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Hedging and Ambiguity

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Abstract

We run an experiment that gives subjects the opportunity to hedge away ambiguity in an Ellsberg-style experiment. Subjects are asked to make two bets on the same draw from an ambiguous urn, with a coin flip deciding which bet is paid. By modifying the timing of the draw, coin flip, and decision, we are able to test the reversal-of-order axiom, particularly as it relates to the ability of the Random-Lottery Incentive System (RLIS) to prevent cross-task contamination in an ambiguity setting. We find that we cannot reject that the reversal-of-order axiom holds. This suggests that hedging could still be possible when carefully implementing RLIS. However, we also find low levels of ambiguity hedging across the board, suggesting the existence of the hedging possibility does not necessarily represent a common problem in ambiguity experiments.

JEL codes: C91, C72, D74.
Keywords: Ellsberg paradox, hedging, reversal of order axiom, experiment.

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1 Introduction

As a thought experiment, the Ellsberg paradox has given rise to a huge theoretical literature on ambiguity.[1] When it comes to implementing the Ellsberg paradox in an incentivized experiment, however, there remain some important challenges. In particular, it is still unclear how to pay subjects for multiple decisions without allowing for hedging.[2] Consider a standard two-color Ellsberg urn consisting of an unknown number of blue and yellow balls and suppose a subject is asked to bet 5 euro on the outcome of one draw from this urn. If he is ambiguity averse, he may attach a low value to betting on a blue ball. In isolation, the same would happen if he is asked to bet on a yellow ball. However, if he is asked to bet twice and both bets are paid (and only one ball is drawn per urn), the subject may combine the two bets and realize that if he bets once on blue and once on yellow, he can guarantee himself a fixed payment of 5€. This already shows that it not a good idea to pay for both questions in this setting.

The Random-Lottery Incentive System (RLIS) was designed to address cross-task-contamination problems (see e.g. Starmer and Sugden, 1991). The RLIS pays for one randomly chosen decision (e.g. chosen by a coin toss). However, as pointed out recently by Oechssler and Roomets (2014), Bade (2015), and Kuzmics (2015) this does not help much in ambiguity settings if the coin toss that determines the decision to be paid out comes at the end of the experiment. In this case, betting once on blue and once on yellow guarantees in total the (objective) 50:50 lottery of winning 5€. This way, all ambiguity is hedged away, an observation which recalls Raiffa's (1961) critique of Ellsberg's experiment.[3] In fact, if Anscombe and Aumann's (1963) "Reversal of order" axiom is assumed, the hedge works regardless of the order in which the urn draw and the coin toss are performed, since the axiom states that it is immaterial for a decision maker whether the coin is tossed first and then the ball is drawn from the urn, or vice versa.

Azrieli et al. (2015) conduct a very careful theoretical analysis of the conditions under which the RLIS is incentive compatible. They show that one of the few conditions under

[1] Schmeidler (1989), Gilboa and Schmeidler (1989), Klibanoff et al. (2005) to name just a few. There is also a large experimental literature, recently surveyed by Trautmann and van de Kuilen (2016) or Oechssler and Roomets (2015).

[2] Most ambiguity experiments involve at least two decisions for each subject in order to establish a contradiction to expected utility theory.

[3] An important difference to Raiffa's critique is that Raiffa imagined that a decision maker randomizes in his head. A frequent counterargument is that there is a commitment problem: once the decision makers performed the randomization, he faces the ambiguity again. With the RLIS, the randomization and the commitment device is provided by the experimenter.

which the RLIS is not incentive compatible is under ambiguity when the reversal of order axiom is maintained. However, the axiom need not be satisfied (see e.g. Seo, 2009, and Saito, 2015, for criticism and alternatives to the reversal of order axiom). Baillon et al. (2015) depart from the reversal of order axiom and show that, theoretically, incentive compatibility can be rescued when the coin is tossed (but not revealed) before decisions are taken. Before we can embrace this procedure for the practical design of experiments, two issues should be tested. (1) Does the order of coin toss, urn draw, and decision matter at all in experiments? So far, to our knowledge there is no experimental test of the reversal of order axiom. (2) Do subjects actually hedge, in particular, when the opportunity is presented to them on a silver platter? This question is important to all experimenters who worry about unintended hedging opportunities in ambiguity experiments.

In this paper we address those two questions using an experiment. The experiment asks subjects to make two bets on a single draw from an Ellsberg urn, only one of which is actually employed. A coin flip is used to decide which of the two bets to employ (the other becomes irrelevant). We then manipulate the order of the decision, the urn draw, and the coin flip. We find that (1) the reversal of order axiom seems to hold. Subjects are not significantly influenced by the order of coin flip, urn draw, and decision. And (2) the majority of subjects do not take up the opportunity to hedge (confirming Dominiak and Schnedler, 2011).

2 Experimental design

In our experiment, subjects had to make bets on the outcomes of a fair coin and a draw from an urn with 24 balls.[4] The urn contained blue and yellow balls in a composition that was unknown to subjects. Subjects were told that any combination from 0 blue balls (and 24 yellow balls) to 24 blue balls (and 0 yellow balls) was possible.

In particular, subjects had to place one bet on the color of the drawn ball if the coin came up heads and one bet if it came up tails (see Appendix A for the decision sheet). This implied that subjects had the choice among four alternatives as illustrated in Table 1. Each subject just made one such (combined) decision. If the correct color was predicted, they received 5€. If not, they received nothing (apart from the show-up fee of 3€, which everyone received). It was important that subjects faced just one incentivized question

[4] In the actual experiment, we used a non-transperant bag and blue and yellow marbles. For expositional reasons, we employ the more customary urns and balls in the text.

in the whole experiment in order to avoid any confounding factors like hedging or income effects.[5]

Table 1: Bets and payoffs

possible choices	coin shows Head		coin shows Tail	
	ball is blue	ball is yellow	ball is blue	ball is yellow
blue if Head & blue if Tail	5€	0€	5€	0€
blue if Head & yellow if Tail	5€	0€	0€	5€
yellow if Head & blue if Tail	0€	5€	5€	0€
yellow if Head & yellow if Tail	0€	5€	0€	5€

Notice that betting on either "blue if Head & blue if Tail" or "yellow if Head & yellow if Tail" results in an ambiguous payoff (determined by the distribution of blue and yellow balls). Meanwhile, betting on either "blue if Head & yellow if Tail" or "yellow if Head & blue if Tail" results in the same objective 50:50 lottery ("roulette wheel"), regardless of which ball is drawn. The latter two options, then, allow for a rather clear opportunity for subjects to hedge away all ambiguity (but not risk). See Section 3 for more details on this issue.

The treatments of our experiment differed with respect to the timing in which the four design elements listed in Table 2 were performed. Accordingly, the three treatments are called DecDrawFlipReveal (first, subjects make a decision, then the ball is drawn from the urn, then the coin is flipped, then the coin is revealed), DecFlipRevealDraw, and FlipDecRevealDraw.

Table 2: Elements of the design

element	description
Coin Flip	A coin is flipped under a cup
Flip Reveal	The result of the coin flip is revealed to subjects
Ball Draw	A ball is drawn from the urn and revealed to subjects
Decision	Subjects make bets contingent on coin flip and ball draw

Instructions (see Appendix A) were written on paper and distributed at the beginning of each session. The draws from the urn and the tosses of coins were performed by different

[5] However, this also meant that we had to forgo the opportunity to gather additional information, e.g. on subjects risk or ambiguity attitudes.

subjects. The urns and the cup were on display during the experiment, so that subjects could be certain that their contents could not be manipulated. Subjects were allowed to verify the urns' contents after the experiment, and some did. Participants were invited from a database using ORSEE (Greiner, 2015). The experiment was conducted in the AWI-lab at the University of Heidelberg, using pen and paper. For each of the three treatments, we have 60 independent observations (58 in DecFlipRevealDraw due to no-shows). Experiments lasted about 30 minutes, including instruction time. Average earnings from the experiment amounted to approximately 5.50 euro.

3 Hypotheses

Saito (2015) nicely illustrates the situation subjects face in our experiment with diagrams like the ones shown in Figure 1. The left tree shows a situation as in our DecDrawFlipReveal treatment. First, the winning ball is drawn from the urn, then the coin is tossed. By choosing to bet on "blue if Head & yellow if Tail" (from now on shortened to "by") or "yellow if Head & blue if Tail" ("yb") the subject can guarantee himself an objective lottery of 50:50 for winning 5€. All ambiguity is hedged away. Regardless of whether a blue ball or a yellow ball is drawn, the subject receives the same Anscombe-Aumann act.[6] If, on the other hand, the subject chooses blue (or yellow) regardless of the coin toss (as in the center tree), then the ambiguity about the number of blue and yellow balls is still very much present. Thus, our first hypothesis is that subjects recognize and prefer the opportunity to hedge.

Hypothesis I Subjects will choose a bet that allows them to hedge (i.e. either "yb"or "by") in treatment DecDrawFlipReveal.

The right panel in Figure 1 shows the situation in our treatment DecFlipRevealDraw. Arguably, after the coin is tossed, the subject faces again an ambiguous situation.[7] Whether or not subjects consider the situation in the left and the right panel as different is an important question that has not been studied experimentally. Theoretically, it depends on whether the Reversal of Order axiom holds. "[The RoO axiom says that] if the prize you

[6]Thus we implicitly assume the Anscombe-Aumann framework. Eichberger and Kelsey (1996) show that in a Savage framework a preference for randomization need not exist.

[7]It is often argued (see e.g. Saito, 2015) that the subject faces a commitment problem after letting the coin decide on which ball to bet. This is correct but we solved this problem for our subjects by providing automatic commitment.

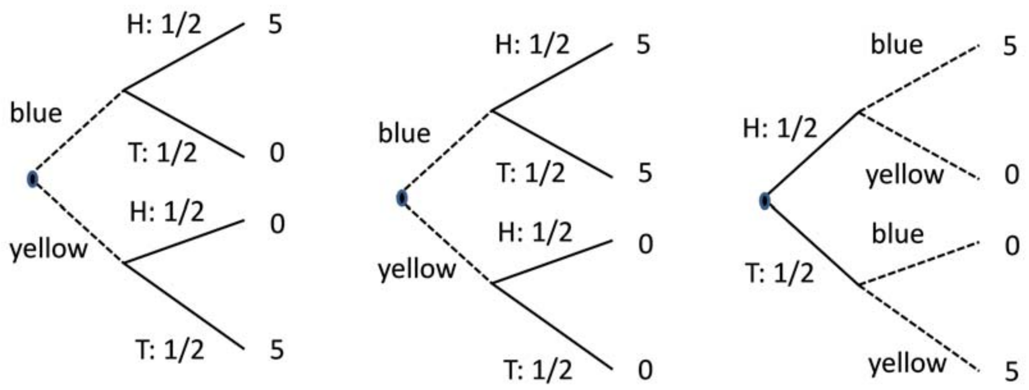

Figure 1: Left panel: The situation induced by betting on "by" in treatment DecDrawFlipReveal. Center panel: betting on "bb" in treatment DecDrawFlipReveal. Right panel: betting on "by" in treatment DecFlipRevealDraw.

Note: The solid lines correspond to the risk introduced by flipping a coin, while the dotted lines correspond to the ambiguity of the color of the drawn ball.

receive is to be determined both by a horse race and the spin of a roulette wheel, then it is immaterial whether the wheel is spun before or after the race." (Anscombe and Aumann, 1963, p. 201). Both Seo (2009) and Saito (2015) express doubts whether the RoO axiom holds and develop their theories under the assumption that it does not. This leads us to

Hypothesis II Subjects will hedge less in treatment DecFlipRevealDraw than in DecDrawFlipReveal.

Predictions for our treatment FlipDecRevealDraw are less clear cut. Baillon et al. (2015) argue that this sequence makes the RLIS incentive compatible. One argument would be that, given that the coin is already tossed (but not revealed), when subjects make their decisions, subjects face a fully ambiguous decision. This implies that hedging would be even less of an issue. Thus, we propose

Hypothesis III Subjects will hedge less in treatment FlipDecRevealDraw than in DecFlipRevealDraw.

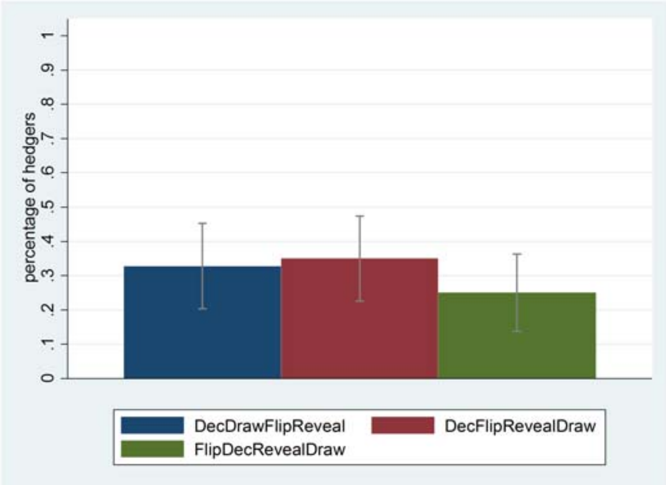

Figure 2: Percentage of hedgers by treatment.
Note: Error bars show 95% confidence intervals based on binomial distributions. Subjects are counted as hedgers if they chose bets "by" or "yb".

4 Results

The results of our experiment can easily be summarized in Figure 2, which shows the percentage of hedgers across treatments.[8] A subject is counted as hedger if he chose "blue if Head & yellow if Tail" or "yellow if Head & blue if Tail".

With respect to Hypothesis I, we find little support. In treatment DecDrawFlipReveal only roughly one third of subjects chose to hedge, a level of hedging which is significantly below 50% (we would expect 50% hedging if subjects chose randomly). A binomial test rejects the null hypothesis that the percentage of hedgers is 50% ($p < 0.05$, two-sided). This suggests an aversion to hedging and supports the finding of Dominiak and Schnedler (2011).[9] Of course some people may still be intentionally hedging, but it would appear that these people are not in the majority even when the opportunity to hedge is as obvious as in our investigation. Moreover, since we avoid the commitment problem (see footnote 7) and since random selection would result in 50% hedging, a level of hedging below 50%

[8]A more detailed analsysis along with a control treatment that controls for order effects can be found in Appendix B.

[9]Dominiak and Schnedler (2011) elicited the willingness to pay for various bets. Their "chameleon ticket" is comparable to the "by" bet in our DecFlipRevealDraw treatment. However, Dominiak and Schnedler (2011) did not vary the timing of the coin toss and the draw from the urn.

is actually rather striking.

With respect to Hypotheses II and III, we also find very little support. The RoO axiom seems to hold, as there is no significant difference in the percentage of hedgers between DecFlipRevealDraw and DecDrawFlipReveal (χ^2-tests, $p = 0.80$). Likewise, there are no significant differences across the other treatments using pairwise or joint χ^2-tests ($p > 0.23$). When the coin is flipped before the decision, as in treatment FlipDecRevealDraw, we find a slightly lower percentage of hedgers. However, this difference is not significant.

5 Conclusion

The Random-Lottery Incentive System (RLIS) was designed to prevent spillovers from one decision in an experiment to another. For ambiguity experiments, several papers (Oechssler and Roomets, 2014, Bade, 2015, Kuzmics, 2015) have argued that this may not be fully successful. Potentially, the typical Ellsberg urn experiment allows subject to hedge away ambiguity if they combine several decisions. This may or may not depend on the order in which the “horse race” (draw from ambiguous urn) and the “roulette wheel” (risky coin toss) are performed. For the latter question, the empirical validity of Anscombe and Aumann’s (1963) Reversal of Order axiom is crucial.

In this paper we addressed two questions using an experiment. (1) Does the order of coin toss, urn draw, and decision matter at all in experiments? To our knowledge this is the first experimental test of the reversal of order axiom. (2) Do subjects actually hedge by combining several decisions in an experiment?

We found that (1) the reversal of order axiom seems to hold. Subjects were not significantly influenced by the order of coin flip, urn draw, and decision. And (2) the majority of subjects did not take up the opportunity to hedge, even when the opportunity was presented to them on a silver platter. Thus, it seems that experimenters do not need to worry too much about potential hedging opportunities, even if they are theoretically interesting.

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Appendix (for online publication only)

A Instructions

[English translation of the German instructions for Treatment FlipDecRevealDraw. The other treatments were modified in the obvious way.]

Instruction Sheet

Welcome to our experiment and thanks for participating.

Please turn off your mobile phone and do not communicate with other participants. If you have any questions, just raise your hand and we will come to you to answer them.

All participants who observe the rules will definitely receive a payment of 3 euros for participation. You can earn an additional 5 euros, depending on your decisions and on the outcome of a lottery. At the end of the experiment you will receive your total payment in cash, with anonymity assured.

On the table the experimenters have a bag containing 24 marbles, each of them either blue or yellow. You do not know how many marbles are blue and how many are yellow. Any combination, from 0 blue marbles (that is, 24 yellow marbles), to 24 blue marbles (that is, 0 yellow marbles) is possible. At the end of the experiment you are free to check the content of the bag.

On the table there is also a dice cup containing a fair coin.

The experiment is executed as follows:

1. A randomly selected participant shakes the dice cup containing the coin.
2. On the decision form you make two decisions, each requiring a choice between two options.
3. The result of the coin flipping (heads or tails) is communicated.
4. Another randomly selected participant draws a marble from the bag without looking at the bag.
5. The color of the marble is communicated.
6. You receive your payoff in accordance with the payment table given below

Your payoff depends on the color of the drawn marble, on the result of the coin flipping, and on the decisions you have made. For example, if you decide on "blue if head" and "yellow if tail", you get 5 euros (in addition to the 3 euros guaranteed) if the drawn marble is blue and the coin shows "head" or if the drawn marble is yellow and the coin shows "tail". If you choose "blue if head" and "blue if tail", then you win 5 euros if the drawn marble is blue and the coin shows "head" or "tail". The two other options are defined accordingly.

The combinations are shown in the payment table below (additionally you receive 3 euros for participating):

Options	Coin shows Head		Coin shows Tail	
	Marble is blue	Marble is yellow	Marble is blue	Marble is yellow
blue if Head & blue if Tail	5€	0€	5€	0€
blue if Head & yellow if Tail	5€	0€	0€	5€
yellow if Head & blue if Tail	0€	5€	5€	0€
yellow if Head & yellow if Tail	0€	5€	0€	5€

Decision Sheet

Please select exactly one of the two options in case the coin shows head:

☐ Blue if “head”

☐ Yellow if “head”

Please select exactly one of the two options in case the coin shows tail:

☐ Blue if “tail”

☐ Yellow if “tail”

Please answer the following two questions:

Your gender:

☐ Female

☐ Male

Your field of study:

☐ Economics (e.g. Political Economy, Business Economics, ...)

☐ Other

After making your decisions, submit the Decision Sheet to the experimenters. Make sure you do not give away the instruction sheet. At the top is printed the Participant No., which you will need to pick up your payoff afterwards.

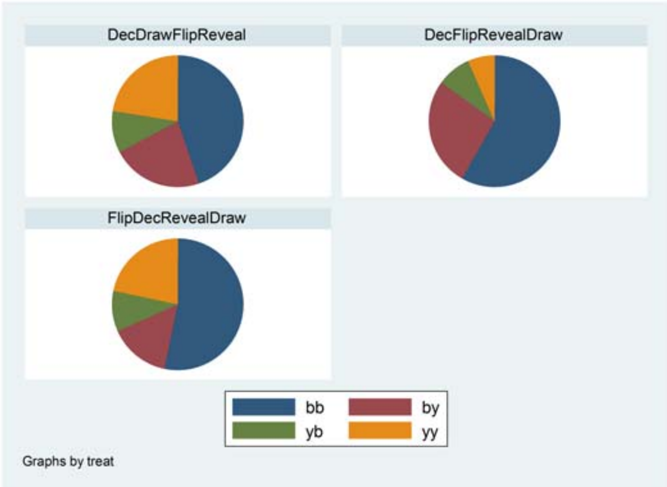

Figure 3: Color choice by subjects; e.g. “yb” means chosing color yellow in th first block (if “head”) and blue in the second (if “tail”).

B Robustness treatment

When analyzing, in more detail, the choice in our three main treatments, a distinct order effect is noticeable. In the decision sheet (see Appendix A), subjects were presented with a 2 blocks of 2 questions. The color blue always came before the color yellow. Figure 3 shows the proportions of subjects choosing the two colors in the two blocks. In all treatments, subjects chose “bb” much more often than “yy” and “by” more often than “yb”. It seems plausible that most subjects who did not want to hedge were indifferent between “bb” and “yy” and chose “bb” because it came first on the questionnaire. Similarly, subjects who wanted to hedge were likely indifferent between “by” and ”yb”. Nevertheless, our order could have introduced a bias, since the non-hedging choice “bb” is the default choice if a subject always chooses the first option on the questionnaire.

To control for this possible bias, we ran a control treatment (DecDrawFlipReveal*), with 57 new subjects, that was identical to DecDrawFlipReveal except that the decision sheet posed the questions in a different order:

Please select exactly one of the two options in case the coin shows head:

☐ Yellow if “head”

☐ Blue if “head”

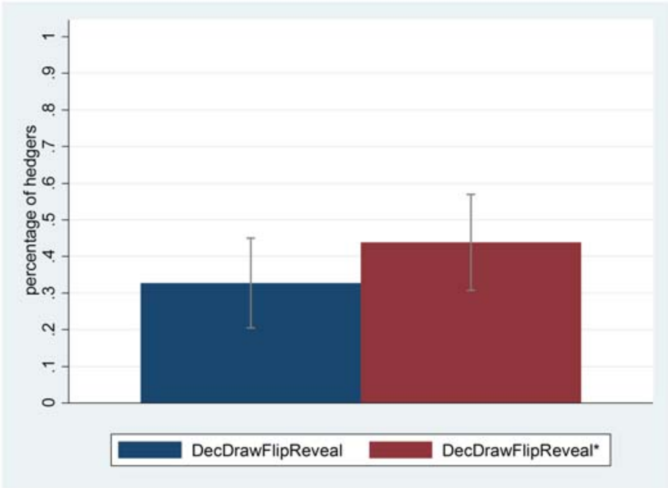

Figure 4: Percentage of hedgers when "bb" is presented first in questionnaire (left bar) or "yb" is presented first (right bar)

Please select exactly one of the two options in case the coin shows tail:

☐ Blue if "tail"

☐ Yellow if "tail"

In treatment DecDrawFlipReveal*, hedging becomes the default choice if a subject always chooses the first option.

Figure 4 compares the proportion of hedgers in treatments DecDrawFlipReveal* and DecDrawFlipReveal. Although hedging increased from 33% to 44%, there is no significant difference according to a χ^2 test ($p = 0.22$). Thus, it seems there may some focality in our questionnaire design, however, any effect seemed to be minor, as it did not significantly impact the results. Importantly, even by making hedging a more focal option in the questionnaire, we still saw less than 50% hedging in absolute terms.