-color Ellsberg urn

Consider a three-color Ellsberg urn experiment with 30 balls and assume subjects employ hedging. The number of green balls is now 9 and the remaining 21 balls can be either blue or yellow. This type of tie-braking has been used in a number of experiments.[10] However, we suggest a novel way of classifying subjects. As before, $\alpha MEU(G, \neg G) = \alpha MEU(B, \neg B) = \alpha MEU(Y, \neg Y) = \frac{1}{2}$. A rational ambiguity averse person ($\alpha = 1$) would thus be indifferent between these bets and strictly prefer them to any other combination. For a SEU-maximizer we obtain

$$\begin{aligned} SEU(Y, \neg G) &= \frac{1}{2}(1 - p(B) - \frac{9}{30}) + \frac{1}{2}\frac{21}{30} \geq \frac{1}{2} \Leftrightarrow p(B) \leq \frac{12}{30} \\ SEU(B, \neg G) &= \frac{1}{2}p(B) + \frac{1}{2}\frac{21}{30} \geq \frac{1}{2} \Leftrightarrow p(B) \geq \frac{9}{30}. \end{aligned}$$

Thus, a SEU-maximizer would strictly prefer either $(Y, \neg G)$ or $(B, \neg G)$ to $(G, \neg G)$, to $(B, \neg B)$, and to $(Y, \neg Y)$ for all priors $p(B)$. But this implies that we can distinguish a SEU-maximizer from a rational ambiguity averse decision maker.

4 The 2-color Ellsberg urn

A very similar argument as in Section 2 can be made for the 2-color Ellsberg urn. Consider two urns with 20 balls, urn K with a *known* distribution of 10 black and 10 white balls, and urn U with an *unknown* proportion of white and black balls. In experiments, usually subjects are asked two questions: 1) to bet on a white ball from urn K (denoted as wK), or on a white ball from urn U (wU). And 2) to bet on a black ball from urn K, or on a black ball from urn U. The choice that is considered typical for ambiguity averse subjects is to choose in both cases a ball from urn K.

Suppose again that questions are incentiviced by throwing a coin to decide which question is being paid out. If subjects consider this as a joint decision, decide according to α-MEU, and satisfy Assumption 3, they will be indifferent between the

[10] See e.g. Cohen et al., 1985; Curley et al., 1986; Einhorn and Hogarth, 1986; Curley and Yates, 1989, Charness et al., 2013.

(unambiguous) combination bets (wK, bK) and (wU, bU). A subject who is rational ambiguity loving would not choose (wU, bU), but rather (wU, bK) or (wK, bU), depending on his set of priors.

5 A more worrisome scenario

So far, the hedging we have discussed is entirely an artifact of experimental design decisions (generally, lotteries imposed by the payment method). However, if we assume as in Kuzmics (2013) that subjects can conduct random lotteries in their head, and that they can commit to act based on the outcomes of these lotteries,[11] then we cannot be sure that subjects who choose $(B, \neg G)$ face ambiguity because this choice could be an outcome of mixing 50:50 between $(B, \neg G)$ and $(G, \neg B)$, which would produce an unambiguous expected value. Clearly, this issue would greatly confound our ability to classify subjects.

Fortunately, it is not necessarily true that subjects can randomize and commit as assumed by Kuzmics (2013). Further, it is not clear, even if they can, that they will recognize and implement these types of lotteries. Still, we believe it is important to recognize that the possibility exists and to understand the implications.

6 Conclusion

We do not want to suggest that most subjects are willing or able to apply the hedging argument. Subjects may actually treat all decisions in an experiment as independent from each other (this is the so-called isolation effect, see Kahneman and Tversky, 1979), or they may be "superstitious." However, we want to point out that there is a potential problem of misclassifying some subjects as ambiguity averse when in fact they are ambiguity neutral, or classifying subjects as ambiguity loving when in fact they are ambiguity averse or neutral. A possible solution to this problem would be letting subjects make just one decision in the whole experiment to prevent hedging. However, strictly speaking, this also prevents identifying a single subject as ambiguity averse since two questions are necessary for this. Thus experimenters necessarily face a trade–off when choosing an experimental design. We hope that our discussion allows experimenters to make more informed decisions.

References

[1] Azrieli, Y., C. P. Chambers, and P.J. Healy. 2012. Incentives in Experiments: A Theoretical Analysis. Mimeo.

[2] Bade, S. 2008. "Stochastic Independence with Maxmin Expected Utilities", mimeo, Penn State University.

[11] These are Kuzmics' (2013) axioms of "creative randomization" and "willpower", respectively.

[3] Charness, G., E. Karni, and D. Levin. 2013. Ambiguity attitudes and social interactions: An experimental investigation, *Journal of Risk and Uncertainty,* 46 (1), 1-25.

[4] Cohen, Michèle, Jean-Marc Tallon, and Jean-Christophe Vergnaud. 2011. An Experimental Investigation of Imprecision Attitude, and Its Relation with Risk Attitude and Impatience. *Theory and Decision*, 71: 81-109.

[5] Curley, Shawn P., J. Frank Yates, and Richard A. Abrams. 1986. Psychological Sources of Ambiguity Avoidance. *Organizational Behavior and Human Decision Processes*, 38: 230-256.

[6] Curley, S. P., & Yates, J. F., 1989. An empirical evaluation of descriptive models of ambiguity reactions in choice situations. Journal of Mathematical Psychology, 33(4), 397-427.

[7] Einhorn, Hillel J., and Robin M. Hogarth. 1986. Decision Making under Ambiguity. *Journal of Business*, 59: S225-S250.

[8] Ellsberg, Daniel. 1961. Risk, Ambiguity and the Savage Axioms. *Quarterly Journal of Economics*, 75: 643-669.

[9] Gilboa, I. and D. Schmeidler. 1989. Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics*, 18, 141-153.

[10] Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci. 2004. Differentiating ambiguity and ambiguity attitude. *Journal of Economic Theory*, 118, 133–173.

[11] Kuzmics, Christoph. 2013. A rational ambiguity averse person will never display her ambiguity aversion. Mimeo, University of Bielefeld.

[12] Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decisions under risk. *Econometrica* 47, 263–291.

[13] Oechssler, Jörg and Alex Roomets. 2013. A Test of Mechanical Ambiguity. Mimeo, University of Heidelberg.

[14] Seo, Kyoungwon. 2009. Ambiguity and Second-order Belief. *Econometrica*, 77 (5), 1575–1605.

[15] Trautmann, Stefan T. and Gijs van de Kuilen. 2013. Ambiguity Attitudes. Mimeo, Tilburg University.