University of Heidelberg

Department of Economics

Discussion Paper Series | No. 510

Population Aging, the Composition of Government Spending, and Endogenous Economic Growth in Politico-Economic Equilibrium

Johanna Kuehnel

February 2011

Population Aging, the Composition of Government Spending, and Endogenous Economic Growth in Politico-Economic Equilibrium

Johanna Kuehnel[1]

University of Heidelberg

Abstract: This paper introduces a democratic voting process into an OLG economy in order to analyze the effects of a rising old-age dependency ratio on the composition of government spending and endogenous economic growth. Forward-looking agents vote each period on the public policy mix between productive government expenditure and public consumption spending that benefits the elderly. Population aging shifts political power from the young to the old. While this does not affect public productive expenditure, it leads to an increase in public spending on the elderly and a slowdown in economic growth. However, the overall effect on long-term economic growth is positive. This is due to reduced capital dilution or increased saving.

Keywords: Demographics, Endogenous Economic Growth, Government Spending, Markov Perfect Equilibrium, Probabilistic Voting

JEL-Classification: D72, E62, O41,

This Version: February 9, 2011

[1]Address: Department of Economics, University of Heidelberg, Bergheimer Str. 58, D-69115 Heidelberg, Germany, E-mail address: kuehnel@uni-hd.de.

1 Introduction

Population aging, i. e., the shift in the distribution of a country's population towards older ages, is one of the most important demographic phenomena of our time. It will neither be confined to the West nor to industrialized economies. Table 1 presents actual data and forecasts of the old-age dependency ratio for selected countries and regions based on data from the United Nations.[1] Roughly speaking, between 2005 and 2050 this ratio is forecasted to double in Europe and Northern America. In Japan, India, Brazil, and Chile its estimated increase is even greater.

Table 1: Old-Age Dependency Ratios in Selected Countries and Regions.

Year	Europe	Northern America	Japan	India	Brazil	Chile
2005	23	19	30	7	9	12
2050	47	36	74	20	36	36

Population aging is likely to alter the distribution of preferences and the support for different types of government spending, thereby affecting economic growth. A typical concern expressed in the public debate is that rising old-age dependency ratios lead to a growing overall tax burden, and thus slow down economic growth. Moreover, it is feared that increased spending on the elderly, e. g., on health and care service, crowds out public investment spending and has negative effects on economic growth. Recent evidence suggests that the concerns with respect to public spending are partly justified. In the United States, for example, the allocation of expenditure is highly skewed towards older members (see, e. g., Rogers et al., 2000; Iqbal and Turnovsky, 2008) and since 1959 public spending on the elderly has grown much faster than other categories of public expenditure (Mulligan and Sala-í-Martin, 1999). Poterba (1997) finds that a similar pattern holds in other OECD countries. Moreover, U.S. federal public spending on infrastructure declined from 5% to 2.5% of GDP over the time period from 1960 to the mid-1990's. However, total productive government spending on infrastructure, educational institutions, and R&D remained stable at 10% of GDP over the same time period (CBO, 1998).

[1] Table 1 is based on the data that appear as the 'medium variant' prediction in United Nations (2008). The old-age dependency ratio is the ratio of the population aged 65 or over to the population aged $15-64$. The ratio is stated as the number of dependents per 100 persons of working age $(15-64)$.

This paper introduces a democratic voting process into a simple two-period overlapping generations model with endogenous growth à la Barro (1990) in order to analyze how population aging, i. e., an increase in the old-age dependency ratio, endogenously changes the composition of government spending and long-term economic growth.

We focus on two public spending categories: productive government expenditure (e. g., on infrastructure, education, or law and order) that increases private production possibilities and (unproductive) public consumption spending that only benefits the elderly (e. g., on health and care services or public infrastructure for the elderly). To finance its expenditure the government levies a uniform, proportional tax on labor and capital income. We solve for the politico-economic equilibrium in which government policy is set each period through voting by rational, forward-looking agents. In particular, voters take into account that current policy choices influence the evolution of the economy and future policies.

As government policy choices are of differing concern to the young and the old, they disagree on which policy mix they prefer to be implemented. We model the resolution of the resulting political conflict under the assumption of probabilistic voting. In contrast to the median voter model, the probabilistic voting model assures that policy proposals represent the interests of both groups of society, reflecting the political process in representative democracies more realistically.

Since elections take place each period, policy makers cannot commit to future policy choices. Therefore, voters have to form expectations about future policy outcomes. We focus on Markov perfect equilibria, i. e., equilibria in which the policy choices expected for a certain period depend only on the value of the fundamental state variable at that time.[2]

Under standard functional form assumptions, we are able to determine the politico-economic equilibrium and the implications of population aging in closed form. (This is in contrast to most of the literature that has to resort to numerical methods to characterize the politico-economic equilibrium. When we relax the functional form assumptions, and thus have to use numerical examples, the qualitative results turn out to be robust.)

We find that in the politico-economic equilibrium both types of government expenditure are chosen as constant shares of output and all variables in per capita terms grow at the same constant rate. The equilibrium share of output devoted to productive purposes corresponds to the exogenous output elasticity of productive public expenditure. In other words, it does not depend on preferences or demographic parameters, and thus is not affected by any form of population aging. By contrast, the equilibrium share of public spending on the elderly balances the interests of the old who support this type of spending

[2]For a discussion of Markov perfect equilibria in the context of endogenous dynamic fiscal policy see, for instance, Krusell et al. (1997).

as long as its benefits outweigh its tax costs and those of the young taxpayers who as net contributors oppose this form of public spending.

Voters internalize only those effects of government policy that materialize during their lifetimes; negative consequences borne by subsequent cohorts (due to higher overall taxes and lower capital accumulation) are not taken into account. By contrast, a benevolent planner with "dynastic" welfare weights (i. e., with welfare weights reflecting the households' discount factor and cohort sizes) also considers the effects on future households. Therefore, the share of public consumption spending on the elderly implemented by such a planner tends to be smaller than the corresponding share in the politico-economic equilibrium.

Population aging in our framework occurs either due to a decline in fertility, which in turn lowers the population growth rate, or due to an increase in life expectancy. Both phenomena increase the old-age dependency ratio and the relative weight that the political process attaches to the interests of the old relative to the young voters. The model predicts an increase in the old-age dependency ratio to be associated with (i) higher public consumption spending on the elderly (as a share of output), (ii) unchanged public productive expenditure (as a share of output), and (iii) a higher distortionary income tax rate. The latter has a negative effect on the economy's growth rate of per capita variables.

However, population aging not only affects economic growth indirectly via the composition and financing of government spending, but also has a direct effect on economic growth. For a given policy mix, an increase in the old-age dependency ratio accelerates economic growth. If the increase in the old-age dependency ratio follows a slowdown in the population growth rate this result is due to reduced capital dilution. In the case of a higher life expectancy the positive growth effect results from an increase in precautionary savings. The same channels are at work in any AK-type OLG growth model. In this paper, we evaluate whether a positive growth bias of population aging persists when an increasing fraction of elderly prefers higher public consumption spending and less economic growth. We find that in both scenarios the direct effect dominates the indirect effect such that population aging overall increases the economy's growth rate of per capita variables.

This paper relates and contributes to at least two strands of the literature. First, it makes a contribution to the recent politico-economic literature on dynamic fiscal policy where rational, forward-looking agents vote repeatedly on the level and financing of different types of government spending. Recent contributions that analyze how population aging endogenously affects government spending include Bassetto (2008), Gonzalez-Eiras and Niepelt (2008), and Song et al. (2009).[3] Hitherto, this literature has not considered a

[3]See, e. g., Hassler et al. (2007, 2005) or Krusell and Ríos-Rull (1999) for insights about the politico-economic determination of taxes, transfers, and/or public consumption spending in environments where agents are heterogeneous in human capital and earnings. However, these papers do not consider the role of population aging.

public policy mix that involves productive government expenditure and public consumption spending that only benefits the elderly. Moreover, the above mentioned papers do not consider an endogenous economic growth framework such that they cannot study the effect of demographic change on long-term economic growth. To the best of our knowledge, the only exception is Gonzalez-Eiras and Niepelt (2007) who quantitatively analyze the effect of population aging on public spending for education, public transfers between workers and retirees, and endogenous productivity growth in a three-period overlapping generations model with human and physical capital accumulation. In their framework, population aging induces a reallocation of public resources from education spending to retirement benefits, which has a negative growth effect. Similar to our results, they also find that reduced capital dilution more than outweighs this effect and that the long-term growth rate overall increases.

Second, this paper complements the theoretical literature on the causal effect of population aging on long-term economic growth in models with overlapping generations and endogenous economic growth. Most contributions in this strand of the literature find this effect to be positive.[4] It results, for instance, from the following channels: (i) reduced capital dilution due to a slowdown in population growth (see, e. g., Gonzalez-Eiras and Niepelt, 2007), (ii) changes in individual saving behavior because of a longer expected lifetime (see, e. g., Futagami and Nakajima, 2001), (iii) more investment in innovations that increase labor productivity because a smaller labor force makes the input factor labor more expensive (see, e. g., Irmen and Heer, 2008), (iv) more private investments into new technologies as they are more likely to pay off when the individual time horizon expands (see, e. g., Prettner, 2009). In the present paper, either channel (i) or (ii) is at work. Additionally, a new channel operates in the opposite direction: population aging by shifting political power from the young to the old leads to an increased demand for public consumption spending and a slowdown of economic growth.[5]

The remainder of this paper is organized as follows. Section 2 describes the model and characterizes the allocation conditional on policy. Section 3 describes the political decision-making process and establishes the politico-economic equilibrium. The allocation chosen by a Ramsey planner, who cares about all future generations, is studied in Section 4. Section 5 analyzes how an increase in the old-age dependency ratio affects the composition of government expenditure and economic growth in the politico-economic equilibrium. While Section 5.1 considers a decline in the population growth rate, Section 5.2 studies

[4]By contrast, Irmen (2009) finds that in the presence of capital-saving technical change population aging does not affect the economy's steady-state growth rate.

[5]There are a few papers (see, e. g., Yakita, 2008; Dioikitopoulos, 2009) that examine the effect of population aging on the growth-maximizing composition of government expenditure. However, in these papers policy is not determined endogenously via a political process and thus does not reflect the distribution of preferences.

the case where the old-age dependency ratio increases because of a higher life expectancy. Section 6 discusses and extends the analysis in two directions. First, in Section 6.1 the Markov perfect equilibrium of Section 3 is compared to two other voting equilibria. Second, numerical examples in Section 6.2 suggest that our main findings are robust to the use of two alternative utility functions. Section 7 concludes. Proofs are relegated to the Appendix.

2 The Economic Environment

Consider an overlapping generations economy in which non-altruistic agents live for two periods: a working period and a retirement period.[6] Individual labor supply when young is inelastic and normalized to one. The size of generation t is denoted by L_t and grows at the exogenous rate $n > (-1)$. The population at any t thus consists of L_t young and L_{t-1} old individuals.[7] Note that n corresponds to the growth rate of the total population and determines the old-age dependency ratio defined as $L_{t-1}/L_t = (1+n)^{-1}$. The economy starts at time 0 with $L_{-1} = 1$.

2.1 Preferences

In the economy at each t there is one private good and one public (consumption) good. The private good delivers utility to the agents when young and when old, whereas the public consumption good only benefits the old agents. For concreteness, one may think of this public good involving publicly-provided health and care services or public infrastructure for the elderly.

The preferences of an individual born at t are described by the following log-linear utility function[8]

$$\ln c_t^y + \beta \left(\ln c_{t+1}^o + b \ln \tilde{h}_{t+1} \right), \tag{2.1}$$

[6]This can be considered the most conservative scenario. A setup with agents that are altruistic towards future generations would represent an intermediate case between the framework presented in this section and the Ramsey planner of Section 4. Thus, it can be expected that altruistic agents would vote for a lower share of public consumption spending and a higher equilibrium growth rate than in the politico-economic equilibrium of Proposition 1.

[7]In the following sections we focus on a deterministic life time. Only in Section 5.2 we reinterpret and extend the setup of Section 2 to incorporate an uncertain life time and the concept of life expectancy.

[8]The choice of logarithmic utility guarantees analytical tractability, but does not affect the qualitative findings. We return to this point in Section 6.2, where the results from the logarithmic utility case are compared to those of (i) a utility function with a constant intertemporal elasticity of substitution different from unity and (ii) non-separable preferences.

where c_t^y and c_{t+1}^o are consumption of the private good of a member of generation t when young and old, respectively, and $\tilde{h}_{t+1}$ is the level of provision of the public good per old agent at $t+1$, i. e., $\tilde{h}_{t+1} \equiv H_{t+1}/L_t$, where H_{t+1} denotes aggregate spending on the public consumption good at $t+1$. The fact that $\tilde{h}_{t+1}$ and not H_{t+1} enters the utility function implies that there is congestion in the public consumption good.[9] Moreover, $\beta \in (0,1)$ denotes the discount factor and $b > 0$ measures the weight an old agent assigns to the public relative to the private consumption good.

2.2 Technology

At each t, the private good is produced by competitive firms operating a technology that uses capital K_t procured by the old, labor L_t supplied by the young, as well as a productivity-enhancing input G_t provided by the government. One may think of G as government expenditure on infrastructure, education, or law and order. More specifically, we assume that total output of the private good at t, Y_t, is manufactured according to

$$Y_t = AK_t^{\alpha} \left(g_t L_t\right)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{2.2}$$

where $A > 0$ denotes the time-invariant total factor productivity and $g_t \equiv G_t/L_t$ is the productive public input per worker at t. Thus, there is also congestion in the productive public input.[10] Given the length of the considered period (one generation) it is assumed that capital fully depreciates after each use.

Let $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$ denote output and capital per worker, respectively. Then, we obtain output per worker from (2.2) as

$$y_t = Ak_t^{\alpha} g_t^{1-\alpha}. \tag{2.3}$$

The initial capital stock per worker is given by $k_0 > 0$.

Note that the technology displays diminishing returns in private capital, but constant returns to scale in private capital and the productive public input. Thus, if g rises with k

[9]In other words, this type of government activity has the character of a utility-enhancing transfer to the old that is not excludable, but rival. Alternatively, we could assume that public consumption spending enters the utility function as a pure public good. The main difference of this modeling approach concerns the long-term effect of population aging on the level of services derived by each old agent from public consumption spending. See Section 5.1, for a more detailed discussion.

[10]The congestion specification assures the existence of a politico-economic equilibrium and a balanced growth path. By contrast, if G were a pure public good, then the interest rate in the politico-economic equilibrium would depend on the aggregate labor supply (see, e. g., Irmen and Kuehnel, 2009, Section 3.1, for a discussion), which in our framework grows over time. However, for an endogenous balanced growth path to exist, the equilibrium interest rate has to be constant. The Barro (1990) literature that uses a pure public good specification avoids this problem by assuming a stationary population.

the diminishing returns to the accumulation of capital do not set in. For this reason, the economy will exhibit endogenous steady-state growth.

At any time t, the private good can either be consumed, saved as capital for the next period, or be converted one-to-one into units of H_t and G_t. We take the private good produced at each period t as the numeraire.

2.3 Government Policy

In each period, the government raises tax revenues and uses the proceeds to purchase private consumption goods to be converted into the public consumption good and the public productive input. Specifically, the government at each t levies a proportional tax $\tau_t \in [0, 1]$ on labor income of the young and capital income of the old. The government cannot issue age-dependent taxes and the government's budget is assumed to balance in each period t, i. e., $G_t + H_t = \tau_t (w_t L_t + R_t K_t)$, where w_t is the pre-tax wage rate at time t and R_t is the rental rate of capital at time t.

Then, the government's budget constraint in per worker terms is given by

$$g_t + h_t = \tau_t (w_t + R_t k_t), \tag{2.4}$$

where $h_t \equiv H_t/L_t = \tilde{h}_t/(1 + n)$ is the level of the public consumption good per worker at t. As h_t is proportional to $\tilde{h}_t$ and n is exogenous, we focus - for notational simplicity - on the policy mix (g_t, h_t).

Then, a feasible government policy at t is a vector $(g_t, h_t) \in \mathbb{R}^2_+$ such that (2.4) holds and $\tau_t \in [0, 1]$.

2.4 Timing

Within each period t the timing of events is as follows: At the beginning of the period, after a new generation of young people has been born, all individuals (the young and the old) democratically elect a political candidate who chooses current policy. When deciding which candidate to support, voters anticipate how each candidate's policy platform would affect subsequent economic and political decisions. Then, firms hire workers and rent capital to produce. The policy vector and the resulting income tax rate together with the wage rate and the rental rate of capital determine the consumption of the old and the disposable income of the young. The young then choose how much to consume and how much to save as capital for the next period. Finally, the old generation dies, while the young generation ages and becomes old in the next period.

In order to solve for the equilibrium we proceed by backward induction. We start in Section 2.5 by analyzing the economic choices of households and firms subject to exogenously given (prices and) government policy. We refer to the allocation that results at time t for a given policy mix as the economic equilibrium. Section 3 then considers the political determination of policy.

2.5 Economic Equilibrium

In an economic equilibrium, each household maximizes her lifetime utility given by (2.1) taking factor prices and the benefits from the public consumption good as given. Each individual that is born at time $t \geq 0$ faces the per-period budget constraints $c_t^y + s_t \leq (1-\tau_t)\, w_t$ and $c_{t+1}^o \leq s_t\,(1-\tau_{t+1})\, R_{t+1}$, where s_t denotes savings at t.

The optimal choices of a member of cohort t are then given by

$$c_t^y = \frac{1}{1+\beta}(1-\tau_t)\, w_t, \tag{2.5}$$

$$c_{t+1}^o = \frac{\beta}{1+\beta}(1-\tau_t)\, w_t\,(1-\tau_{t+1})\, R_{t+1}, \tag{2.6}$$

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)\, w_t. \tag{2.7}$$

Note that optimal saving of a young agent at t, given by (2.7), does neither depend on the population growth rate nor on future fiscal policy.[11]

Moreover, in an economic equilibrium each firm maximizes its profits taking factor prices and the level of provision of the productive public input as given. Thus, the firms' profit maximization problem determines the rental rate of capital and the pre-tax wage rate as

$$R_t = \alpha \frac{y_t}{k_t} \quad \text{and} \quad w_t = (1-\alpha)\, y_t, \tag{2.8}$$

respectively, where y_t is given by (2.3). Due to constant returns to scale in private inputs the firm sector makes zero profits, i. e., $Y_t = w_t L_t + R_t K_t$. This in turn implies that the government budget constraint (2.4) may be written as

$$g_t + h_t = \tau_t y_t. \tag{2.9}$$

For the capital market to clear it has to hold at all t that

$$K_{t+1} = s_t L_t, \tag{2.10}$$

i. e., the aggregate capital stock in period $t+1$ corresponds to aggregate saving in period t.

[11] The latter independence is a direct consequence of the logarithmic utility and greatly simplifies the analysis. We relax this restriction in the numerical sensitivity analysis of Section 6.2.

Combining conditions (2.5) - (2.10), the equilibrium allocation in t can be expressed in terms of government policy and the capital stock per worker

$$
\begin{aligned}
c_t^y &= \frac{1-\alpha}{1+\beta}\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right), && (2.11)\\
c_t^o &= \alpha(1+n)\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right), && (2.12)\\
k_{t+1} &= \frac{s_t}{1+n} = \frac{\tilde{B}}{1+n}\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) \equiv \pi^k(g_t, h_t, k_t), && (2.13)
\end{aligned}
$$

where $\tilde{B} \equiv \beta(1-\alpha)/(1+\beta)$.[12]

The function $\pi^k(\cdot)$ is the economic equilibrium condition that describes how young agents optimally choose their savings and thus determine the next period's capital stock per worker for given g_t, h_t, and k_t. Equation (2.13) also reveals how the composition and financing of government spending affects capital accumulation. First, the income tax financing of both types of public expenditure has a negative effect on the accumulation of capital (negative terms in brackets). Second, g_t has an additional positive effect on the accumulation of capital by raising the productivity of private capital.

In an economic equilibrium, the indirect utility of a young and an old agent alive at t, respectively, can be expressed as functions of government policy and the capital stock per worker:

$$
\begin{aligned}
U_t^Y &= \ln c_t^y + \beta\left(\ln c_{t+1}^o + b\ln \tilde{h}_{t+1}\right) = \ln\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right)\\
&\quad +\beta\ln\left(Ak_{t+1}^\alpha g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1}\right) + \beta b \ln h_{t+1} + \text{t.i.p.}, && (2.14)
\end{aligned}
$$

$$
U_t^O = \ln c_t^o + b\ln\tilde{h}_t = \ln\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) + b\ln h_t + \text{t.i.p.}, \quad (2.15)
$$

s.t. $k_{t+1} = \pi^k(g_t, h_t, k_t)$. Here, t.i.p. denotes terms independent of the policy choice.

3 Politico-Economic Equilibrium

In the politico-economic equilibrium, the government policy mix (g_t, h_t) is chosen through voting at the beginning of each period t. Electoral competition is modeled under the assumption of probabilistic voting. As elections take place each period, policy makers cannot commit to future policy choices. Therefore, voters have to form expectations about future policy outcomes. In order to limit the set of potential equilibria, we restrict attention to Markov perfect equilibria, i. e., equilibria in which the policy choices expected for a certain period depend only on the value of the fundamental state variables expected

[12] Note that the above equilibrium conditions imply that the market for the private good clears at all t, i. e., $L_t c_t^y + L_{t-1}c_t^o + K_{t+1} + G_t + H_t = Y_t$.

at that time, and not on the past history of policies or artificial state variables sustaining trigger strategy equilibria. In the present setup, the only state variable is the level of the private capital stock per worker; it affects future wages and returns, and therefore income of future voters.[13]

3.1 Probabilistic Voting

The political process is represented via a two-candidate probabilistic voting model. In this model agents cast their votes on one of two candidates, who maximize their probability of becoming elected. Voters support a candidate not only for her policy platform, but also for other characteristics like "ideology" that are orthogonal to the fundamental policy dimensions of interest. The evaluation of these features differs across voters and is subject to random aggregate shocks, realized after candidates have chosen their platforms.[14]

In a probabilistic-voting Nash equilibrium, two candidates maximizing their respective vote shares both propose the same policy platform and each of them has a 50 % probability of winning. The proposed policy platform maximizes a "political objective function" which is a weighted average utility of all voters, with the weights reflecting the group size and the sensitivity of voting behavior to policy changes. Groups that have a low concern for the orthogonal policy dimension have more political influence since they are more likely to alter their support in response to small changes in the proposed platform. In other words, these groups of "swing voters" are more attractive to power-seeking candidates and exert a stronger influence on the equilibrium policy outcome. Formalizing the foregoing discussion, we assume that the "political" aggregation of different preferences is summarized by the following political objective function

$$U_t = (1+n)\, U_t^Y + \omega U_t^O, \tag{3.1}$$

where U_t^Y and U_t^O are given by (2.14) and (2.15), respectively, $\omega > 0$ represents the per-capita political weight of the old relative to the young, and $(1+n)$ the relative group size of the young compared to the old. Thus, the political objective function (3.1) to be maximized in the political process attaches a positive weight to the welfare of the elderly, even if the median voter is a young agent. This appears to be a realistic implication. In fact, it is often argued that the old are more policy-focused, i. e., care less about ideology and have more swing voters, and thus even exert a stronger political influence per capita

[13]Note that the population growth rate n as well as life expectancy in Section 5.2, i. e., the variables that determine the old-age dependency ratio, will affect the actual policy choice. However, in their decision-making process all agents treat these variables as exogenous.

[14]For a more detailed discussion of the probabilistic voting model, see Lindbeck and Weibull (1987) or Persson and Tabellini (2000).

than the young (see e. g., Rhodebeck (1993, p.357), Dixit and Londregan (1996, p.1144) or Grossman and Helpman (1998, p.1309)). This case would correspond to an $\omega > 1$.

Using the expressions for U_t^Y and U_t^O, the political objective function obtains as

$$\begin{aligned} U\left(g_t, h_t, k_t, g_{t+1}, h_{t+1}, k_{t+1}\right) &= (1+n+\omega)\ln\left(Ak_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) + \omega b \ln h_t \\ &+ (1+n)\,\beta \ln\left(Ak_{t+1}^{\alpha} g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1}\right) \\ &+ (1+n)\,\beta b \ln h_{t+1} \end{aligned}$$

subject to $k_{t+1} = \pi^k(g_t, h_t, k_t)$.

3.2 Definition of the Politico-Economic Equilibrium

As mentioned above, we look for Markov perfect equilibria, i. e., for equilibria in which the policy choices are functions only of the level of private capital per worker in the economy. The dynamic aspect of the voting game stems from the fact that current policy affects capital accumulation, and thus income and the strategic position of the currently young in the next period. Agents are assumed to be fully forward-looking. Thus, when voting over today's policy, young agents correctly anticipate how future policy will depend on current policy via the state of the economy.

Definition 1 *The Politico-Economic Equilibrium is defined as a pair of functions* $\left\langle \pi^g, \pi^h \right\rangle$, *where* π^g *and* π^h *are public policy rules,* $g_t = \pi^g\left(k_t\right)$ *and* $h_t = \pi^h\left(k_t\right)$, *such that the following functional equation holds:*

$\left\langle \pi^g\left(k_t\right), \pi^h\left(k_t\right)\right\rangle = \arg \ \max_{\{g_t, h_t\}} U\left(g_t, h_t, k_t, g_{t+1}, h_{t+1}, k_{t+1}\right)$, *subject to*

$$\begin{aligned} k_t & \quad \textit{given}, \\ k_{t+1} &= \pi^k\left(g_t, h_t, k_t\right), \\ g_{t+1} &= \pi^g\left(k_{t+1}\right) = \pi^g\left(\pi^k\left(g_t, h_t, k_t\right)\right), \\ h_{t+1} &= \pi^h\left(k_{t+1}\right) = \pi^h\left(\pi^k\left(g_t, h_t, k_t\right)\right). \end{aligned}$$

The equilibrium condition requires the political mechanism in t to choose g_t and h_t to maximize the political objective function U, for a given k_t, taking into account that future government policies, g_{t+1} and h_{t+1}, depend on the current policy mix (g_t, h_t) via the state of economy, k_{t+1}, as described by the economic equilibrium decision rule π^k. Moreover, the above definition of the politico-economic equilibrium has the usual fixed point structure induced by a rational expectations equilibrium: the anticipated policy functions coincide with the optimal ones. In other words, suppose that agents believe future government policy to be set according to $g_{t+1} = \pi^g\left(k_{t+1}\right)$ and $h_{t+1} = \pi^h\left(k_{t+1}\right)$. Then, we require that the same functions $g_t = \pi^g\left(k_t\right)$ and $h_t = \pi^h\left(k_t\right)$ define optimal spending today.

3.3 Solving for the Politico-Economic Equilibrium

To solve for the politico-economic equilibrium we need to find two functions π^g and π^h satisfying Definition 1. Guided by the fact that government expenditure is financed by a proportional tax on income, we conjecture that π^g and π^h are linear functions in the capital stock. Specifically, we make the following guess for future policy variables: $\pi^g(k_{t+1}) = \eta^g k_{t+1}$ and $\pi^h(k_{t+1}) = \eta^h k_{t+1}$, with some yet undetermined coefficients η^g and η^h.[15] We derive the equilibrium choice of government policy in period t under this conjecture and show that the spending shares in t are indeed linear in the capital stock, thereby verifying the conjecture.

First of all, note that with this guess the production function (2.3) at $t+1$ becomes linear in the capital stock $y_{t+1} = A(\eta^g)^{1-\alpha} k_{t+1}$ and we can write $Ak_{t+1}^{\alpha} g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1} = \left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right) k_{t+1}$.

Using these results and omitting terms independent of the policy choice, the program characterizing equilibrium policy choices in period t can be expressed as

$$\max_{\{g_t,h_t\}} \bar{U}(g_t,h_t,k_t) \quad \text{s.t.} \quad k_t \text{ given}, \quad \text{where}$$

$$\bar{U}(g_t,h_t,k_t) \equiv [(1+n)(1+\beta(1+b))+\omega] \ln\left(Ak_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) + \omega b \ln h_t. \tag{3.2}$$

After some algebra, the first-order conditions of the program (3.2) with respect to g_t and h_t yield

$$g_t = (1-\alpha) y_t \quad \text{and} \quad h_t = \frac{\alpha\omega b}{(1+n)(1+\beta(1+b))+\omega(1+b)} y_t, \tag{3.3}$$

$$\text{where} \quad y_t = A(\eta^g)^{1-\alpha} k_t. \tag{3.4}$$

Equations (3.3) and (3.4) verify the tentative guess as a fixed point of the functional equation of Definition 1 if $\eta^g = (A(1-\alpha))^{1/\alpha}$ and $\eta^h = \alpha\omega b\left[(1+n)(1+\beta(1+b))+\omega(1+b)\right]^{-1} A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$, which allow us to establish the following proposition.

[15] Note that the above decision problem is a stationary Markov decision problem because the problem facing voters looks the same (contingent on the state) at each t. Moreover, note that guessing a policy function that does not depend on time per se is not the same as imposing ex-ante that the expenditure has to be a constant fraction of the capital stock. For details on this see Section 6.1.1.

Proposition 1 *(Politico-Economic Equilibrium)*

The politico-economic equilibrium is characterized as follows:

$$\begin{aligned} \pi^g(k_t) &= (1-\alpha)\, y_t \quad \textit{and} \quad \pi^h(k_t) = \tau_h^{\mathcal{P}} y_t, \\ \textit{with} \quad y_t &= A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} k_t \end{aligned}$$

and $\alpha > \tau_h^{\mathcal{P}} \equiv \alpha\omega b \left[(1+n)(1+\beta(1+b)) + \omega(1+b)\right]^{-1} > 0$ *such that* $0 < \tau = 1-\alpha+\tau_h^{\mathcal{P}} < 1$ *for all* t.

Moreover, the equilibrium growth factor of the capital stock per worker, $\gamma_{t+1} \equiv k_{t+1}/k_t$*, is constant and given by*

$$\gamma_{t+1} = \frac{B}{1+n}\left(\alpha - \tau_h^{\mathcal{P}}\right) \equiv \gamma, \tag{3.5}$$

with $B \equiv A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}} \tilde{B}$*. The economy immediately settles on its steady-state path on which the economy's relevant variables such as per capita consumption, per capita output, the per capita capital stock, government spending as well as wages all grow at the same constant rate* $\gamma - 1$*.*

According to Proposition 1, under rational voting both types of government expenditure are chosen as constant shares of output. The equilibrium share of output devoted to the productive public input corresponds to $1-\alpha$, which is the output elasticity of the productive public input. Thus, productive government expenditure satisfies the so-called natural condition of productive efficiency, i. e., the marginal contribution of government expenditure to aggregate output is one (see, e.g., Barro, 1990). In the present context, as aggregate output is $Y = AK^{\alpha}G^{1-\alpha}$, we have $dY/dG = (1-\alpha)(Y/G) = (1-\alpha)(y/g) = 1$. This also implies that the young and the old prefer the same share of output devoted to productive purposes. In other words, there is no conflict about this type of public spending. The reason for this is that g_t symmetrically affects the labor income of the young and the capital income of the old.

The equilibrium share of output spent on the public consumption good benefiting the old is given by $\tau_h^{\mathcal{P}}$ and depends on preferences, technology, and demographic parameters. Intuitively, it balances the interests of the elderly who support public consumption spending as long as the related benefits outweigh the tax costs and those of the young taxpayers who oppose this form of spending as they are net contributors to the system. This reflects the intergenerational conflict more realistically than what would be observed under simple majority voting. For instance, assume that the median voter is a young agent.[16] Then, if we anticipate that all agents will prefer the same share of productive government

[16] In a two-period OLG model there are always more young people than old as long as $n > 0$.

spending such that the voting problem becomes one-dimensional, we find that the median voter would set public consumption spending on the old equal to zero. In our probabilistic voting setup, $\tau_h^{\mathcal{P}} = 0$ could only occur if the old had no political influence at all (i. e., if $\omega = 0$) or if they did not care about the public consumption good (i. e., if $b = 0$).[17]

The equilibrium income tax rate corresponds to the sum of the two public expenditure shares and turns out to be strictly smaller than one such that the equilibrium policy mix is feasible. Moreover, note that the income tax rate in equilibrium is time-invariant. In other words, it is independent of the economy's endogenous state variable, i. e., the capital stock per worker. Nevertheless, the equilibrium tax rate will be affected by population aging because it depends on demographic parameters.

Finally, Proposition 1 reveals that in the politico-economic equilibrium the economy's relevant variables in per capita terms grow at the same constant rate given by $\gamma - 1$. There is no guarantee that this rate is positive for all parameter combinations. However, a positive steady-state growth rate can be assured if we assume that the economy is sufficiently productive, i. e., if A is large enough.

The following corollary verifies that the Markov perfect equilibrium derived above is the limit of a unique finite-horizon equilibrium.[18]

Corollary 1 *(Limit of Finite-Horizon Economy)*

The equilibrium policy functions $g_t = (1-\alpha)y_t$ and $h_t = \tau_h^{\mathcal{P}} y_t$ of Proposition 1 represent the unique equilibrium policy mix in (the limit of) the corresponding finite-horizon economy. In the last period, the policy function for h_t is different, but also unique.

4 The Ramsey Allocation

This section compares the politico-economic equilibrium with the Ramsey allocation chosen by a benevolent planner who has the ability to commit to all its future policy choices at the beginning of time, but is constrained by the same economic equilibrium conditions.

Specifically, we consider the Ramsey solution in the case where the planner's weight on generation $t \geq 0$ is $\beta^{t+1}(1+n)^{t+1}$, i. e., the planner's weights on future generations reflect the discount factor of households as well as the cohort size ("dynastic discounting").[19]

[17] In this paper we abstract from these polar cases.

[18] This allows us to rule out potential reputation-like equilibria that can only be supported if the horizon is infinite.

[19] See, e. g., Gonzalez-Eiras and Niepelt (2008) for a discussion.

The planner's decision problem is therefore to choose the sequence $\{g_t, h_t\}_{t=0}^{\infty}$ in order to maximize

$$W\left(k_0, \{g_t, h_t\}_{t=0}^{\infty}\right) \equiv \beta U_0^O + \sum_{t=0}^{\infty} \left(\beta\left(1+n\right)\right)^{t+1} U_t^Y$$
$$\text{subject to} \quad (2.13) - (2.15) \quad \text{and} \quad k_0 \quad \text{given.} \tag{4.1}$$

In the following we assume that $\beta(1+n) < 1$ such that the planner's objective function W is finite.

The main difference to the program solved by the political candidates is that the Ramsey program (4.1) involves the choice of an entire sequence of policy mixes. Moreover, the Ramsey planner values the welfare of all households, not only of those currently alive and voting.

In order to solve for the Ramsey allocation it is helpful to first establish the following lemma.

Lemma 1 *The Ramsey program (4.1) is equivalent to the following recursive program:*

$$V\left(k_t\right) = \max_{\{g_t, h_t, k_{t+1}\}} \left\{T_t\left(g_t, h_t, k_t\right) + \left(1+n\right)\beta V\left(k_{t+1}\right)\right\} \quad \textit{for} \quad t \geq 0, \tag{4.2}$$

subject to (2.13), where $T_t\left(g_t, h_t, k_t\right) \equiv \beta\left(2+n\right)\ln\left(Ak_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) + \beta b \ln h_t$.

The fact that the planner's problem admits the standard recursive formulation of Lemma 1 reveals that its solution is time consistent. Intuitively, the generational weights (in the case of dynastic discounting) are such that the Ramsey plan is dynamically consistent (see, e. g., Heijdra, 2009, p.656-658).

The following proposition summarizes the solution of the Ramsey problem.

Proposition 2 *(The Ramsey Allocation)*

Let $\beta(1+n) < 1$. *Then, the solution of the Ramsey program (4.1) involves for* $t \geq 0$

$$g_t = \left(1-\alpha\right) y_t \quad \textit{and} \quad h_t = \tau_h^{\mathcal{R}} y_t,$$

where $y_t = A^{1/\alpha}\left(1-\alpha\right)^{(1-\alpha)/\alpha} k_t$ *and* $0 < \tau_h^{\mathcal{R}} \equiv \alpha b\left(1-(1+n)\beta\right)\left(2+n+b\right)^{-1} < \alpha$. *Moreover,* g_t, h_t, y_t *and* k_t *grow at the same constant rate determined by (2.13).*

Proposition 2 reveals that the Ramsey planner sets the levels of both types of government expenditure proportional to output. He chooses the same share of output, namely $1-\alpha$, to be devoted to the productive public input as in the politico-economic equilibrium. The optimal share of output spent on the public consumption good benefiting the old is given by $\tau_h^{\mathcal{R}}$ and depends again on preferences, technology, and demographic parameters. The following corollary compares $\tau_h^{\mathcal{R}}$ to $\tau_h^{\mathcal{P}}$ of the politico-economic equilibrium.

Corollary 2 *(Ramsey Allocation vs. Politico-Economic Equilibrium)*

It holds that

$$\tau_h^{\mathcal{R}} \leq \tau_h^{\mathcal{P}} \Leftrightarrow \omega \geq 1 - \beta(1+n).$$

Corollary 2 shows that the share of public spending on the elderly chosen by the Ramsey planner falls short of the corresponding share in the politico-economic equilibrium whenever ω or $\beta\,(1+n)$ are sufficiently large. For instance, this is the case for any $\omega \geq 1$, i. e., if the old have at least the same per capita political weight as the young.

The intuition for $\tau_h^{\mathcal{R}} < \tau_h^{\mathcal{P}}$ is that voters in their optimization problem only consider the effects of their policy choice that materialize during their lifetimes. Negative consequences borne by subsequent generations due to higher taxes and lower capital accumulation are not taken into account. By contrast, the Ramsey planner internalizes the effects of policy on all current *and* future households.

5 Implications of Population Aging

5.1 Declining Population Growth

This section studies the effect of a permanent decline in the population growth rate n on government policy and economic growth in the politico-economic equilibrium. The decline in the population growth rate is due to a decline in fertility and causes a rise in the old-age dependency ratio $(1+n)^{-1}$. Increases in the latter are meant to capture the tendencies shown in Table 1. The decline in n materializes at the beginning of the period (see Section 2.4) and is then taken into account by all agents alive in that period. Note that it does not affect the results whether or not the decline in n is anticipated by the generation born in the previous period as their savings decision is independent of n (see equation 2.7).

In the politico-economic equilibrium of Proposition 1 the economy at all t grows at the constant rate $\gamma - 1$ given by (3.5). Recall that there are no transitional dynamics in the economy. Denote τ_g the fraction of current output devoted to productive public services, i. e., $\tau_g = g_t/y_t = 1 - \alpha$. Then, the results of the comparative static analysis described above can be summarized in the following corollary.

Corollary 3 *(Population Growth, Government Spending, and Economic Growth)*

1. *If $\omega \neq (1+n)$, then it holds that*

$$\frac{d\tau_h^{\mathcal{P}}}{dn} < 0, \quad \frac{d\tau_g}{dn} = 0, \quad \textit{and} \quad \frac{d\tau}{dn} < 0.$$

2. It holds that

$$\frac{d\gamma}{dn} < 0.$$

The first statement of Corollary 3 reveals that an increase in the old-age dependency ratio (due to a decline in n) raises $\tau_h^{\mathcal{P}}$, i. e., the fraction of output used for the provision of public services that benefit the old. The reason is that a decline in n reduces the share of young agents relative to old agents in the population, and thus their weight in the political objective function (3.1). Intuitively, the old prefer greater spending on the public consumption good than the young. In the non-generic case $\omega = 1 + n$, i. e., when both groups have exactly the same weight in the political objective function, $\tau_h^{\mathcal{P}}$ does not depend on n. The share of output devoted to the productive public input corresponds to $1 - \alpha$, and is thus always independent of the population growth rate. Overall, the income tax rate, τ, which is levied on households to finance government expenditure, has to increase in the politico-economic equilibrium. Statement 1 of Corollary 3 also implies that population aging increases the share of public consumptive expenditure in total government expenditure, i. e., $\tau_h^{\mathcal{P}}/\tau$, and decreases the share of productive government expenditure in total government expenditure given by $(1 - \alpha)/\tau$.

According to the second statement of Corollary 3, an increase in the old-age dependency ratio leads to a higher equilibrium growth rate of per capita variables. This is the result of two opposing forces. On the one hand, reduced labor force growth weakens the effect of capital dilution, i. e., a given amount of capital implies a higher capital stock per worker at each t and a rise in the equilibrium growth rate of per capita variables. Intuitively, a lower n reduces the break-even investment, the amount of investment necessary for k to grow at a constant rate, without affecting saving at any given level of capital.[20] On the other hand, there is a negative, indirect tax effect via $\tau_h^{\mathcal{P}}$. As discussed in the previous paragraph, an increase in the old-age dependency ratio raises spending for the public consumption good, and thus taxes. Since taxes are levied on capital *and* labor income, they reduce the incentive to save and to accumulate capital, and hence have a negative effect on the steady-state growth rate. The point of the second statement of Corollary 3 is that the former effect dominates the latter. Therefore, population aging accelerates the economy's growth rate of per capita variables.

Finally, Corollary 3 implies that an increase in the old-age dependency ratio in the long run raises the benefits derived by each old agent from aggregate public consumption spending. To see this note that $\tilde{h}_t$ is given by

$$\tilde{h}_t = (1 + n)h_t = (1 + n)\tau_h^{\mathcal{P}} y_t = (1 + n)\tau_h^{\mathcal{P}} y_0 e^{(\gamma-1)t}. \tag{5.1}$$

[20]In the context of a conventional neoclassical growth model, Cutler et al. (1990) refer to this channel as the "Solow effect".

From the definition of $\tau_h^{\mathcal{P}}$ in Proposition 1 one readily verifies that $(1+n)\tau_h^{\mathcal{P}} y_0$ declines if n decreases. Thus, the level of benefits per old initially falls, but then grows at a higher rate (as $d\gamma/dn < 0$) and at some point reaches a higher level than what would have be attainable without a change in n.[21] The initial decline is due to congestion effects; intuitively, the benefits of the public consumption good have to be spread over more old people.

5.2 Increasing Life Expectancy

In the previous section, we studied population aging as a decline in the population growth rate. With a slight reinterpretation of the analytical framework, we can also analyze the effect of an increasing life expectancy on government policy and economic growth.

For this purpose, suppose that each individual faces an exogenous probability of dying at the end of its first period of life equal to $(1-v) \in (0,1)$. This implies that the old-age dependency ratio at t becomes vL_{t-1}/L_t and increases in v.[22]

Let $\beta_v \in (0,1)$ denote the pure discount factor, i. e., the discount factor that the individual would apply if he or she were sure to reach the retirement age. Moreover, normalize the utility after death to zero. Then, we may interpret the utility function of (2.1) as the expected utility of a member of generation t with $\beta \equiv \beta_v v$ as the effective discount factor of the agent and with $\tilde{h}_{t+1} \equiv H_{t+1}/vL_t$ as the provision of the public consumption good per surviving old agent.

Against the survival risk individuals may buy annuity assets with which they receive insurance payments if they are alive and nothing if dead in the retirement period. Assuming that the private annuity markets are perfectly competitive, insurance payments are actuarially fair.

Finally, let ω_v denote the pure per capita political weight of the old, i. e., the political power the old would exert if all individuals survived. Then, the political objective function remains given by (3.1) with $\omega \equiv \omega_v v$ as the effective political weight of the old and the economy inherits the properties established in Proposition 1 and Corollary 1.

[21]By contrast, if aggregate public consumption spending H entered the utility function (2.1) as a pure public good, then an increase in the old-age dependency ratio would lower the level of H in the long run. To see this note that in this case the level of public consumption services would be given by $H_t = \tau_h^{\mathcal{P}} Y_t = \tau_h^{\mathcal{P}} Y_0 e^{g_K t}$, where $g_K = B(\alpha - \tau_h^{\mathcal{P}}) - 1$ corresponds to the growth rate of aggregate variables. As $d\tau_h^{\mathcal{P}}/dn < 0$ and $dg_K/d\tau_h^{\mathcal{P}} < 0$, a decline in n thus implies that H_t initially increases, but then grows at a lower rate. Therefore, population aging here leads to an increase in taxes and at the same time to a decline in the long-term provision of H. The reason for this is that the tax base is lower at all t.

[22]Note that the population growth rate in this framework still corresponds to n.

A permanent increase in life expectancy due to a permanent rise in the survival probability, v, increases the effective discount factor, β, and the effective political weight of the old, ω. The following proposition summarizes the effects of such an increase on government policy and the equilibrium growth rate.

Proposition 3 *(Life Expectancy, Government Spending, and Economic Growth)*

Consider an economy that at $t = 1$ experiences a small but permanent increase in its life expectancy, i. e., $\hat{v} > v$ for all generations $t = 1, 2, 3, ..., \infty$. Assume that this change is unexpected, i. e., it is anticipated by all generations $t = 2, 3, ..., \infty$, but not by generation 1. Denote variables associated with an evolution under $\hat{v}$ by a hat such that the politico-economic equilibrium at t is characterized by $\hat{\tau}_{h_t}^{\mathcal{P}}$ and $\hat{\gamma}_{t+1}$.

Then, it holds that

$$\hat{\tau}_{h_1}^{\mathcal{P}} = \tau_h^{\mathcal{P}}, \quad \hat{\gamma}_2 = \gamma, \quad \textit{and}$$

$$\hat{\tau}_{h_t}^{\mathcal{P}} = \hat{\tau}_h^{\mathcal{P}} > \tau_h^{\mathcal{P}}, \qquad \hat{\gamma}_{t+1} = \hat{\gamma} > \gamma \quad \textit{for} \quad t = 2, 3, ..., \infty.$$

Proposition 3 reveals that an increase in life expectancy increases the share of public consumption spending and the equilibrium growth rate. Intuitively, a higher life expectancy increases savings per next period's worker since the weight on the expected old-age utility increases. This has a positive effect on capital accumulation and dominates the negative tax effect that results from a greater political weight of the old.

However, contrary to the case of a permanent decline in the population growth rate, this effect is delayed by one generation. The reason for this is that the increase in life expectancy is unexpected, i. e., generation 1 makes its consumption and savings plan anticipating an effective discount factor of β instead of $\hat{\beta}$.[23]

Arguably, this is a realistic assumption as expectations of one's own life expectancy are usually myopic, i. e., coincide with the actual life expectancy of the previous generation. Moreover, it is reasonable to assume that the choice of s_1 is made by the young agents before the change in the survival probability is experienced.

Consequently, a permanent increase in life expectancy affects public spending and economic growth in the same direction as a decline in the population growth rate, but with a period delay.

[23] If the increase in life expectancy were anticipated by generation 1, then savings would already increase in $t = 1$. However, the effective political weight of the elderly ($\omega_v v$) and the public policy choice in $t = 1$ are not affected by an anticipated change in life expectancy. In $t = 1$, the young of generation 1 and the old of generation 0, whose size is determined by the initial life expectancy v, vote on government policy. Therefore, the equilibrium growth rate would first jump to a level $\hat{\gamma}_2 > \hat{\gamma}$ and then from $t = 2$ onwards correspond to $\hat{\gamma}$. For a more detailed discussion see the proof of Proposition 3 in the Appendix.

6 Discussion and Extensions

This section discusses and extends the analysis in two directions. First, Section 6.1 compares the politico-economic equilibrium to two other voting equilibria. Second, Section 6.2 presents numerical examples for two alternative preference specifications.

6.1 Other Voting Equilibria

In this section we compare the politico-economic equilibrium of Section 3 to (i) the voting equilibrium that results when voters ex-ante are restricted to choose constant policy paths and (ii) the myopic voting equilibrium.

6.1.1 Voting Equilibrium under Commitment to Constant Policy Paths

This section analyzes a voting equilibrium in which taxes and expenditure shares are ex-ante restricted to a constant path. In other words, we assume that the political candidates in period t propose and fully commit to policies that set government expenditure as the same constant fraction of output.

For this purpose, suppose that a feasible government policy is a vector $(\tau_g^c, \tau_h^c) \in [0,1] \times [0,1]$ such that $g_t = \tau_g^c y_t$, $h_t = \tau_h^c y_t$ and $\tau^c = (\tau_g^c + \tau_h^c) \in [0,1]$. Otherwise the economic environment is identical to that of Section 2.

Then, following the same steps as in Section 2.5 the economic equilibrium in period t, i. e., the allocation conditional on the policy mix (τ_g^c, τ_h^c) and for a given k_t is characterized by

$$
\begin{aligned}
c_t^y &= \frac{1-\alpha}{1+\beta}\left(1-\tau_g^c-\tau_h^c\right) y_t, && (6.1)\\
c_t^o &= \alpha(1+n)\left(1-\tau_g^c-\tau_h^c\right) y_t, && (6.2)\\
k_{t+1} &= \frac{\tilde{B}}{1+n}\left(1-\tau_g^c-\tau_h^c\right) y_t, && (6.3)
\end{aligned}
$$

with $y_t = A^{1/\alpha}\left(\tau_g^c\right)^{(1-\alpha)/\alpha} k_t$.

Using equations (6.1) - (6.3) and dropping terms independent of policy yields the indirect utilities of a young and an old agent at t as

$$
\begin{aligned}
U_t^Y &\simeq [1+\beta(2+b)]\ln\left(1-\tau_g^c-\tau_h^c\right) + [1+2\beta(1+b)]\frac{1-\alpha}{\alpha}\ln\tau_g^c \\
&\quad + \beta b\ln\tau_h^c, && (6.4)
\end{aligned}
$$

and

$$
U_t^O \simeq \ln\left(1-\tau_g^c-\tau_h^c\right) + (1+b)\frac{1-\alpha}{\alpha}\ln\tau_g^c + b\ln\tau_h^c, \qquad (6.5)
$$

respectively.

The political candidates in period t then choose (τ_g^c, τ_h^c) to maximize the political objective function (3.1) with U_t^Y and U_t^O given by (6.4) and (6.5). The following proposition establishes the equilibrium policy mix and the resulting economic growth rate.

Proposition 4 *(Voting Equilibrium under Commitment to Constant Policy Paths)*

The equilibrium policy mix under commitment to constant policy paths is given by

$$\tau_g^c = 1 - \alpha \quad \textit{and} \quad \tau_h^c = \frac{((1+n)\,\beta + \omega)\,\alpha b}{(1+n)\,(1 + 2\beta\,(1+b)) + \omega\,(1+b)} < \alpha. \tag{6.6}$$

Under this policy mix, the economy's growth factor of all per capita variables, government spending and wages is given by

$$\gamma^c = \frac{B}{1+n}(\alpha - \tau_h^c). \tag{6.7}$$

Proposition 4 reveals that policy makers in this voting equilibrium also set the share of output devoted to the productive public input equal to $1 - \alpha$.[24] The following corollary concerns public consumption spending on the elderly.

Corollary 4 *(Commitment to Constant Policy Paths vs. Politico-Economic Equilibrium)*

Comparing the share of output spent on the public consumption good for the elderly, τ_h^c of (6.6), to the corresponding expenditure share in the politico-economic equilibrium of Proposition 1 yields

$$\tau_h^c > \tau_h^{\mathcal{P}}.$$

Thus, policy makers that commit to a constant tax path will opt for a higher share of public consumption spending in output than in the politico-economic equilibrium without commitment. Intuitively, when voting on government policy today the current young are aware that they decide about the expenditure share for public good provision that benefits them tomorrow. Hence, they choose a higher share of output to be spent on these services than in the politico-economic equilibrium.

The implications of population aging on the voting equilibrium of Proposition 4 are summarized in the following corollary.

[24] In other words, they choose the same share as in the politico-economic equilibrium and as a Ramsey planner.

Corollary 5 *(Implications of Population Aging under Commitment to Constant Policy Paths)*

An increase in the old-age dependency ratio - either due to a permanent decline in the population growth rate or due to a permanent, but unexpected increase in life expectancy - does not affect τ_g^c*, but increases* τ_h^c *and* γ^c*. In the case of an increase in life expectancy the latter effects are delayed by one period.*

Thus, we can conclude that qualitatively the effects of population aging on government spending and economic growth in this voting equilibrium are the same as in the politico-economic equilibrium (see Corollary 3 and Proposition 3).

6.1.2 Myopic Voting Equilibrium

This section derives the equilibrium policy mix when agents vote myopically, i. e., when they ignore the effect of the current political decision on future political outcomes, and then compares it to the politico-economic equilibrium of Section 3.

More specifically, in a myopic voting equilibrium agents at t treat future policy variables, i. e., g_{t+1}, h_{t+1}, and τ_{t+1}, as given. However, they are aware that their policy choice today affects tomorrow's capital stock and output per worker. Then, the economic equilibrium at t continues to be characterized by equations (2.11) - (2.13). Moreover, using (2.13) we obtain consumption of an agent that is old in period $t+1$ as

$$\begin{aligned} c_{t+1}^o &= (1+n)\alpha\left(1-\tau_{t+1}\right)Ak_{t+1}^{\alpha}g_{t+1}^{1-\alpha} \\ &= (1+n)^{1-\alpha}\alpha A\tilde{B}^{\alpha}\left(Ak_t^{\alpha}g_t^{1-\alpha}-g_t-h_t\right)^a g_{t+1}^{1-\alpha}\left(1-\tau_{t+1}\right). \end{aligned}$$

Omitting all terms independent of policy and those that involve future policy variables (as they are treated as exogenous), the relevant indirect utilities of a young and an old agent at t are

$$U_t^Y \simeq (1+a\beta)\ln\left(Ak_t^{\alpha}g_t^{1-\alpha}-g_t-h_t\right) \tag{6.8}$$

and

$$U_t^O \simeq \ln\left(Ak_t^{\alpha}g_t^{1-\alpha}-g_t-h_t\right)+b\ln h_t, \tag{6.9}$$

respectively. The political candidates at t then choose (g_t, h_t) to maximize the political objective function (3.1) with U_t^Y and U_t^O given by (6.8) and (6.9). The following proposition provides the equilibrium policy mix and the resulting economic growth rate.

Proposition 5 *(Myopic Voting Equilibrium)*

The equilibrium policy mix under myopic voting for all t is given by

$$g_t = (1-\alpha)y_t \tag{6.10}$$

and

$$h_t = \tau_h^m y_t, \quad \textit{where} \quad \tau_h^m = \frac{\alpha\omega b}{(1+n)(1+\alpha\beta)+\omega(1+b)} < \alpha \tag{6.11}$$

and $y_t = A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}k_t$.

Under this policy mix, the economy's growth factor of all per capita variables, government spending and wages is given by

$$\gamma^m = \frac{B}{1+n}(\alpha - \tau_h^m). \tag{6.12}$$

Proposition 5 reveals that policy makers in this voting equilibrium again choose the same share of output to be devoted to the productive public input, namely $1-\alpha$. With respect to the equilibrium share of public consumption spending, we can establish the following corollary.

Corollary 6 *(Myopic Voting Equilibrium vs. Politico-Economic Equilibrium)*

Comparing the share of output spent on the public consumption good for the elderly given by (6.11) to the corresponding expenditure share in the politico-economic equilibrium of Proposition 1 yields

$$\tau_h^m > \tau_h^{\mathcal{P}}.$$

Thus, if agents vote myopically the equilibrium share of government expenditure for the public consumption good exceeds the one of the politico-economic equilibrium. The reason for this is that the young agents at t neglect that the choice of h_t via savings and the accumulation of capital affects tomorrow's provision, h_{t+1}. Therefore, they agree to a too high spending level today.

The implications of population aging on the myopic voting equilibrium of Proposition 5 are summarized in the following corollary.

Corollary 7 *(Implications of Population Aging on the Myopic Voting Equilibrium)*

An increase in the old-age dependency ratio - either due to a permanent decline in the population growth rate or due to a permanent, but unexpected increase in life expectancy - does not affect τ_g^m, but increases τ_h^m and γ^m. In the case of an increase in life expectancy the latter effects are delayed by one period.

Thus, we can conclude that qualitatively the effects of population aging on government spending and economic growth in a myopic voting equilibrium are the same as in the politico-economic equilibrium (see Corollary 3 and Proposition 3).

6.2 Alternative Utility Functions

This section presents several numerical examples to gauge the sensitivity of the comparative static effects of population aging to the specification of the utility function. In particular, we consider two alternative specifications: one with a constant intertemporal elasticity of substitution and the other with non-separable preferences between private and public consumption when old. Both specifications encompass the benchmark separable, log utility function of (2.1) as a special case. Otherwise the economic framework is as described in Section 2.

A necessary price of this sensitivity analysis is that (at some point) we must adopt specific parameters for the model. For this purpose, let a period represent 30 years. Then, set $\beta = 0.55$, implying a 2% annual discount rate. The parameter that measures the weight of public relative to private consumption in the utility function is $b = 0.1$. As there is no strong prior on ω, we simply assume equal political weights on the young and the old ($\omega = 1$).[25] Moreover, a parameter value for the output elasticity of productive government expenditure, $1 - \alpha$, is needed. As one period represents 30 years, it seems acceptable to suppose that an estimate of the output elasticity of public capital is a good proxy for $1 - \alpha$. Glomm and Ravikumar (1997) review the empirical results related to the output elasticity of public capital and find estimates in the range of zero to 0.39. Therefore, we choose $1-\alpha = 0.2$ as an intermediate value. This implies that the elasticity of output with respect to private capital corresponds to $\alpha = 0.8$. This appears reasonable if we consider that private capital encompasses physical as well as human capital.

We start with the assumption that the population growth rate is 2% annually. This annual rate corresponds to growth of 81% over a model period ($n = 1.02^{30} - 1 \simeq 0.81$). This in turn implies an old-age dependency ratio of $(1+n)^{-1} = 0.55$. Note that in a model where agents live for two periods, it is impossible to match the actual population growth rate and the old-age dependency ratio of a country. The above choice reflects this trade-off, with both the population growth rate and the dependency ratio being somewhat higher than currently in Europe or in the US.[26] Then, we investigate the comparative static effect resulting from a shift in the population growth rate from 2.0% to 1.0%. In other words, n declines to 0.35 and the dependency ratio rises to 0.74. Finally, the productivity parameter A is set such that the annual growth rate of per capita variables is 1.8% for the benchmark utility (2.1) when $n = 0.81$.

[25] However, we have solved for a range of economies with ω different from unity and holding constant the other parameters. The comparative static results are qualitatively unchanged. Moreover and equivalently to the log utility case, the results suggest that $d\tau_h^{\mathcal{P}}/d\omega > 0$.

[26] Introducing a survival probability $v \neq 1$ as in Section 5.2 allows - conditional on n - to calibrate the ratio of retirees to workers.

6.2.1 Constant Intertemporal Elasticity of Substitution Utility Function

This section generalizes the analysis to a more general constant intertemporal elasticity of substitution utility function. Assume that the preferences of an individual born at t are described by

$$\frac{\left(c_t^y\right)^{1-\sigma}-1}{1-\sigma}+\beta\left[\frac{\left(c_{t+1}^o\right)^{1-\sigma}-1}{1-\sigma}+b\frac{\left(\tilde{h}_{t+1}\right)^{1-\sigma}-1}{1-\sigma}\right], \tag{6.13}$$

where $\sigma > 0$ and $1/\sigma$ is the intertemporal elasticity of substitution. This specification includes the benchmark log utility for $\sigma \to 1$.

One aim of this generalization is to analyze whether there is a third channel (besides the two discussed in Section 5.1) by which a decline in the population growth rate potentially affects the steady-state growth rate. In a standard two-period OLG model under (6.13) with $b = 0$ and a neoclassical production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$ an increase in the capital stock per worker (e. g. due to decline in n) lowers the rental rate of capital. If the intertemporal elasticity of substitution is different from unity this in turn affects savings, and thus the accumulation of capital. However, in the present framework the interest rate (independent of the utility specification) turns out to be constant in the politico-economic equilibrium. Hence, this third channel is mute and we will see that the qualitative comparative static results are unchanged.

To see this, we first derive the economic equilibrium at t and then define the politico-economic equilibrium. Finally, numerical results for the equilibrium policy mix are presented. We consider the three cases: $\sigma = 0.5$, $\sigma = 1$, and $\sigma = 2$, with the other parameters as described above.

The Economic Equilibrium

Maximizing the lifetime utility of an individual born at t given by (6.13) subject to her per-period budget constraints, and then taking into account the remaining equilibrium conditions of Section 2.5, i. e., equations (2.8) - (2.10), yields the equilibrium allocation at t as

$$c_t^y = \frac{(1-\tau_t)\,w_t}{1+\beta^{\frac{1}{\sigma}}\left[(1-\tau_{t+1})\,R_{t+1}\right]^{\frac{1-\sigma}{\sigma}}} = \frac{(1-\alpha)\,(y_t-g_t-h_t)}{1+\beta^{\frac{1}{\sigma}}\left[\frac{\alpha(y_{t+1}-g_{t+1}-h_{t+1})}{k_{t+1}}\right]^{\frac{1-\sigma}{\sigma}}}, \tag{6.14}$$

$$c_t^o = k_t(1+n)\,(1-\tau_t)\,R_t = \alpha\,(1+n)\,(y_t-g_t-h_t)\,, \tag{6.15}$$

$$k_{t+1} = \frac{(1+n)^{-1}\,(1-\tau_t)\,w_t}{1+\beta^{-\frac{1}{\sigma}}\left[(1-\tau_{t+1})\,R_{t+1}\right]^{\frac{\sigma-1}{\sigma}}} = \frac{(1+n)^{-1}\,(1-\alpha)\,(y_t-g_t-h_t)}{1+\beta^{-\frac{1}{\sigma}}\left[\frac{\alpha(y_{t+1}-g_{t+1}-h_{t+1})}{k_{t+1}}\right]^{\frac{\sigma-1}{\sigma}}}. \tag{6.16}$$

The Politico-Economic Equilibrium

In a politico-economic equilibrium the public policy rules $\langle \pi^g, \pi^h \rangle$ have to maximize the political objective function $U_t = (1+n)\, U_t^Y + \omega U_t^O$ with

$$\begin{aligned} U_t^Y &= \frac{\left(c_t^y\right)^{1-\sigma} - 1}{1-\sigma} + \beta \frac{\left(c_{t+1}^o\right)^{1-\sigma} - 1}{1-\sigma} + \beta b \frac{\left(h_{t+1}(1+n)\right)^{1-\sigma} - 1}{1-\sigma} \\ &\text{and} \\ U_t^O &= \frac{\left(c_t^o\right)^{1-\sigma} - 1}{1-\sigma} + b \frac{\left(h_t(1+n)\right)^{1-\sigma} - 1}{1-\sigma}, \end{aligned}$$

subject to (6.14) - (6.16).

Making the same policy guess as in Section 3.3, i. e., $\pi^g(k_{t+1}) = \eta^g k_{t+1}$ and $\pi^h(k_{t+1}) = \eta^h k_{t+1}$, the economic equilibrium conditions (6.14) - (6.16) yield

$$\begin{aligned} c_t^y &= Y(y_t - g_t - h_t), \quad Y \equiv \frac{1-\alpha}{1 + \beta^{\frac{1}{\sigma}} \left[\alpha \left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)\right]^{\frac{1-\sigma}{\sigma}}}, \\ c_{t+1}^o &= Z(y_t - g_t - h_t), \quad Z \equiv \frac{(1-\alpha)\,\alpha \left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}{1 + \beta^{-\frac{1}{\sigma}} \left[\alpha \left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)\right]^{\frac{\sigma-1}{\sigma}}}, \\ k_{t+1} &= \frac{X(y_t - g_t - h_t)}{1+n}, \quad X \equiv \frac{1-\alpha}{1 + \beta^{-\frac{1}{\sigma}} \left[\alpha \left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)\right]^{\frac{\sigma-1}{\sigma}}}. \end{aligned}$$

Using the latter conditions and omitting additive constant terms, the political objective function simplifies to

$$\begin{aligned} U_t &= \left((1+n)\left(Y^{1-\sigma} + \beta Z^{1-\sigma} + \beta b \left(\eta^h X\right)^{1-\sigma}\right) + \omega\left(\alpha(1+n)\right)^{1-\sigma}\right) \frac{\left(y_t - g_t - h_t\right)^{1-\sigma}}{1-\sigma} \\ &+ \omega b (1+n)^{1-\sigma} \frac{\left(h_t\right)^{1-\sigma}}{1-\sigma}, \end{aligned} \tag{6.17}$$

and the equilibrium policy mix (g_t, h_t) has to maximize (6.17). The first-order conditions of this optimization problem with respect to g_t and h_t are

$$\frac{\left((1+n)\left(Y^{1-\sigma} + \beta Z^{1-\sigma} + \beta b \left(\eta^h X\right)^{1-\sigma}\right) + \omega\left(\alpha(1+n)\right)^{1-\sigma}\right)\left((1-\alpha)\frac{y_t}{g_t} - 1\right)}{\left(y_t - g_t - h_t\right)^{\sigma}} = 0$$

and

$$\frac{-\left((1+n)\left(Y^{1-\sigma} + \beta Z^{1-\sigma} + \beta b \left(\eta^h X\right)^{1-\sigma}\right) + \omega\left(\alpha(1+n)\right)^{1-\sigma}\right)}{\left(y_t - g_t - h_t\right)^{\sigma}} + \frac{\omega b (1+n)^{1-\sigma}}{\left(h_t\right)^{\sigma}} = 0,$$

respectively.

The former condition is fulfilled if and only if $g_t = (1-\alpha)\, y_t$ which verifies our guess for $\eta^g = A^{1/\alpha}(1-\alpha)^{1/\alpha}$. If a political equilibrium exists, i. e., if the guess for h_t can also be verified, then the above result implies that the equilibrium interest rate is constant and given by $R = \alpha A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$.[27]

Using $g_t = A^{1/\alpha}(1-\alpha)^{1/\alpha} k_t$ and the guess $h_t = \eta^h k_t$ in the second first-order condition then yields

$$\eta^h = \frac{\alpha D}{1+\left(\omega b\,(1+n)^{1-\sigma}\right)^{-\frac{1}{\sigma}}\left((1+n)\left(Y^{1-\sigma}+\beta\left(Z^{1-\sigma}+b\,(\eta^h X)^{1-\sigma}\right)\right)+\omega\,(\alpha\,(1+n))^{1-\sigma}\right)^{\frac{1}{\sigma}}} \tag{6.18}$$

where $D \equiv A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha} = \left(A\,(\eta^g)^{1-\alpha} - \eta^g\right)/\alpha$. For the guess to be correct condition (6.18) needs to have a unique solution for η^h in the interval $(0, \alpha D)$. As this problem cannot be solved analytically, the following section considers numerical examples for $\sigma = 0.5$ and $\sigma = 2$ and compares them to the benchmark case of $\sigma = 1$.

Numerical Results

In this section, we set $A = 34.5$, implying an annual growth rate of per capita variables of 1.8% if $\sigma = 1$. For both choices of σ exists a unique $\eta^h \in (0, \alpha D)$ that solves (6.18). The results are summarized in Table 2. Note that $\tau_h^{\mathcal{P}} \equiv \eta^h/D$ denotes the share of public consumption spending that benefits the elderly in aggregate output.[28]

Table 2: Comparative Static Analysis of Demographic Change: CIES Utility

σ		$n = 0.81$	$n = 0.35$
0.5	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0009	0.0011
	annual p.c. growth rate	5.16%	6.19%
1	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0200	0.0245
	annual p.c. growth rate	1.80%	2.78%
2	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0475	0.0621
	annual p.c. growth rate	-2.11%	-1.19%

[27] For this reason, the savings decision even in the CIES case with $\sigma \neq 1$ is independent of n and it does not matter whether the change in n is anticipated or not.

[28] All examples were computed using Maple. All files are available upon request.

Table 2 suggests that an intertemporal elasticity of substitution different from unity does not alter the qualitative comparative static results of Section 5.1, i. e., $d\tau_h^{\mathcal{P}}/dn < 0$ and $d\gamma/dn < 0$. Nevertheless, the equilibrium ratio of public spending on the elderly and the equilibrium growth rate depend on σ. The numerical examples reveal that $d\tau_h^{\mathcal{P}}/d\sigma > 0$ and $d\gamma/d\sigma < 0$. Intuitively, a greater intertemporal elasticity of substitution (i. e., a smaller σ) implies a stronger negative substitution effect of a higher tax rate on savings such that households prefer a lower tax rate, which in turn involves a higher growth rate.

6.2.2 Non-Separable Preferences

This section generalizes the analysis to non-separable preferences between private and public consumption when old. Assume that the preferences of an individual born at t are given by

$$\ln c_t^y + \beta \ln \left(\left[\frac{1}{1+b} \left(c_{t+1}^o \right)^{\rho} + \frac{b}{1+b} \left(\tilde{h}_{t+1} \right)^{\rho} \right]^{\frac{1}{\rho}} \right), \tag{6.19}$$

where $\rho < 1$. This specification encompasses the benchmark separable log utility as $\rho \rightarrow 0$.[29] Private and public consumption when old are substitutes if $\rho > 0$ and complements if $\rho < 0$. This generalization has interesting implications: for instance, if agents can substitute private for public health services when old they will be less concerned for public good provision and vote for a lower tax rate. Nevertheless, the qualitative comparative static results with respect to population aging will not be affected by this generalization.

Equivalently to Section 6.2.1, we first determine the economic equilibrium and then analyze the politico-economic equilibrium analytically. To analyze the comparative static effects of a decline in the population growth rate we consider three numerical examples for $\rho = -0.1$, $\rho = 0$, and $\rho = 0.1$, with the other parameters as before.

The Economic Equilibrium

Maximizing the lifetime utility (6.19) with respect to an individual's per-period budget constraints delivers the following implicit characterization of optimal savings at t

$$\beta \left(1 - \tau_t\right) w_t \quad = \quad s_t \left[1 + \beta + b \left(\frac{\tilde{h}_{t+1}}{s_t \left(1 - \tau_{t+1}\right) R_{t+1}} \right)^{\rho} \right]. \tag{6.20}$$

Optimal consumption of a young and an old agent at t then follow from the respective per-period budget constraints.

Taking into account the equilibrium conditions (2.8) - (2.10), equation (6.20) becomes

$$\beta \left(1 - \alpha\right) \left(y_t - g_t - h_t\right) = k_{t+1} \left(1 + n\right) \left[1 + \beta + b \left(\frac{h_{t+1}}{\alpha \left(y_{t+1} - g_{t+1} - h_{t+1}\right)} \right)^{\rho} \right]. \tag{6.21}$$

[29]Note that for $\rho \rightarrow 0$ (6.19) reduces to $\ln c_t^y + \beta/(1+b) \left(\ln c_{t+1}^o + b \ln \tilde{h}_{t+1} \right)$. This specification only differs from the benchmark utility (2.1) by a constant factor which does not affect the qualitative results.

The Politico-Economic Equilibrium

In a politico-economic equilibrium the public policy rules $\langle \pi^g, \pi^h \rangle$ have to maximize the political objective function $U_t = (1+n)\,U_t^Y + \omega U_t^O$ with the indirect utilities of the young and the old at t (disregarding terms independent of policy) given by

$$\begin{aligned} U_t^Y &\simeq \ln c_t^y + \frac{\beta}{\rho} \ln \left[\left(c_{t+1}^o\right)^\rho + b\,(h_{t+1})^\rho\,(1+n)^\rho \right] \\ &\text{and} \\ U_t^O &\simeq \frac{1}{\rho} \ln \left[(c_t^o)^\rho + b\,(h_t)^\rho\,(1+n)^\rho \right]. \end{aligned}$$

With the linear policy guess, $\pi^g(k_{t+1}) = \eta^g k_{t+1}$ and $\pi^h(k_{t+1}) = \eta^h k_{t+1}$, condition (6.21) can be written as

$$k_{t+1}(1+n) = \frac{\beta(1-\alpha)(y_t - g_t - h_t)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}\right)^\rho}. \tag{6.22}$$

Moreover, using $s_t = k_{t+1}(1+n)$ and (6.21) in the per-period budget constraints yields consumption of a young and an old agent at t as

$$c_t^y = X\,(y_t - g_t - h_t), \text{ where } X \equiv \frac{(1-\alpha)\left(1+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}\right)^\rho\right)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}\right)^\rho} \tag{6.23}$$

and

$$c_t^o = (1+n)\,\alpha\,(y_t - g_t - h_t), \tag{6.24}$$

respectively. Additionally, we obtain the levels of private and public consumption of an old agent at $t+1$ as

$$c_{t+1}^o = Y\,(y_t - g_t - h_t), \text{ where } Y \equiv \frac{\beta(1-\alpha)\,\alpha\left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}\right)^\rho} \tag{6.25}$$

and

$$\tilde{h}_{t+1} = Z\,(y_t - g_t - h_t), \text{ where } Z \equiv \frac{\eta^h \beta(1-\alpha)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right)}\right)^\rho}, \tag{6.26}$$

respectively. Using conditions (6.23) - (6.26) in the indirect utility functions and omitting terms independent of policy variables, the political objective function simplifies to

$$U_t = (1+n)(1+\beta)\ln(y_t - g_t - h_t) + \frac{\omega}{\rho}\ln\left[\alpha^\rho (y_t - g_t - h_t)^\rho + b\,(h_t)^\rho\right] \tag{6.27}$$

and the equilibrium policy mix (g_t, h_t) has to maximize (6.27). The first-order conditions of this optimization problem with respect to g_t and h_t are

$$\left[(1-\alpha)\frac{y_t}{g_t}-1\right]\left[\frac{(1+n)(1+\beta)}{y_t-g_t-h_t}+\frac{\omega\alpha^\rho (y_t-g_t-h_t)^{\rho-1}}{\alpha^\rho (y_t-g_t-h_t)^\rho + b(h_t)^\rho}\right]=0$$

and

$$\frac{-(1+n)(1+\beta)}{y_t-g_t-h_t}+\omega\frac{-\alpha^\rho (y_t-g_t-h_t)^{\rho-1}+b(h_t)^{\rho-1}}{\alpha^\rho (y_t-g_t-h_t)^\rho + b(h_t)^\rho}=0,$$

respectively.

The first condition is fulfilled if and only if $g_t = (1-\alpha)y_t$ which verifies our guess for $\eta^g = A^{1/\alpha}(1-\alpha)^{1/\alpha}$. If a political equilibrium exists, i. e., if the guess for h_t can also be verified, then the above result again implies that the equilibrium interest rate is constant and given by $R = \alpha D$, where $D \equiv A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$.

Then, using $g_t = A^{1/\alpha}(1-\alpha)^{1/\alpha} k_t$ and the guess $h_t = \tau_h^{\mathcal{P}} y_t$, where $\tau_h^{\mathcal{P}} \equiv \eta^h / D$, in the second first-order condition and rearranging yields

$$(1+n)(1+\beta) \quad = \quad \omega\frac{-1+\alpha^{-\rho}b\left(\tau_h^{\mathcal{P}}\right)^{\rho-1}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{1-\rho}}{1+\alpha^{-\rho}b\left(\tau_h^{\mathcal{P}}\right)^{\rho}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{-\rho}}. \tag{6.28}$$

For the guess to be correct, condition (6.28) needs to have a unique solution $\tau_h^{\mathcal{P}}$ in the interval $(0,\alpha)$. This is the case for not too large values of ρ. For a proof of this see the Appendix.

Numerical Results

To analyze the comparative static effect of a decline in the population growth rate, this section considers numerical examples for $\rho = -0.1$ and $\rho = 0.1$ and compares them to the benchmark case of $\rho = 0$.

In the examples of this section, we set $A = 36.198$ in order to again obtain an annual growth rate of per capita variables of 1.80% if $\rho = 0$. For both choices of ρ, there exists a unique $\tau_h^{\mathcal{P}} \in (0,\alpha)$ that solves (6.28). The results are summarized in Table 3.

Table 3 suggests that allowing for non-separable preferences between private and public consumption when old does not change the qualitative comparative static results of Section 5.1, i. e., $d\tau_h^{\mathcal{P}}/dn < 0$ and $d\gamma/dn < 0$.

Table 3: Comparative Static Analysis of Demographic Change: Non-Separable Preferences

ρ		$n = 0.81$	$n = 0.35$
-0.1	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0254	0.0308
	annual p.c. growth rate	1.70%	2.68%
0	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0191	0.0235
	annual p.c. growth rate	1.80%	2.78%
0.1	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0134	0.0168
	annual p.c. growth rate	1.89%	2.87%

Moreover, the numerical examples reveal that the equilibrium ratio of public spending on the elderly declines in ρ, i. e., $d\tau_h^{\mathcal{P}}/d\rho < 0$. Intuitively, a higher degree of substitutability between private and public consumption goods makes the old less concerned for the public consumption good and induces them to vote for a lower spending ratio.

7 Concluding Remarks

What is the role of population aging for the composition of government spending and long-term economic growth? This paper addressed this question in an overlapping generations model in which economic growth is endogenous and agents each period vote on the composition of government spending between productive public expenditure and public consumption spending on the elderly. Population aging corresponds either to a decline in the population growth rate or to an increase in life expectancy. Both phenomena increase the economy's old-age dependency ratio.

The model predicts that population aging, by increasing the relative weight of the old in the political process, leads to an increase in public spending on the elderly (as a share of output), but does not affect the share of public productive expenditure in output. This is in line with recent evidence (see Section 1). To finance the additional government spending, the income tax rate has to increase, which in turn has a negative effect on the economy's growth rate of per capita variables. However, the model also suggests that population aging overall accelerates the economy's growth rate. If the increase in the old-age dependency ratio is due to a decline in the population growth rate, then reduced capital dilution is at the source of this acceleration of growth. By contrast, an increase in life expectancy generates higher long-term growth by strengthening the incentives to save.

The present analysis leaves scope for future research. For instance, for analytical tractability this paper introduced the productive public input as a flow into production. Considering that the length of a model period corresponds to one generation, this appears to be a good benchmark. Alternatively, one could treat the publicly-provided productive input as a stock rather than as a flow, thereby introducing public as well as private capital. In this case the advantages of a larger public investment today only materialize tomorrow whereas the tax costs have to be borne today. Then, the young and the old are no longer symmetrically affected by current public productive spending. Additionally, the stock approach introduces transitional equilibrium dynamics into the analysis. This would allow us to study the effects of the projected demographic transition not only on the steady state but also on the dynamics of transition between steady states. A second suggestion for future research is to disentangle the uniform income tax rate into a separate labor and capital income tax rate. This introduces another dimension of policy choice and a further source of potential conflict between the young and the old. It would be interesting to see whether in this case the public consumption good that benefits the elderly will be entirely financed via capital income taxes and the productive public input via both types of taxes.

Appendix

Detailed Derivation of Condition (3.3)

The first-order conditions of the program (3.2) with respect to g_t and h_t are

$$\bar{U}_{g_t} = \frac{(1-\alpha)\, y_t - g_t}{g_t\,(y_t - g_t - h_t)}\left[(1+n)\,(1+\beta\,(1+b)) + \omega\right] = 0 \quad (.1)$$

$$\bar{U}_{h_t} = -\frac{(1+n)\,(1+\beta\,(1+b)) + \omega}{y_t - g_t - h_t} + \frac{\omega b}{h_t} = 0, \quad (.2)$$

where $\bar{U}_x \equiv \partial \bar{U}/\partial x$. The first condition is fulfilled if and only if $g_t = (1-\alpha)y_t$. Using this in the second condition and rearranging yields

$$\left[(1+n)\,(1+\beta\,(1+b)) + \omega\right] h_t = \omega b\,(\alpha y_t - h_t)\,, \quad (.3)$$

and thus

$$h_t = \underbrace{\frac{\alpha\omega b}{(1+n)\,(1+\beta\,(1+b)) + \omega\,(1+b)}}_{\equiv \tau_h^{\mathcal{P}}} y_t. \quad (.4)$$

Thus, the unique interior solution is given by $g_t = (1-\alpha)y_t$ and $h_t = \tau_h^{\mathcal{P}} y_t$ as stated in the main text.

Proof of Proposition 1

In addition to what is stated in the text, it remains to be verified that (i) the first-order conditions are sufficient for a global maximum, (ii) the economy's relevant variables grow at the rate $\gamma - 1$.

(i) The unique interior solution derived above is a global maximum if

$$\bar{U}_{g_t g_t} < 0, \bar{U}_{h_t h_t} < 0 \text{ and } \bar{U}_{g_t g_t}\bar{U}_{h_t h_t} - \left(\bar{U}_{g_t h_t}\right)^2 > 0, \quad \text{for any} \quad (g_t, h_t),$$

where $\bar{U}_{xy} \equiv \partial^2 \bar{U}/\partial x \partial y$. First, note that

$$\begin{aligned}
\bar{U}_{g_t g_t} &= -\left[(1+n)(1+\beta(1+b))+\omega\right] \frac{\alpha(1-\alpha) y_t (y_t - g_t - h_t) + \left[(1-\alpha) y_t - g_t\right]^2}{g_t^2 (y_t - g_t - h_t)^2} \\
&< 0 \\
\bar{U}_{h_t h_t} &= \frac{-\left[(1+n)(1+\beta(1+b))+\omega\right]}{(y_t - g_t - h_t)^2} - \frac{\omega b}{(h_t)^2} < 0.
\end{aligned}$$

Then, $\bar{U}_{g_t g_t}\bar{U}_{h_t h_t}$ can be written as

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t} = \left[(1+n)(1+\beta(1+b))+\omega\right]^2 \frac{\left[(1-\alpha)\frac{y_t}{g_t} - 1\right]^2}{(y_t - g_t - h_t)^4} + X + Y + Z,$$

where X, Y, and Z are positive constants. Moreover,

$$\bar{U}_{g_t h_t} = -\left[(1+n)(1+\beta(1+b))+\omega\right] \frac{\left[(1-\alpha)\frac{y_t}{g_t} - 1\right]}{(y_t - g_t - h_t)^2},$$

and thus

$$\left(\bar{U}_{g_t h_t}\right)^2 = \left[(1+n)(1+\beta(1+b))+\omega\right]^2 \frac{\left[(1-\alpha)\frac{y_t}{g_t} - 1\right]^2}{(y_t - g_t - h_t)^4}$$

such that

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t} - \left(\bar{U}_{g_t h_t}\right)^2 = X + Y + Z > 0, \quad \text{for any} \quad (g_t, h_t).$$

■

(ii) First, it is straightforward that, as in the standard AK model, there are no transitional dynamics such that the economy immediately jumps onto its steady-state path. Moreover, note that output per worker in equilibrium is linear in the capital stock per worker k, and thus has to grow at the same rate as k, namely at rate $\gamma - 1$.

Then, output per capita at t is given by

$$\frac{Y_t}{L_t + L_{t-1}} = \frac{y_t L_t}{L_t + L_{t-1}} = \frac{y_t}{1 + L_{t-1}/L_t} = \frac{1+n}{2+n} y_t,$$

and is thus proportional to output per worker and has to grow at the same rate. Using (2.11) and (2.12), consumption per capita at t obtains as

$$\begin{aligned}\frac{C_t}{L_t+L_{t-1}} &= \frac{c_t^y L_t + c_t^o L_{t-1}}{L_t+L_{t-1}} = \frac{L_t}{L_t+L_{t-1}}\left(c_t^y + \frac{c_t^o}{1+n}\right) \\ &= \frac{1+n}{2+n}\frac{1+\alpha\beta}{1+\beta}(1-\tau)y_t,\end{aligned}$$

and is also proportional to output per worker. Similar arguments apply to all other relevant variables such as government spending and wages. ■

Proof of Corollary 1

Assume that the economic environment is identical to that of the previous sections, except in a final period T where there is a generation of newborns that lives only for one period. The consumption of old and young households in this period are given by

$$\begin{aligned} c_T^y &= (1-\tau_T)\, w_T = (1-\alpha)\left(Ak_T^\alpha g_T^{1-\alpha} - g_T - h_T\right), &(.5)\\ c_T^o &= (1-\tau_T)\, R_T s_{T-1} = \alpha(1+n)\left(Ak_T^\alpha g_T^{1-\alpha} - g_T - h_T\right), &(.6)\end{aligned}$$

respectively. The policymaker then chooses g_T and h_T to maximize the political objective function $U_T = (1+n)\,U_T^Y + \omega U_T^O = (1+n)\ln c_T^y + \omega \ln c_T^o + \omega b \ln h_T$. Omitting terms independent of the policy choices g_T and h_T, the political objective function reduces to

$$U_T \simeq (1+n+\omega)\ln\left(Ak_T^\alpha g_T^{1-\alpha} - g_T - h_T\right) + \omega b \ln h_T. \tag{.7}$$

The first-order conditions of maximizing (.7) with respect to g_T and h_T yield

$$\begin{aligned} g_T &= (1-\alpha)y_T \quad \text{and} \quad h_T = \frac{\omega b\alpha}{1+n+\omega\,(1+b)}y_T, &(.8)\\ &\text{with} \quad y_T = A(1-\alpha)^{1-\alpha}k_T. \end{aligned}$$

Now we can proceed by backward induction. In period $T-1$ voters choose g_{T-1} and h_{T-1}, correctly anticipating g_T and h_T , to maximize

$$\begin{aligned} U_{T-1} &= (1+n)\,U_{T-1}^Y + \omega U_{T-1}^O \\ &= (1+n)\ln c_{T-1}^y + \beta\,(1+n)\left[\ln c_T^o + b\ln h_T\right] + \omega\left[\ln c_{T-1}^o + b\ln h_{T-1}\right] \end{aligned}$$

with c_T^o given by (.6), c_{T-1}^y follows from (2.11) for $t = T-1$ and c_{T-1}^o from (2.12) for $t = T-1$. Using g_T and h_T of (.8) as well as k_T of (2.13) for $t = T-1$ and omitting terms independent of policy variables the political objective function at $T-1$ can be written as

$$U_{T-1} \simeq \left[(1+n)\,(1+\beta\,(1+b)) + \omega\right]\ln\left(Ak_{T-1}^\alpha g_{T-1}^{1-\alpha} - g_{T-1} - h_{T-1}\right) + \omega b \ln h_{T-1}. \tag{.9}$$

After some algebra, the first-order conditions of maximizing (.9) with respect to g_{T-1} and h_{T-1} yield

$$g_{T-1} = (1-\alpha)y_{T-1} \tag{.10}$$

and

$$h_{T-1} = \frac{\alpha\omega b}{(1+n)(1+\beta(1+b))+\omega(1+b)} y_{T-1} = \tau_h^{\mathcal{P}} y_{T-1}, \tag{.11}$$

where $y_{T-1} = A(1-\alpha)^{1-\alpha}k_{T-1}$. The policy functions (.10) and (.11) correspond to the equilibrium policy functions of the infinite-horizon economy (see equation 3.3). Proceeding in the same way for all preceding periods one readily verifies that this equilibrium policy mix results for all periods $t = 0, 1, ..., T-1$. ■

Proof of Lemma 1

First note that the indirect utility of a young agent of generation t given by (2.14) is additively separable in (h_t, g_t, k_t) and $(h_{t+1}, g_{t+1}, k_{t+1})$, i. e.,

$$U_t^Y(h_t, g_t, k_t, h_{t+1}, g_{t+1}, k_{t+1}) = P_t(g_t, h_t, k_t) + Q_{t+1}(g_{t+1}, h_{t+1}, k_{t+1}),$$

where

$$\begin{aligned} P_t(g_t, h_t, k_t) &\equiv \ln\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) \quad \text{and} \\ Q_{t+1}(g_{t+1}, h_{t+1}, k_{t+1}) &\equiv \beta \ln\left(Ak_{t+1}^\alpha g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1}\right) + \beta b \ln h_{t+1}. \end{aligned}$$

Then, the Ramsey planner's objective function in (4.1) can be expressed as

$$\begin{aligned} &\max_{\{g_t,h_t,k_{t+1}\}_{t=0}^\infty} W(\cdot) \equiv \max_{\{g_t,h_t,k_{t+1}\}_{t=0}^\infty} \left\{\beta U_0^O + \sum_{t=0}^\infty ((1+n)\beta)^{t+1} U_t^Y\right\} \\ &= \max_{g_0,h_0,k_1} \left\{\beta U_0^O + \max_{\{g_t,h_t,k_{t+1}\}_{t=1}^\infty} \sum_{t=0}^\infty ((1+n)\beta)^{t+1} U_t^Y\right\} \\ &= \max_{g_0,h_0,k_1} \left\{\beta U_0^O + \max_{g_1,h_1,k_2} \left\{(1+n)\beta U_0^Y + \max_{\{g_t,h_t,k_{t+1}\}_{t=2}^\infty} \sum_{t=1}^\infty ((1+n)\beta)^{t+1} U_t^Y\right\}\right\} \\ &= \max_{g_0,h_0,k_1} \left\{\beta U_0^O + \max_{g_1,h_1,k_2} \left\{(1+n)\beta\left[P_0(\cdot) + Q_1(\cdot)\right] + \max_{\{\cdot\}_{t=2}^\infty} \sum_{t=1}^\infty ((1+n)\beta)^{t+1} U_t^Y\right\}\right\} \\ &= \max_{g_0,h_0.k_1} \left\{\beta\left(U_0^O + (1+n)P_0(\cdot)\right) + \max_{\{\cdot\}_{t=1}^\infty} \sum_{t=1}^\infty (1+n)^t \beta^t \left[Q_t(\cdot) + (1+n)\beta P_t(\cdot)\right]\right\}, \end{aligned} \tag{.12}$$

where the argument of $\{\cdot\}$ is g_t, h_t, k_{t+1}.

Now let $T_t(g_t, h_t, k_t) \equiv Q_t(g_t, h_t, k_t) + (1+n)\beta P_t(g_t, h_t, k_t)$ and note that from (2.15) and the definition of P_t follows $\beta U_0^O + (1+n)\beta P_0(\cdot) = \beta Q_0(\cdot) + (1+n)\beta P_0(\cdot) = T_0(\cdot)$

such that (.12) can be written as

$$\max_{\{g_t,h_t,k_{t+1}\}_{t=0}^{\infty}} W(\cdot) = \max_{g_0,h_0,k_1} \left\{ T_0\left(g_0,h_0,k_0\right) + \max_{\{g_t,h_t,k_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} (1+n)^t \beta^t T_t\left(g_t,h_t,k_t\right) \right\}. \tag{.13}$$

Defining the value function

$$V\left(k_t\right) \equiv \max_{\{g_{t+s},h_{t+s},k_{t+1+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} (1+n)^s \beta^s T_{t+s}\left(g_{t+s},h_{t+s},k_{t+s}\right),$$

standard recursion on (.13) yields the functional Bellman equation (4.2). ■

Proof of Proposition 2

In order to solve the Ramsey problem, we start by guessing that the solution to the functional equation (4.2) takes the form of $V(k) = a_0 + a_1 \ln k$ for all k, where a_0 and a_1 are yet undetermined coefficients. Then, the Bellman equation becomes

$$\begin{aligned} a_0 + a_1 \ln k_t = \quad & \max_{\{g_t,h_t,k_{t+1}\}} \quad \{(2+n)\,\beta \ln\left(Ak_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) + \beta b \ln h_t \\ & +(1+n)\beta a_0 + (1+n)\beta a_1 \ln k_{t+1}\} \end{aligned}$$

subject to (2.13). Substituting for k_{t+1}, the Bellman equation reduces to

$$\begin{aligned} a_0 + a_1 \ln k_t = \quad & \max_{\{g_t,h_t\}} \quad \{\beta\left(1+(1+n)\left(1+a_1\right)\right) \ln\left(Ak_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) \\ & +\beta b \ln h_t + (1+n)\beta a_0 + (1+n)\beta a_1 \ln \tilde{B}\}. \end{aligned} \tag{.14}$$

After some algebra, the first-order conditions with respect to g_t and h_t yield

$$g_t \quad = \quad (1-\alpha) y_t \quad \text{and} \quad h_t = \frac{b\alpha}{1+b+(1+n)\left(1+a_1\right)} y_t, \tag{.15}$$

with $y_t = A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$. Using this in (.14) and collecting the terms that multiply $\ln k_t$ results in

$$\begin{aligned} a_0 + a_1 \ln k_t \quad = \quad & \beta\left(1+b+(1+n)\left(1+a_1\right)\right) \ln k_t + (1+n)\beta a_0 + (1+n)\beta a_1 \ln \tilde{B} \\ & +\beta\left(1+(1+n)\left(1+a_1\right)\right) \ln\left[\frac{1+(1+n)\left(1+a_1\right)}{1+b+(1+n)\left(1+a_1\right)} \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}\right] \\ & +\beta b \ln\left[\frac{b\alpha}{\left(1+b+(1+n)\left(1+a_1\right)\right)} A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]. \end{aligned}$$

The functional equation holds for all k if and only if $a_1 = \beta\left(1+b+(1+n)\left(1+a_1\right)\right)$. This in turn implies that

$$a_1 \quad = \quad \frac{\beta\left(2+b+n\right)}{1-\beta(1+n)}$$

is required for a solution. This expression can then be used to solve for a_0. Thus, it has been verified that the tentative guess is indeed a solution to the functional equation. Substitution of a_1 in (.15) then yields $\tau_h^{\mathcal{R}}$ of Proposition 2. ■

Proof of Corollary 2

The result follows directly from comparing $\tau_h^{\mathcal{R}}$ of Proposition 2 to $\tau_h^{\mathcal{P}}$ of Proposition 1. ■

Proof of Corollary 3

1. From Proposition 1 we have

$$\tau_h^{\mathcal{P}} = \frac{\alpha\omega b}{(1+n)\left[1+\beta(1+b)\right]+\omega(1+b)}. \tag{.16}$$

   Partial derivation of (.16) with respect to n immediately yields $d\tau_h^{\mathcal{P}}/dn < 0$. Moreover, $\tau_g = 1-\alpha$ such that $d\tau_g/dn = 0$. The comparative static result for τ immediately follows from the definition of τ and from the first two results.

2. Using (.16) in (3.5) yields the equilibrium growth rate as

$$\gamma = \alpha B \frac{\left[1+\beta(1+b)\right]+\omega/(1+n)}{(1+n)\left[1+\beta(1+b)\right]+\omega(1+b)}.$$

   Then, partial derivation immediately yields $d\gamma/dn < 0$. ■

Proof of Proposition 3

In the presence of a perfect annuity market, an individual born at t chooses the plan (c_t^y, c_{t+1}^o, s_t) to maximize her lifetime utility (2.1) subject to $c_t^y + s_t = (1-\tau_t)w_t$ and $c_{t+1}^o = s_t(1-\tau_{t+1})R_{t+1}/v$. Writing the problem like this uses the fact that the assets at $t+1$ of a member of generation t are equal to $s_t + (1-v)s_t/v = s_t/v$. Moreover, it incorporates the results of Yaari (1965) and Sheshinski and Weiss (1981) according to which individuals without a bequest motive want to annuitize all their wealth. The optimal choices of a member of cohort t are given by (2.5), (2.7), and $c_{t+1}^o = \beta(1-\tau_t)w_t(1-\tau_{t+1})R_{t+1}/v(1+\beta)$ with $\beta \equiv \beta_v v$. Then, one readily verifies that all other equations in Sections 2 and 3 remain valid.[30]

As an increase in the survival probability v raises the effective discount factor β, equation (2.7) yields $\partial s_t/\partial v > 0$. Thus, for a given government policy, an increase in v raises savings per worker. This, in turn has a positive effect on the growth rate of capital per worker, see equation (3.5). However, we assume that the increase in life expectancy is unexpected for generation 1 such that it makes its plan (c_1^y, s_1, c_2^o) without anticipating the increase of the survival probability from v to $\hat{v}$. Hence, the positive growth effect only materializes from generation 2 onwards.

[30]The only exception is equation (2.12) that modifies to $c_t^o = \alpha(1+n)\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right)/v$.

The second effect of an increase in life expectancy is that the effective weight of the old, $\omega = \omega_v v$, in the political objective function (3.1) increases. However, this effect only becomes effective from period 2 onwards too. In period $t = 1$, the young of generation 1 and the old of generation 0, whose size is determined by the initial life expectancy v, vote on government policy. Thus, in $t = 1$ government policy is unaffected by an increase in the survival probability. This in turn implies that the accumulation rule that determines the capital stock per worker in period 2 is unchanged. Thus, $\hat{\tau}_{h_1}^{\mathcal{P}}$ and $\hat{\gamma}_2$ correspond to $\tau_h^{\mathcal{P}}$ and γ of Proposition 1 with $\omega = \omega_v v$.

By contrast, from period 2 onwards the relevant effective discount factor and the effective political weight are $\hat{\beta} \equiv \beta_v \hat{v}$ and $\hat{\omega} \equiv \omega_v \hat{v}$. Then, the politico-economic equilibrium for any $t = 2, 3, ..., \infty$ is characterized by

$$\hat{\tau}_{gt} = 1 - \alpha \equiv \hat{\tau}_g, \tag{.17}$$

$$\hat{\tau}_{ht}^{\mathcal{P}} = \frac{\alpha \omega_v \hat{v} b}{(1+n)\left[1 + \hat{v}\beta_v (1+b)\right] + \omega_v \hat{v}(1+b)} \equiv \hat{\tau}_h^{\mathcal{P}}, \tag{.18}$$

$$\hat{\gamma}_{t+1} = \frac{\beta_v \hat{v}}{1 + \beta_v \hat{v}} \left(\alpha - \hat{\tau}_h^{\mathcal{P}}\right) X \equiv \hat{\gamma} \tag{.19}$$

where $X \equiv A^{1/\alpha} (1-\alpha)^{1/\alpha} / (1+n)$.

Partial derivation of (.18) with respect to $\hat{v}$ gives

$$\begin{aligned} \frac{d\hat{\tau}_h^{\mathcal{P}}}{d\hat{v}} &= \frac{\alpha \omega_v b \left[(1+n)(1 + \hat{v}\beta_v (1+b)) + \omega_v \hat{v}(1+b) - (1+n)\hat{v}\beta_v (1+b) - \omega_v \hat{v}(1+b)\right]}{\left[(1+n)\left[1 + \hat{v}\beta_v (1+b)\right] + \omega_v \hat{v}(1+b)\right]^2} \\ &= \frac{\alpha \omega_v b (1+n)}{\left[(1+n)\left[1 + \hat{\beta}(1+b)\right] + \hat{\omega}(1+b)\right]^2} > 0. \end{aligned} \tag{.20}$$

Moreover,

$$\frac{\partial \hat{\gamma}}{\partial \hat{v}} = \frac{\beta_v}{\left(1 + \hat{\beta}\right)^2} \left(\alpha - \hat{\tau}_h^{\mathcal{P}}\right) X - \frac{\hat{\beta}}{1 + \hat{\beta}} \frac{d\hat{\tau}_h^{\mathcal{P}}}{d\hat{v}} X = \frac{X \beta_v}{1 + \hat{\beta}} \left(-\hat{v} \frac{d\hat{\tau}_h^{\mathcal{P}}}{d\hat{v}} + \frac{\alpha - \hat{\tau}_h^{\mathcal{P}}}{1 + \hat{\beta}}\right). \tag{.21}$$

Using (.20) in (.21) and rearranging yields

$$\begin{aligned} \frac{\partial \hat{\gamma}}{\partial \hat{v}} &= \frac{X \beta_v \alpha}{1 + \hat{\beta}} \frac{(1+n)^2 \left(1 + \hat{\beta}(1+b)\right)^2 + \hat{\omega}^2 (1+b) + \hat{\omega}(1+n)\left(2\left(1 + \hat{\beta}(1+b)\right) + \hat{\beta} b^2\right)}{\left[(1+n)\left(1 + \hat{\beta}(1+b)\right) + \hat{\omega}(1+b)\right]^2} \\ &> 0. \end{aligned}$$

Thus, $\hat{\tau}_h^{\mathcal{P}} > \tau_h^{\mathcal{P}}$ and $\hat{\gamma} > \gamma$ for all $t = 2, 3, ..., \infty$. ■

Proof of Proposition 4

Substituting (6.4) and (6.5) in the political objective function (3.1) yields the program to be solved by the political mechanism as $\max_{\{\tau_g^c,\tau_h^c\}} \bar{U}_t$ with

$$\begin{aligned}\bar{U}_t &= [(1+n)(1+\beta(2+b))+\omega]\ln\left(1-\tau_g^c-\tau_h^c\right)+[(1+n)\beta+\omega]\,b\ln\tau_h^c \\ &+ [(1+n)(1+2\beta(1+b))+\omega(1+b)](1-\alpha)/\alpha\ln\tau_g^c.\end{aligned}$$

The first-order conditions of the above program with respect to τ_g^c and τ_h^c yield

$$\frac{(1+n)(1+\beta(2+b))+\omega}{1-\tau_g^c-\tau_h^c} = \frac{(1-\alpha)[(1+n)(1+2\beta(1+b))+\omega(1+b)]}{\alpha\tau_g^c} \tag{.22}$$

and

$$\frac{(1+n)(1+\beta(2+b))+\omega}{1-\tau_g^c-\tau_h^c} = \frac{(1+n)\beta b+\omega b}{\tau_h^c}. \tag{.23}$$

Combining (.22) and (.23) yields

$$\tau_g^c = \frac{(1-\alpha)[(1+n)(1+2\beta(1+b))+\omega(1+b)]}{[(1+n)\beta+\omega]\alpha b}\tau_h^c. \tag{.24}$$

Substituting (.24) in (.23) and solving for τ_h^c yields

$$\tau_h^c = \frac{((1+n)\beta+\omega)\alpha b}{(1+n)(1+2\beta(1+b))+\omega(1+b)} < \alpha,$$

which is τ_h^c of (6.6). Finally, using (6.6) in (.24) yields $\tau_g^c = 1-\alpha$.

The policy mix of (6.6) is the global maximizer of the political objective function. To see this note that

$$\begin{aligned}\bar{U}_{\tau_g^c\tau_g^c} &= -\frac{(1+n)(1+\beta(2+b))+\omega}{\left(1-\tau_g^c-\tau_h^c\right)^2} \\ &\quad -\frac{(1-\alpha)[(1+n)(1+2\beta(1+b))+\omega(1+b)]}{\alpha\left(\tau_g^c\right)^2} < 0 \\ \bar{U}_{\tau_h^c\tau_h^c} &= -\frac{(1+n)(1+\beta(2+b))+\omega}{\left(1-\tau_g^c-\tau_h^c\right)^2} - \frac{[(1+n)\beta+\omega]b}{\left(\tau_h^c\right)^2} < 0 \\ \left(\bar{U}_{\tau_g^c\tau_h^c}\right)^2 &= \frac{[(1+n)(1+\beta(2+b))+\omega]^2}{\left(1-\tau_g^c-\tau_h^c\right)^4} \\ \bar{U}_{\tau_g^c\tau_g^c}\bar{U}_{\tau_h^c\tau_h^c} &= \frac{[(1+n)(1+\beta(2+b))+\omega]^2}{\left(1-\tau_g^c-\tau_h^c\right)^4} + X + Y + Z,\end{aligned}$$

where X, Y, and Z are positive constants. Then,

$$\bar{U}_{\tau_g^c\tau_g^c}\bar{U}_{\tau_h^c\tau_h^c} - \left(\bar{U}_{\tau_g^c\tau_h^c}\right)^2 = X+Y+Z > 0, \quad \text{for any} \quad (\tau_g^c, \tau_h^c).$$

■

Proof of Corollary 4

Proof by contradiction. Suppose that $\tau_h^c \leq \tau_h^{\mathcal{P}}$, then

$$\begin{aligned}
& \frac{((1+n)\beta+\omega)\,\alpha b}{(1+n)\,(1+2\beta\,(1+b))+\omega\,(1+b)} \leq \frac{\alpha b\omega}{(1+n)\,(1+\beta\,(1+b))+\omega\,(1+b)} \\
\Leftrightarrow \quad & [(1+n)\beta+\omega]\,(1+n)\,(1+\beta\,(1+b))+(1+n)\beta\omega\,(1+b) \\
& \leq (1+n)\,(1+2\beta\,(1+b))\,\omega \\
\Leftrightarrow \quad & (1+n)\,[-\beta\,(1+b)\,\omega+(1+n)\beta\,(1+\beta\,(1+b))+\beta\omega\,(1+b)] \leq 0 \\
\Leftrightarrow \quad & (1+n)\beta\,(1+\beta\,(1+b)) \leq 0,
\end{aligned}$$

which is a contradiction. Thus, it has to hold that $\tau_h^c > \tau_h^{\mathcal{P}}$.

■

Proof of Corollary 5

1. Comparative statics for a change in n

   Partial derivation of each of the expenditures shares of (6.6) with respect to n yields

$$\begin{aligned}
\frac{\partial \tau_g^c}{\partial n} &= 0 \\
\frac{\partial \tau_h^c}{\partial n} &= \frac{-\alpha b\omega\,[1+\beta\,(1+b)]}{[(1+n)\,(1+2\beta\,(1+b))+\omega\,(1+b)]^2} < 0.
\end{aligned}$$

   Using τ_h^c in (6.7) we obtain the equilibrium growth factor as

$$\gamma^c = \alpha B \frac{[1+\beta\,(2+b)]+\omega/\,(1+n)}{(1+n)\,[1+2\beta\,(1+b)]+\omega\,(1+b)}.$$

   Then, partial derivation immediately yields $d\gamma^c/dn < 0$.

2. Comparative statics for an increase in life expectancy

   Consider the reinterpretation of the economic framework as described in Section 5.2. Then, τ_h^c and γ^c can be rewritten as

$$\tau_h^c = \frac{((1+n)\,\beta_v+\omega_v)\,\alpha b}{(1+n)\,(1/v+2\beta_v\,(1+b))+\omega_v\,(1+b)}$$

   and

$$\gamma^c = \frac{\beta_v v X}{1+\beta_v}\,(\alpha-\tau_h^c)\,,$$

where $X \equiv A^{1/\alpha}(1-\alpha)^{1/\alpha}/(1+n)$. Then, a permanent increase in the survival probability v has the following effects on government policy and economic growth:

$$
\begin{aligned}
\frac{\partial \tau_g^c}{\partial v} &= 0 \\
\frac{\partial \tau_h^c}{\partial v} &> 0 \\
\frac{\partial \gamma^c}{\partial v} &= \alpha\beta_v X \frac{\beta_v(1+n)^2\left[(2+b)(1+2v)+(1+b)(3+b)\omega+2\beta(1+b)^2\right]}{(1+\hat{\beta})^2\left[\frac{1+n}{v}+2\beta_v(1+n)(1+b)+\omega(1+b)\right]^2} \\
&\quad + \frac{(1+n)\left[2\omega_v/v+\beta_v\omega_v+(1+n)/v^2\right]+(1+b)\omega_v^2}{(1+\hat{\beta})^2\left[\frac{1+n}{v}+2\beta_v(1+n)(1+b)+\omega_v(1+b)\right]^2} > 0.
\end{aligned}
$$

However, as the increase in the life expectancy is unexpected these effects only materialize with a period delay; for an intuition see the proof of Proposition 3.

■

Proof of Proposition 5

Substituting (6.8) and (6.9) in the political objective function (3.1) yields the program to be solved by the political mechanism as

$$
\max_{\{g_t,h_t\}} \bar{U} \quad \text{with} \quad \bar{U} \equiv [(1+n)(1+\alpha\beta)+\omega]\ln\left(Ak_t^\alpha g_t^{1-\alpha}-g_t-h_t\right)+\omega b\ln h_t.
$$

The first-order conditions of the above program with respect to g_t and h_t are

$$
\begin{aligned}
\bar{U}_{g_t} &= \frac{(1-\alpha)y_t-g_t}{g_t(y_t-g_t-h_t)}[(1+n)(1+\alpha\beta)+\omega]=0 \\
\bar{U}_{h_t} &= -\frac{(1+n)(1+\alpha\beta)+\omega}{y_t-g_t-h_t}+\frac{\omega b}{h_t}=0.
\end{aligned}
$$

The first condition is fulfilled if and only if $g_t=(1-\alpha)y_t$. Using this in the second condition and rearranging immediately yields τ_h^m of (6.11).

The policy mix of Proposition 5 is the global maximizer of the political objective function. To see this note that

$$
\begin{aligned}
\bar{U}_{g_tg_t} &= -[(1+n)(1+\alpha\beta)+\omega]\frac{\alpha(1-\alpha)y_t(y_t-g_t-h_t)+[(1-\alpha)y_t-g_t]^2}{g_t^2(y_t-g_t-h_t)^2}<0 \\
\bar{U}_{h_th_t} &= \frac{-[(1+n)(1+\alpha\beta)+\omega]}{(y_t-g_t-h_t)^2}-\frac{\omega b}{(h_t)^2}<0.
\end{aligned}
$$

Then, $\bar{U}_{g_tg_t}\bar{U}_{h_th_t}$ can be written as

$$
\bar{U}_{g_tg_t}\bar{U}_{h_th_t} = [(1+n)(1+\alpha\beta)+\omega]^2\frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]^2}{(y_t-g_t-h_t)^4}+X+Y+Z,
$$

where X, Y, and Z are positive constants. Moreover,

$$\bar{U}_{g_t h_t} = -\left[(1+n)(1+\alpha\beta)+\omega\right] \frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]}{(y_t-g_t-h_t)^2},$$

and thus

$$\left(\bar{U}_{g_t h_t}\right)^2 = \left[(1+n)(1+\alpha\beta)+\omega\right]^2 \frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]^2}{(y_t-g_t-h_t)^4}$$

such that

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t} - \left(\bar{U}_{g_t h_t}\right)^2 = X+Y+Z>0, \quad \text{for any} \quad (g_t, h_t).$$

■

Proof of Corollary 6

Proof by contradiction. Suppose that $\tau_h^m \leq \tau_h^{\mathcal{P}}$, then

$$\frac{\alpha\omega b}{(1+n)(1+\alpha\beta)+\omega(1+b)} \leq \frac{\alpha\omega b}{(1+n)(1+\beta(1+b))+\omega(1+b)}$$

$$\Leftrightarrow \quad 1+b \leq \alpha,$$

which is a contradiction. Thus, it has to hold that $\tau_h^m > \tau_h^{\mathcal{P}}$.

■

Proof of Corollary 7

1. Comparative statics for a change in n

   Partial derivation of (6.11) with respect to n immediately yields $d\tau_h^m/dn<0$. Moreover, $d(g_t/y_t)/dn=0$. Then, using (6.11) in (6.12) we obtain the equilibrium growth factor as

   $$\gamma^m = \alpha B \frac{(1+\alpha\beta)+\omega/(1+n)}{(1+n)(1+\alpha\beta)+\omega(1+b)}.$$

   Then, partial derivation immediately yields $d\gamma^m/dn<0$.

2. Comparative statics for an increase in life expectancy

   Consider the reinterpretation of the economic framework as described in Section 5.2. Then, τ_h^m and γ^m can be rewritten as

   $$\tau_h^m = \frac{\alpha\omega_v b}{(1+n)(1/v+\alpha\beta_v)+\omega_v(1+b)}$$

   and

   $$\gamma^m = \frac{\beta_v v X}{1+\beta_v}\left(\alpha-\tau_h^m\right),$$

where $X \equiv A^{1/\alpha}(1-\alpha)^{1/\alpha}/(1+n)$. Then, a permanent increase in the survival probability v has the following effects on government policy and economic growth:

$$\begin{aligned}
\frac{\partial(g_t/y_t)}{\partial v} &= 0 \\
\frac{\partial\tau_h^m}{\partial v} &> 0 \\
\frac{\partial\gamma^m}{\partial v} &= \frac{\beta_v X}{1+\hat{\beta}}\left(-v\frac{\partial\tau_h^m}{\partial v}+\alpha-\tau_h^m\right) \\
&= \frac{X\beta_v\alpha}{1+\hat{\beta}}\frac{(1+n)\left(1+\alpha\hat{\beta}\right)\left((1+n)(1+\alpha\hat{\beta})+\hat{\omega}\right)}{\left[(1+n)\left(1+\alpha\hat{\beta}\right)+\hat{\omega}(1+b)\right]^2} \\
&\quad +\frac{X\beta_v\alpha\hat{\omega}}{1+\hat{\beta}}\frac{(1+b)\hat{\omega}+(1+n)\left(1+(1+b)\alpha\hat{\beta}\right)}{\left[(1+n)\left(1+\alpha\hat{\beta}\right)+\hat{\omega}(1+b)\right]^2}>0.
\end{aligned}$$

However, as the increase in the life expectancy is unexpected these effects only materialize with a period delay; for an intuition see the proof of Proposition 3.

■

Existence proof for numerical example of Section 6.2.2

To see that a unique $\tau_H^{\mathcal{P}}$ exists if ρ is sufficiently small, rewrite equation (6.28) as

$$(1+n)(1+\beta)+\omega=\alpha^{-\rho}b\left[\omega\left(\tau_h^{\mathcal{P}}\right)^{\rho-1}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{1-\rho}-(1+n)(1+\beta)\left(\tau_h^{\mathcal{P}}\right)^{\rho}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{-\rho}\right]. \tag{.25}$$

Denote the right-hand side of (.25) by $RHS(\tau_h^{\mathcal{P}},\rho)$ and the left-hand side, which does not depend on τ_h, by LHS. One readily verifies that $\partial RHS(\tau_h^{\mathcal{P}},\rho)/\partial\tau_h<0$ for any $\rho<1$. Moreover, for a given ρ, $RHS(\tau_h^{\mathcal{P}},\rho)>0$ when $\tau_h^{\mathcal{P}}$ is sufficiently small (i. e. close to zero) and $RHS(\tau_h^{\mathcal{P}},\rho)<0$ when $\tau_h^{\mathcal{P}}$ is sufficiently close to α. Therefore, there is a unique value of $\tau_h^{\mathcal{P}}\in(0,\alpha)$ which satisfies (.25) if and only if $RHS(\tau_h^{\mathcal{P}},\rho)$ for $\tau_h^{\mathcal{P}}$ close to zero is greater than LHS. Now note that for a given $\tau_h^{\mathcal{P}}$

$$\frac{\partial RHS(\tau_h^{\mathcal{P}},\rho)}{\partial\rho}=\frac{b\left(\tau_h^{\mathcal{P}}\right)^{\rho-1}}{\alpha^{\rho}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{\rho}}\ln\left(\frac{\left(\alpha-\tau_h^{\mathcal{P}}\right)\alpha}{\tau_h^{\mathcal{P}}}\right)\left(\tau_h^{\mathcal{P}}(1+\beta)(1+n)-\omega\left(\alpha-\tau_h^{\mathcal{P}}\right)\right).$$

Thus, $lim_{\tau_h^{\mathcal{P}}\to 0}\left(\partial RHS(\tau_h^{\mathcal{P}},\rho)/\partial\rho\right)<0$ and $lim_{\tau_h^{\mathcal{P}}\to 0}RHS(\tau_h^{\mathcal{P}},\rho)$ is greater the smaller ρ. Therefore, we can conclude that a solution to (.25) only exists if ρ is not too large.

■

References

BARRO, R. J. (1990): "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, 98, 103–125.

BASSETTO, M. (2008): "Political Economy of Taxation in an Overlapping-Generations Economy," *Review of Economic Dynamics*, 11, 18–43.

CBO (1998): "The Economic Effects of Federal Spending on Infrastructure and Other Investments," *Congressional Budget Office Paper.*

CUTLER, D. M., J. M. POTERBA, L. M. SHEINER, AND L. H. SUMMERS (1990): "An Aging Society: Opportunity or Challenge," *Brookings Papers on Economic Activity*, 90, 1–73.

DIOIKITOPOULOS, E. (2009): "Ageing and the Allocation of Public Expenditures on Health and Education," Working Paper.

DIXIT, A. AND J. LONDREGAN (1996): "The Determinants of Success of Special Interests in Redistributive Politics," *The Journal of Politics*, 58, 1132–1155.

FUTAGAMI, K. AND T. NAKAJIMA (2001): "Population Aging and Economic Growth," *Journal of Macroeconomics*, 23, 31–44.

GLOMM, G. AND B. RAVIKUMAR (1997): "Productive Government Expenditure and Long-Run Growth," *Journal of Economic Dynamics and Control*, 21, 183–294.

GONZALEZ-EIRAS, M. AND D. NIEPELT (2007): "Population Ageing, Government Budgets, and Productivity Growth in Politico-Economic Equilibrium," *CEPR Discussion Paper No. 6581.*

GONZALEZ-EIRAS, M. AND D. NIEPELT (2008): "The Future of Social Security," *Journal of Monetary Economics*, 55, 197–218.

GROSSMAN, G. M. AND E. HELPMAN (1998): "Intergenerational Redistribution with Short-Lived Governments," *The Economic Journal*, 108, 1299–1329.

HASSLER, J., P. KRUSELL, K. STORESLETTEN, AND F. ZILIBOTTI (2005): "The Dynamics of Government," *Journal of Monetary Economics*, 52, 1331–1358.

HASSLER, J., K. STORESLETTEN, AND F. ZILIBOTTI (2007): "Democratic Public Good Provision," *Journal of Economic Theory*, 133, 127–151.

HEIJDRA, B. J. (2009): *Foundations of Modern Macroeconomics*, Oxford: Oxford University Press, 2^{nd} ed.

IQBAL, K. AND S. J. TURNOVSKY (2008): "Intergenerational Allocation of Government Expenditures and Optimal Taxation," *Journal of Public Economic Theory*, 10, 27–53.

IRMEN, A. (2009): "Population Aging and the Direction of Technical Change," *University of Heidelberg Discussion Paper Series No. 493.*

IRMEN, A. AND B. HEER (2008): "Population, Pensions, and Endogenous Economic Growth," *CESifo Working Paper No. 2480.*

IRMEN, A. AND J. KUEHNEL (2009): "Productive Government Expenditure and Economic Growth," *Journal of Economic Surveys*, 23, 692–733.

KRUSELL, P., V. QUADRINI, AND J. RÍOS-RULL (1997): "Politico-Economic Equilibrium and Economic Growth," *Journal of Economic Dynamics and Control*, 21, 243–272.

KRUSELL, P. AND J.-V. RÍOS-RULL (1999): "On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model," *The American Economic Review*, 89, 1156–1181.

LINDBECK, A. AND J. W. WEIBULL (1987): "Balanced-Budget Redistribution as the Outcome of Political Competition," *Public Choice*, 52, 273–297.

MULLIGAN, C. B. AND X. SALA-Í-MARTIN (1999): "Gerontocracy, Retirement, and Social Security," *NBER Working Paper No. 7117.*

PERSSON, T. AND G. TABELLINI (2000): *Political Economics: Explaining Economic Policy*, Cambridge, MA: The MIT Press.

POTERBA, J. (1997): "Demographic Structure and the Political Economy of Public Education," *Journal of Policy Analysis and Management*, 16, 48–66.

PRETTNER, K. (2009): "Population Ageing and Endogenous Economic Growth," Working Papers 8/2009, Vienna Institute of Demography of the Austrian Academy of Sciences in Vienna.

RHODEBECK, L. A. (1993): "The Politics of Greed? Political Preferences among the Elderly," *Journal of Politics*, 55, 342–364.

ROGERS, D. L., E. TODER, AND L. JONES (2000): "Economic Consequences of an Aging Population," Urban Institute, The Retirement Project, Occasional Paper No.6.

SHESHINSKI, E. AND Y. WEISS (1981): "Uncertainty and Optimal Social Security Systems," *The Quarterly Journal of Economics*, 96, 189–206.

SONG, Z., K. STORESLETTEN, AND F. ZILIBOTTI (2009): "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt," *Working Paper.*

United Nations (2008): "World Population Prospects: The 2008 Revision," *United Nations, New York, http://esa.un.org/unpp/, accessed on December 10, 2009.*

Yaari, M. E. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, 32, 137–150.

Yakita, A. (2008): "Ageing and Public Capital Accumulation," *International Tax and Public Finance*, 15, 582–598.