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# Modeling the link between US inflation and output: the importance of the uncertainty channel\*

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## Abstract

This paper employs an augmented version of the UECCC GARCH specification proposed in Conrad and Karanasos (2010) which allows for lagged in-mean effects, level effects as well as asymmetries in the conditional variances. In this unified framework we examine the twelve potential intertemporal relationships between inflation, growth and their respective uncertainties using US data. We find that high inflation is detrimental to output growth both directly and indirectly via the nominal uncertainty. Output growth boosts inflation but mainly indirectly through a reduction in real uncertainty. Our findings highlight that macroeconomic performance affects nominal and real uncertainty in many ways and that the bidirectional relation between inflation and growth works to a large extent indirectly via the uncertainty channel.

**Keywords:** Bivariate GARCH process, volatility feedback, inflation uncertainty, output variability.

**JEL Classification:** C32, C51, E31.

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# 1 Introduction

One of the most fiercely debated issues in macroeconomics is the nature of the relation between inflation and output or unemployment (see, for example, Gillman and Kejak, 2005a,b, 2009). While much of the debate has been with a focus on the levels of the two series, there are many economic theories that highlight the importance of the effects which are due to the interaction of the levels and the volatilities.<sup>1</sup> E.g. Friedman's (1977) famous argument about the negative welfare effects of inflation consists of two legs: higher inflation increases nominal uncertainty which then decreases output growth. Thus, the negative welfare effects of inflation may (at least partly) work indirectly via nominal uncertainty. On the other hand, Cukierman and Meltzer (1986) argue that in the presence of higher nominal variability there is an incentive for the central bank to increase the inflation rate.

Moreover, as pointed out by Blackburn and Pelloni (2005) the existence of a relation between growth and its volatility adds a new dimension to the design and evaluation of macroeconomic policies aimed at stabilizing fluctuations. A monetary stabilization policy aimed at reducing stochastic fluctuations may work either for or against the promotion of long-run growth depending on whether the volatility-growth link is negative or positive.

Finally, Fuhrer (1997) defines an optimal monetary policy as a policy that minimizes the variability of the central bank's ultimate objectives around their targets. As Fuhrer (1997) puts it: "It is difficult to imagine a policy that embraces targets for the level of inflation or the output gap without caring about their variability around their target levels". His theory implies a trade-off between the variabilities of inflation and growth.

As mentioned by Stock and Watson (2007) inflation is much less volatile than it was in the 1970s and early 1980s. Kumar and Okimoto (2007) point out that there was also a marked increase in concerns about deflation in the early part of the decade. A number of studies have examined the extent to which a decline in the average rate of inflation and its volatility may reflect improved monetary policy design and implementation, increasing globalization, as well as the role of the informational technology revolution (Kumar and Okimoto, 2007). Moreover, many recent studies in macroeconomics have found growing stability in the U.S. economy. For example, Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) find that there was reduction in the volatility of output since 1984.

A series of papers, published during the last thirty years (see, for example, Logue and Sweeney, 1981, Evans, 1991, Brunner, 1993, Evans and Wachtel, 1993, Ungar and Zilberfarb, 1993, Holland, 1993, 1995, Fuhrer, 1997, Elder, 2004), highlights how important are the aforementioned causal relations for policy making and macroeconomic modeling.

Brunner and Hess (1993) was one of the first papers to employ a univariate GARCH model in order to test for the first leg of the Friedman hypothesis (see also Baillie et al., 1996). During the last decade researchers have employed various bivariate GARCH-in-mean models to investigate the relation between the two uncertainties (see, for example, Conrad et al., 2010b) and/or to examine their impact on the levels of inflation and growth (see, for example, Elder, 2004 and Grier et al., 2004). However, the econometric specifications which are employed in most of these studies are typically characterized by one or

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<sup>1</sup>We will use the terms variance, variability, uncertainty and volatility interchangeably in the remainder of the text.

more of the following three limitations.

First, the impact from the variabilities on the levels (the so-called in-mean effects) is typically restricted to being contemporaneous (as, for example, in Shields et al., 2005). However, since the theoretical rationale for the in-mean effects usually suggests that it takes some time for them to materialize (e.g. in the Cukierman and Meltzer, 1986, theory it requires a change in monetary policy), it appears more appropriate to investigate such effects within a specification that includes several lags of the variances in the mean equations (see also Elder, 2004).

Second, the existing literature focuses almost exclusively on the impact of macroeconomic uncertainty on performance, but neglects the effects in the opposite direction (level effects). Moreover, the few studies that take level effects into account, focus on own but not cross level effects. For example, Ungar and Zilberfarb (1993) provide a theoretical model for the effect of inflation on its unpredictability and specify the necessary conditions for a positive impact. Logue and Sweeney (1981) argue that such heightened uncertainty produces greater variability of real growth. That is, inflation, via the nominal uncertainty channel, affects not only growth (the Friedman hypothesis) but real variability as well (Dotsey and Sarte, 2000). In addition, Brunner (1993) points out that while the second leg of Friedman's hypothesis is plausible, the negative causation between nominal uncertainty and growth could also work in the opposite direction. Therefore, the possible negative effect of growth on its variability could come in two parts. In the first leg high growth rates reduce inflation uncertainty (the Bruner conjecture). In the second leg this reduced inflation variability lowers real uncertainty (the Logue-Sweeney theory). Thus, a meaningful empirical analysis should allow for bidirectional causality between the four variables.

Third, the two most commonly used specifications are the diagonal (constant conditional correlation) CCC model (see, for example Grier and Perry, 2000, and Fountas et al., 2006) and the BEKK representation (see, for example, Shields et al., 2005, and Grier and Grier, 2006, and Bredin and Fountas, 2009). Both specifications are characterized by rather restrictive assumptions regarding potential volatility interaction. At the one extreme, the former assumes that there is no link between the two uncertainties, whereas, near the other extreme, the latter only allows for a positive variance relationship. In sharp contrast, several economic theories predict either a positive or a negative association between the two volatilities (for more details and a review of the literature, see, Arestis et al., 2002, and Karanasos and Kim, 2005). Obviously, the extent to which there is an interaction of either sign between the two variances is an issue that cannot be resolved on merely theoretical grounds. These considerations reinforce a widespread awareness of the need for more empirical evidence, but also make clear that a good empirical framework is lacking.

Finally, as shown in Hamilton (2010) the correct modeling of the conditional variances of inflation and growth is important even when one is only interested in testing hypothesis about the mean of the series. First, this is because hypothesis tests about the mean in a model in which the variance is misspecified will be invalid. Second, substantially more efficient estimates of the conditional mean can be obtained by correctly modeling the conditional heteroskedasticity.

In this paper we investigate the interactions between US inflation, growth, and their

respective uncertainties using the bivariate unrestricted extended constant conditional correlation (UECCC) GARCH model, defined in Conrad and Karanasos (2010).<sup>2</sup> This model has the advantage that it allows for feedback effects between the two volatilities that can be of either sign, i.e. positive or negative. Further, we augment the UECCC GARCH model in two ways: i) we estimate a system of equations that allows various lags of the two variabilities to affect the conditional means and ii) we include lagged values of inflation and growth in the two variance specifications and, thereby, control for *own as well as cross* level effects. Thus, we can examine in a unified empirical framework all the possible causal relations among the four variables that are predicted by economic theory.<sup>3</sup>

In short, our main results can be summarized as follows. First, inflation is a negative determinant of real growth. This effect takes place both directly and indirectly, via the nominal variability channel, as put forward by Friedman (1977). That is, we find that the impact of inflation on its uncertainty is positive and nominal variability itself has a contemporaneous negative in-mean effect on output growth. This finding highlights the harmful effects of inflation which was found to lead to less predictability. Since this unpredictability can reduce economic activity and lead to a misallocation of resources in the economy, the incentive for lowering inflation is clear. Second, the theoretical and empirical literature is divided on the effects of nominal uncertainty on inflation and this paper offers considerable evidence to resolve that dispute. In particular, our results appear to support the view that the FED does tend to cause inflation surprises in the presence of more nominal variability, as argued by Cukierman and Meltzer (1987). That is, we find strong evidence that higher nominal uncertainty increases the average inflation rate. As expected, this effect does not occur contemporaneously but takes three month to materialize. If the source for high inflation is erratic government policies our findings call for a consistent and stable economic policy.

Third, we find that real variability has a positive contemporaneous effect on growth. This finding is in line with the positive correlation between real uncertainty and output growth which emerges from the model considered in Blackburn and Pelloni (2004) when real shocks predominate. Thus, policies that increase real variability contribute to higher economic growth and, therefore, the claim that increased stabilization of the business cycle is a requirement for long-run growth does not appear to find support in our study. Moreover, contrary to the Cukierman-Gerlach (2003) theory higher real uncertainty, with a time delay of one month, reduces inflation. This effect can be rationalized by negative growth shocks which increase real uncertainty. The heightened real variability then reduces inflation. These results show that the behavior of macroeconomic performance is influenced by its volatility, but also that the significance and the sign of the in-mean effects depend on the correct modeling of the lag length.

Although much has been written in applied theory regarding the potential welfare costs of inflation, surprisingly little work has been carried out to identify how it might affect the variability of output. This issue is of particular importance, because we can

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<sup>2</sup>The specification is termed ‘unrestricted extended’ because it can be viewed as an unrestricted version of the extended CCC (ECCC) specification of Jeantheau (1998) which allows for positive volatility feedback only.

<sup>3</sup>Since our framework is very general, it is potentially useful in numerous other contexts, such as investigating the relation between trading volume and stock volatility.

think of inflation as being primarily influenced by central bank policy. Of significant relevance is our finding that inflation has a positive impact on real uncertainty as predicted by Dotsey and Sarte (2000). Thus we provide strong evidence in support of the argument that inflation breeds uncertainty in many forms. We also find that growth affects inflation variability negatively thus supporting the Brunner (1993) conjecture. The potential for reverse causation to have influenced the nominal uncertainty-growth link has not been considered by researchers. Our results suggest the importance of devoting explicit attention to the effects of macroeconomic performance on its variability.

Dotsey and Sarte (2000) highlight the fact that the volatilities of inflation and of growth are directly linked and that this observation deserves empirical attention since it has important implications for the analysis of the impact of macroeconomic performance on the real uncertainty. Regarding this relation, our results are strongly in favor of the prediction by Logue and Sweeney (1981) that higher nominal variability increases real uncertainty. Therefore inflation through its variability affects i) growth negatively as predicted by Friedman (1977), and ii) real uncertainty positively as predicted by Dotsey and Sarte (2000). In other words not only the Friedman hypothesis consists of two legs but the Dotsey-Sarte conjecture as well. The first leg is identical in both whereas the second leg of the latter is the Logue and Sweeney theory. Equally important, growth affects its variability negatively via the inflation uncertainty channel. This negative indirect effect also comes in two parts. The first leg is the Brunner conjecture and the second one is identical with that of the Dotsey-Sarte conjecture. Finally, there is also strong indirect evidence (via either the inflation or growth channel) that growth uncertainty has a negative impact on the volatility of inflation which is in line with the Fuhrer (1997) theory. However, we do not find evidence for a direct effect.

The remainder of the article is organized as follows. Section 2 introduces the bivariate UECCC GARCH model, presents its properties, and sets out assumptions and notation. In Section 3 we present an overview of the theories that link inflation, growth and their respective uncertainties. Sections 4 and 5 present and discuss the significance of the empirical results. In Section 6 we present a sensitivity analyzes of our results with respect to the specification of the model, subsamples and the data frequency. Section 5 compares our findings to the results in the previous literature. Finally, Section 9 concludes the article.

## 2 The Bivariate GARCH Model

We use a bivariate model to simultaneously estimate the conditional means, variances, and covariances of inflation and output growth. Let  $\mathbf{y}_t = (\pi_t \ y_t)'$  represent the  $2 \times 1$  vector with the inflation rate and real output growth. Further,  $\mathcal{F}_{t-1} = \sigma(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots)$  is the filtration generated by the information available up through time  $t-1$ , and  $\mathbf{h}_t = (h_{\pi,t} \ h_{y,t})'$  denotes the vector of  $\mathcal{F}_{t-1}$  measurable conditional variances. We estimate the following bivariate AR( $p$ )-GARCH(1, 1)-in-mean model

$$\mathbf{y}_t = \mathbf{\Gamma}_0 + \sum_{l=1}^p \mathbf{\Gamma}_l \mathbf{y}_{t-l} + \sum_{r=0}^s \mathbf{\Delta}_r \mathbf{h}_{t-r} + \boldsymbol{\varepsilon}_t, \quad (1)$$

where  $\mathbf{\Gamma}_0 = [\gamma_i]_{i=\pi,y}$ ,  $\mathbf{\Gamma}_l = [\gamma_{ij}^{(l)}]_{i,j=\pi,y}$  and  $\mathbf{\Delta}_r = [\delta_{ij}^{(r)}]_{i,j=\pi,y}$ . Let  $\mathbf{I}$  be the  $2 \times 2$  identity matrix and  $L$  the lag operator. We assume that the roots of  $|\mathbf{I} - \sum_{l=1}^p \mathbf{\Gamma}_l L^l|$  lie outside the unit circle. Note that our specification allows the conditional variances to affect the level variables contemporaneously and up to lag  $s > 0$ . Controlling for both autoregressive terms as well as lagged conditional variances is important, because as shown in Ghysels et al. (2005) and Conrad et al. (2010a) the omission of autoregressive terms/lagged conditional variances may lead to spuriously significant in-mean/autoregressive terms.<sup>4</sup>

The residual vector is defined as  $\boldsymbol{\varepsilon}_t = (\varepsilon_{\pi,t} \ \varepsilon_{y,t})' = \mathbf{z}_t \odot \mathbf{h}_t^{\wedge 1/2}$ , where the symbols  $\odot$  and  $\wedge$  denote the Hadamard product and the elementwise exponentiation respectively. The stochastic vector  $\mathbf{z}_t = (z_{\pi,t} \ z_{y,t})'$  is assumed to be independently and identically distributed (*i.i.d.*) with mean zero, finite second moments, and  $2 \times 2$  correlation matrix  $\mathbf{R} = [\rho_{ij}]_{i,j=\pi,y}$  with diagonal elements equal to one and off-diagonal elements absolutely less than one. Thus, we have  $\mathbf{E}[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0}$  and  $\mathbf{H}_t = \mathbf{E}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \mathcal{F}_{t-1}] = \text{diag}\{\mathbf{h}_t\}^{1/2} \mathbf{R} \text{diag}\{\mathbf{h}_t\}^{1/2}$ , where  $\rho_{\pi y} = h_{\pi y,t} / \sqrt{h_{\pi,t} h_{y,t}}$  is the constant conditional correlation.

Following Conrad and Karanasos (2010), we impose the UECCC GARCH(1,1) structure on the conditional variances:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A} \boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B} \mathbf{h}_{t-1}, \quad (2)$$

where  $\boldsymbol{\omega} = [\omega_i]_{i=\pi,y}$ ,  $\mathbf{A} = [a_{ij}]_{i,j=\pi,y}$  and  $\mathbf{B} = [b_{ij}]_{i,j=\pi,y}$ . We assume that the above model is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2010). The invertibility condition implies that the inverse roots of  $|\mathbf{I} - \mathbf{B}L|$ , denoted by  $\phi_1$  and  $\phi_2$ , lie inside the unit circle. Following Conrad and Karanasos (2010) we also impose the four conditions which are necessary and sufficient for  $\mathbf{h}_t > 0$  for all  $t$ : (i)  $(1 - b_{yy})\omega_\pi + b_{\pi y}\omega_y > 0$  and  $(1 - b_{\pi\pi})\omega_y + b_{y\pi}\omega_\pi > 0$ , (ii)  $\phi_1$  is real and  $\phi_1 > |\phi_2|$ , (iii)  $\mathbf{A} \geq 0$  and (iv)  $[\mathbf{B} - \max(\phi_2, 0)\mathbf{I}]\mathbf{A} > 0$ . Note, that these constraints do not place any *a priori* restrictions on the signs of the coefficients in the  $\mathbf{B}$  matrix. In particular, this implies that negative volatility spillovers are possible.

The UECCC specification nests the diagonal CCC model when  $\mathbf{A}$  and  $\mathbf{B}$  are diagonal matrices and Jeantheau's (1998) ECCC model when  $a_{ij} \geq 0$  and  $b_{ij} \geq 0$ . Arestis et al. (2002) and Arestis and Mouratidis (2004) correctly argue that any multivariate GARCH model which imposes positive volatility feedback cannot be used to estimate and test for a volatility trade-off.<sup>5</sup> However, although this is true for both the BEKK representation and the restricted ECCC specification of Jeantheau (1998), this is no longer the case for the unrestricted version of the latter formulation. More specifically, the necessary and sufficient conditions derived in Conrad and Karanasos (2010) ensure the positive definiteness of the conditional covariance matrix even in the case of negative volatility feedback. While negative values of the GARCH coefficients have commonly been thought of as resulting either from sampling error or model misspecification, they show that this is not necessarily the case. Interestingly, negative volatility spillovers may be in line with economic theory (see Section 3).

<sup>4</sup>In Section 6 on robustness, we also consider a specification in which the mean is a function of the conditional standard deviations, i.e.  $\mathbf{h}_{t-r}^{\wedge 1/2}$ , instead of the conditional variances.

<sup>5</sup>As an alternative, it is possible to use either a stochastic volatility or an EGARCH model, both of which assume an exponential specification of the conditional variance and, thereby, allow to estimate the model parameters without any positivity restrictions.

Finally, we augment the variance specification in order to allow for asymmetries and to include level effects:

$$\mathbf{h}_t = \boldsymbol{\omega} + (\mathbf{A} + \mathbf{G}\mathbf{1}_{\{\varepsilon_t > 0\}})\varepsilon_{t-1}^2 + \mathbf{B}\mathbf{h}_{t-1} + \mathbf{e}^{\Lambda\mathbf{y}_{t-1}}, \quad (3)$$

where  $\mathbf{G} = [g_{ij}]_{i,j=\pi,y}$ ,  $\mathbf{1}_{\{\varepsilon_t > 0\}} = [1_{\{\varepsilon_{i,t} > 0\}}]_{i=\pi,y}$  is an indicator function and  $\Lambda = [\lambda_{ij}]_{i,j=\pi,y}$ . We choose the exponential specification for the level effects, because it ensures that our non-negativity conditions are still sufficient for guaranteeing positive conditional variances.<sup>6</sup> Note, that we can easily control for lagged level effects by adding the respective terms to equation (3). An alternative approach to introducing level effects in an exponential fashion as in equation (3) would be by adding the lagged inflation rates either linearly (see Conrad et al., 2010b) or quadratically (see Brunner and Hess, 1993). In the following, we will term the asymmetric GARCH (AGARCH) in-mean-level (ML) model by AGARCH-ML.

It is worth reiterating in just a few sentences what we see to be the main benefits of our model. First, it does not require us to make the dubious assumption that there is a positive link between the two variabilities. That is, the sign of the coefficients that capture the volatility-relation ( $b_{\pi y}$ ,  $b_{y\pi}$ ) is not restricted a priori. Second, several lags of the conditional variances are added as regressors in the mean equation. Third, distinguishing empirically between the in-mean and level effects found in theoretical models is extremely difficult in practice so it makes sense to emphasize that both are relevant.

### 3 Economic Theories

This section provides an overview of the economic theories which rationalize a link between inflation, output growth and their respective uncertainties. Table 1 presents a summary of the signs implied by the respective theories.

#### 3.1 The Inflation-Growth link

Mean inflation and output growth are interrelated. Temple (2000) presents a critical review of the emerging literature which tends to discuss how inflation affects growth. Most empirical literature finds that inflation affects growth negatively:  $\pi \bar{\rightarrow} y$ . Recent findings, for example, of Barro (2001) compound the evidence of a strongly significant negative impact. Moreover, the summary of the findings in Gillman and Kejak (2005a) establishes clearly a robust significant negative inflation-growth effect across a range of growth models.

Briault (1995) argues that there is a positive relation between growth and inflation, at least over the short run, with the direction of causation running from higher growth (at least in relation to productive potential) to higher inflation:  $y \overset{\pm}{\rightarrow} \pi$ . For simplicity, in what follows we will refer to this positive influence as the Briault conjecture.

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<sup>6</sup>As mentioned below, Ungar and Zilberfarb (1993) argue that higher levels of inflation can lead to more inflation uncertainty. They also point out that there are reasons to expect nonlinearities: the positive impact of mean inflation should increase as mean inflation rises. This is another important reason why one should use the exponential form. Notice that in equation (3) the derivative of  $h_{\pi,t}$  with respect to  $\pi_{t-1}$  is given by  $\lambda_{\pi\pi} e^{\lambda_{\pi\pi}\pi_{t-1}}$  which, if  $\lambda_{\pi\pi} > 0$ , increases as  $\pi_{t-1}$  rises.



### 3.2 The relation between the variabilities of inflation and output

There are some reasons to suspect a relation between nominal uncertainty and the volatility of real growth. In particular, Logue and Sweeney (1981) argue that producers operating in a highly inflationary economy might be unable to distinguish real shifts in demand from nominal shifts. Real growth in investment, and all other economic activity will be more variable than it would be in an environment where less guessing as to the source of an increase in nominal demand was necessary. For this reason, greater variability of inflation leads to greater uncertainty in production, investment, and marketing decisions, and greater variability in real growth:  $h_\pi \overset{+}{\rightarrow} h_y$ .<sup>7</sup>

In sharp contrast, in Fuhrer's (1997) models the short run trade-off between inflation and output implies a long-run trade-off in variability. For example, he argues that if the Fed wishes to make the variance in output small, it must allow shocks that affect inflation to persist, thus increasing the variance in inflation. On the other hand, in order to make the variance in inflation small, in the face of demand and supply shocks, the Fed must vary real output a great deal in order to stabilize inflation (Fuhrer, 1997):  $h_\pi \overset{-}{\leftrightarrow} h_y$ . It is easy to show that in our bivariate UECCC-GARCH(1,1) model, given by equation (2), when there is a volatility feedback the two conditional variances will be correlated even if  $\rho_{\pi y} = 0$ . Details are available from the authors upon request.

### 3.3 The impact of macroeconomic uncertainty on performance

Macroeconomists have placed considerable emphasis on the impact of economic uncertainty on the state of the macroeconomy. The profession seems to agree that the objectives of monetary policy are inflation and output stabilisation around some target levels. A detailed survey of the theories is provided, e.g., in Fountas et al. (2006) and Fountas and Karanasos (2007).

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<sup>7</sup>In addition, models with stable inflation-unemployment trade-off imply a positive relation between the two variabilities (Logue and Sweeney, 1981). Moreover, in Devereux's (1989) model higher output variability is associated with more inflation uncertainty:  $h_y \overset{+}{\rightarrow} h_\pi$ . Karanasos and Kim (2005) discuss a number of arguments, advanced over the last 30 years, that predict a positive association between the two variables.

Table 1: Economic Theories

	$\pi$	$y$	$h_\pi$	$h_y$
$\pi$		Briault: + (conjecture)	<u>Cukierman-Meltzer: +</u> Holland: - (conjecture)	<u>Fuhrer(-)/</u> Cukierman-Meltzer (+): - Cukierman-Gerlach: +
$y$	Gillman-Kejak: -		<u>Pindyck: -</u> (Friedman 2nd lag) Dotsey-Sarte: +	Blackburn-Pelloni: $\pm$
$h_\pi$	Ungar-Zilberfarb: $\pm$ (Friedman 1st lag: +)	Brunner: - (conjecture)		<u>Fuhrer: -</u> Devereux: + (conjecture)
$h_y$	<u>Dotsey-Sarte</u> (conjecture): + Ball et al. : -	Brunner(-)/ Logue-Sweeney(+): -	<u>Logue-Sweeney: +</u> Fuhrer: -	

**Notes:** The effects of inflation (nominal uncertainty) on the other three variables are presented in the first (third) column. The effects of growth (real variability) on the other three variables are presented in the second (fourth) column. +/-: the effect is positive/negative.

### 3.3.1 The effects of inflation variability

Variability about future inflation affects the average rate of inflation. However, the direction of the effect is ambiguous from a theoretical point of view. Cukierman and Meltzer's (1986) model explains the positive association between the two variables. In the words of Holland (1995): 'The policy maker chooses monetary control procedures that are less precise, so that uncertainty about inflation is higher. The reason is that greater ambiguity about the conduct of monetary policy makes it easier for the government to create the monetary surprises that increase output. This causes the rate of inflation to be higher on average':  $h_\pi \overset{+}{\rightarrow} \pi$ . On the other hand, one possible reason for greater nominal variability to precede lower inflation is that an increase in uncertainty is viewed by policymakers as costly, inducing them to reduce inflation in the future (Holland, 1995). We will refer to this negative effect as the Holland conjecture:  $h_\pi \overset{-}{\rightarrow} \pi$ .

The impact of nominal uncertainty on output growth, has received considerable attention in the literature. Friedman (1977) argues that higher uncertainty about inflation distorts the effectiveness of the price mechanism in allocating resources efficiently, thus leading to negative output effects. According to Pindyck (1991) the effect might work through its impact on investment. Inflation variability increases the uncertainty regarding the potential returns of investment projects and therefore provides an incentive to delay these projects, thus contributing to lower investment and output growth:  $h_\pi \overset{-}{\rightarrow} y$ .<sup>8</sup>

### 3.3.2 The effects of growth variability

Next, real variability may affect the rate of inflation. First, it would be expected to have a negative impact on inflation via a combination of the Fuhrer and the Cukierman-Meltzer effects:  $h_y \overset{-}{\rightarrow} h_\pi \overset{+}{\rightarrow} \pi$ .<sup>9</sup>

Finally, of particular interest has been the relation between growth and its variance with different analyses reaching different conclusions depending on what type of model is employed, what values for parameters are assumed and what types of disturbance are considered (see Blackburn and Pelloni, 2005, and the references therein). The conclusions reached, on the question of how the structure of the business cycle (the volatility, frequency and persistence of fluctuations) might affect long-term growth, differ markedly between models and depend essentially on the mechanism responsible for generating technological progress. In one class of models, where the mechanism is 'creative destruction' the relation/correlation between short-term volatility and long-term growth is positive. In sharp contrast, in models where the mechanism is 'learning-by-doing' the same relation is negative (see Blackburn, 1999 and the references therein). Correlation between the two variables does not imply a causal link. However, in our analysis correlation is a consequence of the impact of real uncertainty on growth:  $h_y \overset{\pm}{\rightarrow} y$ . It is easy to show that in our bivariate AR( $p$ )-UECCC-GARCH(1,1)-M model if  $\delta_{yy}^{(r)} \neq 0$  then there will be

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<sup>8</sup>In sharp contrast, Dotsey and Sarte (2000) employ a model where money is introduced via a cash-in-advance constraint and find that nominal variability increases average growth through a precautionary savings motive:  $h_\pi \overset{+}{\rightarrow} y$ .

<sup>9</sup>In sharp contrast, the approach in Cukierman and Gerlach (2003) implies a positive relation between inflation and the variance of growth where causality runs from the latter to the former:  $h_y \overset{+}{\rightarrow} \pi$ .

correlation between growth and its conditional variance. Details are available from the authors upon request.

### 3.4 The impact of macroeconomic performance on uncertainty

#### 3.4.1 The impact of inflation

Ungar and Zilberfarb (1993) provide a theoretical model of the relation between inflation and its unpredictability and specify the necessary conditions for a positive link. It may be positive as argued by, among others, Friedman (1977) ( $\pi \overset{+}{\rightarrow} h_\pi$ ) or negative if higher inflation may induce the relevant economic agents to invest more in generating accurate predictions. Ungar and Zilberfarb find that the positive effect of inflation on its unpredictability is stronger in high-inflation periods, in contrast with the hypothesis that higher inflation leads economic agents to invest more in generating accurate predictions, thus reducing their prediction error. Notice that in equation (3) the derivative of  $h_{\pi,t}$  with respect to  $\pi_{t-1}$  is given by  $\lambda_{\pi\pi} e^{\lambda_{\pi\pi}\pi_{t-1}}$  which, if  $\lambda_{\pi\pi} > 0$  increases as  $\pi_{t-1}$  rises. Holland (1993) points out that unpredictable changes in inflation regime of the type considered in Evans and Wachtel (1993) lead to a relation between inflation and its uncertainty. In particular, if regime changes causes unpredictable changes in the persistence of inflation, then lagged inflation squared is positively related to inflation uncertainty.

Dotsey and Sarte (2000) analyze the effects of inflation and its uncertainty on growth and real uncertainty in a linear neoclassical growth model where money is introduced via a cash-in-advance constraint. In their setting they control for the fraction of investment, in both physical and human capital, which is subject to the cash-in-advance constraint. They show that when a liquidity constraint applies only to a small fraction of investment, then as the average inflation rate increases, and as a result the variance of inflation also rises, the degree of substitution between consumption and investment becomes more intensive creating a wider dispersion between the possible levels of state contingent growth rates.<sup>10</sup> Thus, their model suggests that as average money growth rises, nominal variability increases and real growth rates become more volatile:  $\pi \overset{+}{\rightarrow} h_\pi \overset{+}{\rightarrow} h_y$ . The fact that variable monetary policy has implications for the volatility of growth rates has thus been overlooked in empirical studies (Dotsey and Sarte, 2000).<sup>11</sup>

#### 3.4.2 The impact of output growth

The sign of the impact of output growth on macroeconomic volatility is negative. Consider first the influence on nominal uncertainty. As Brunner (1993) puts it: ‘While Friedman’s hypothesis is plausible, one could also imagine that when economic activity falls off, there is some uncertainty generated about the future path of monetary policy, and consequently, about the future path of inflation’. We will use the term ‘Brunner conjecture’ as a

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<sup>10</sup>They assume that the shocks to money growth follow a two-state Markov process, with low and high shocks, and they also derive the associated growth rates.

<sup>11</sup>The models developed by Ball et al. (1988) assume menu costs and imply that the slope of the short-run Phillips curve should be steeper when average inflation is higher. In their New Keynesian model, nominal shocks have real effects because nominal prices change infrequently. Higher average inflation reduces the real effects of nominal disturbances and hence also lowers the variance of output:  $\pi \rightarrow h_y$ .

shorthand for this negative effect:  $y \bar{\rightarrow} h_\pi$ . Finally, consider now the effect of growth on its variability. An increase in growth, given that the Brunner conjecture and the Logue-Sweeney hypothesis hold, decreases its variance:  $y \bar{\rightarrow} h_\pi \overset{+}{\rightarrow} h_y$ .

## 4 Empirical Results

In order to work with a reasonable number of observations in the empirical analysis, we employ deseasonalized monthly data obtained from the FRED database at the Federal Reserve Bank of St. Louis. The annualized inflation and output growth series are calculated as 1200 times the monthly difference in the natural log of the Consumer Price Index and the Industrial Production Index, respectively. The data range from 1960:01 to 2010:01 and, hence, comprise 600 usable observations. Applying various unit root tests to both series, we came to the conclusion that inflation as well as output growth can be treated as stationary variables.

Within the bivariate UECCC GARCH-in-mean framework we analyze the dynamic adjustments of the conditional means and variances of US inflation and output growth, as well as the implications of these dynamics for the direction of causality between the two variables and their respective uncertainties. Parameter estimates are obtained by quasi-maximum likelihood estimation (QMLE). To check for the robustness of our estimates we used a range of starting values and, hence, ensured that the estimation procedure converged to a global maximum. The best model was chosen on the basis of Likelihood Ratio (LR) tests and three alternative information criteria. In the inflation/output equations the best model includes 12/4 lags of inflation/output. Significant cross effects between inflation and output are found at lags 2 and 3 only. For reasons of brevity, we refrain from presenting the estimation results for the autoregressive parameters. Instead, in Table 2 we concentrate on the main parameters of interest.

### 4.1 Baseline Specification

First, from equations (4) and (5) in Table 2 there is strong evidence supporting the Gillman-Kejak theory and the Briault conjecture. That is, there is strong bidirectional feedback between the levels of inflation and output growth. In particular, with a delay of two lags inflation affects growth negatively, whereas growth has a positive effect on inflation after three lags.

Second, the two variance expressions in equation (6) in Table 2 allow us to analyze the potential spillover effects between the two volatilities. The coefficients  $a_{\pi y}$ ,  $b_{\pi y}$  and  $b_{yy}$  were found to be insignificant and, hence, excluded from the specification. That is, inflation uncertainty obeys a GARCH(1,1) structure, while real variability is best characterized as an ARCH(1) process. Because  $a_{y\pi}$  and  $b_{y\pi}$  are significantly estimated, both inflation shocks and nominal volatility affect real uncertainty but not vice versa.<sup>12</sup>

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<sup>12</sup>More precisely, squared inflation shocks  $\varepsilon_{\pi,t-1}^2$  have a direct effect (through  $a_{y\pi}$ ) on output uncertainty  $h_{y,t}$ . They also have an indirect effect by increasing  $h_{\pi,t}$  (through  $a_{\pi\pi}$ ) and thereby  $h_{y,t+1}$  (through  $b_{y\pi}$ ) in the next period (and thereafter because  $h_{\pi,t}$  is persistent). Also note, that the conditional heteroskedasticity in growth is mainly due to the transmission of the conditional heteroskedasticity from

Since  $b_{y\pi}$  is positive and significant there is strong evidence that nominal uncertainty has a positive impact on real volatility, as predicted by Logue and Sweeney (1981). We find no evidence for a significant direct impact in the opposite direction. However, as shown below, real variability has a negative *indirect* effect on nominal uncertainty which works via either the inflation or growth channel. This indirect influence is in line with the Fuhrer (1997) theory. Note, that the parameter restrictions established in Conrad and Karanasos (2010) are naturally satisfied, since all ARCH/GARCH parameters are positive.

Table 2: Bivariate AR-UECCC-GARCH(1, 1)-in-mean Model.

$$\pi_t = \dots + \underset{(0.011)}{0.025} y_{t-3} \dots + \underset{(0.016)}{0.032} h_{\pi,t-3} - \underset{(0.0008)}{0.0018} h_{y,t-1} + \varepsilon_{\pi,t} \quad (4)$$

$$y_t = \dots - \underset{(0.087)}{0.359} \pi_{t-2} \dots - \underset{(0.062)}{0.226} h_{\pi,t} + \underset{(0.010)}{0.030} h_{y,t} + \varepsilon_{y,t} \quad (5)$$

$$\begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} = \begin{pmatrix} \underset{(0.249)}{0.985} \\ \underset{(3.895)}{35.785} \end{pmatrix} + \begin{pmatrix} \underset{(0.044)}{0.303} & - \\ \underset{(0.282)}{0.839} & \underset{(0.059)}{0.301} \end{pmatrix} \begin{pmatrix} \varepsilon_{\pi,t-1}^2 \\ \varepsilon_{y,t-1}^2 \end{pmatrix} \\ + \begin{pmatrix} \underset{(0.055)}{0.592} & - \\ \underset{(0.478)}{0.727} & - \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \quad (6)$$

$$h_{\pi y,t} = \underset{(0.044)}{0.044} \sqrt{h_{\pi,t} h_{y,t}} \quad (7)$$

Residual Diagnostics

	$Q(4)$	$Q^2(4)$	$Q(10)$	$Q^2(10)$
Inflation Eq.	3.21 [0.52]	1.95 [0.75]	9.41 [0.49]	7.44 [0.68]
Output Eq.	4.90 [0.30]	1.86 [0.76]	9.14 [0.52]	12.13 [0.28]

**Notes:** The table reports the quasi-maximum likelihood parameter estimates of the bivariate AR(p)-UECCC-GARCH(1, 1)-in-mean model for the US inflation ( $\pi_t$ ) and output growth ( $y_t$ ) data.  $h_{\pi,t}$  and  $h_{y,t}$  denote the conditional variances of inflation and output, respectively. The numbers in parentheses are robust standard errors, the number in brackets are  $p$ -values.  $Q(s)$  and  $Q^2(s)$  are the Ljung-Box tests for  $s$ th-order serial correlation in the standardized and squared standardized residuals.

Next, we discuss the parameter estimates for the in-mean terms in equations (4) and (5) in Table 2. Whether higher nominal uncertainty increases or decreases inflation depends on the central bank's reaction function. If a central bank is sufficiently independent and primarily focused on achieving price stability, the central bank will react to higher nominal variability by reducing the inflation rate (see Holland, 1995). If on the other hand the central bank is targeting inflation as well as output growth, then the reaction of the central bank will depend on the respective weights that are given to the two targets. If the weight on growth is sufficiently large, the central bank has an incentive to increase inflation in the presence of higher nominal uncertainty (see Cukierman and Meltzer, 1986). The in-mean parameter estimate in equation (4),  $\delta_{\pi\pi}^{(3)}$ , suggests that - with a lag of three inflation.

months - higher nominal uncertainty indeed leads to more inflation in the US. This finding is in line with the observation that the Fed is targeting both inflation and growth and, hence, suggests that across our sample considerable weight has been given to the latter.<sup>13</sup>

The finding that  $\delta_{y\pi}^{(0)}$  is negative and significant in equation (5) supports the second leg of the Friedmann (1977) hypothesis that increasing inflation uncertainty affects output growth negatively and is also consistent with the argument by Pindyck (1991). Interestingly, the two in-mean effects of real uncertainty are also significant. With a time delay of one month, increasing output volatility appears to lower the average inflation rate ( $\delta_{\pi y}^{(1)}$  in equation (4) is negative). Note that this is in line with the indirect effect which works via growth and nominal uncertainty [see Section 5.2.2 below (third row in Panel B of Table 6)]. In addition, higher real variability appears to increase output growth ( $\delta_{yy}^{(0)}$  is positive and significant in equation (5) in Table 2). This finding is consistent with the theoretical predictions in Blackburn and Pelloni (2004) who study the relation between output growth and its variability in a stochastic monetary growth model.

It is important to highlight again that the effects from the two uncertainties to inflation arise with some time delay (insignificant contemporaneous parameters are not presented), which is to be expected given the economic theories and the fact that we work with monthly data.<sup>14</sup>

We also investigated the effect of omitting the autoregressive terms from the mean equations. In this case the impact of nominal uncertainty on inflation is estimated to be considerably stronger. To the contrary, the effect of real uncertainty on growth becomes insignificant. These changes can be explained by the positive/negative relation between lagged inflation/growth and nominal/real uncertainty (discussed in more detail below). If the autoregressive terms are omitted, this generates a sort of ‘omitted variables bias’. Similarly, omitting the lagged conditional variances from the mean equations leads to biased estimates of the autoregressive terms. Thus, it is important to control for both (see Conrad et al., 2010a).

Finally, note that from the Ljung-Box tests it appears that our model is well specified, i.e. there is no evidence for serial correlation in the standardized and squared standardized residuals at lags 4 and 10.

## 4.2 The Model with Asymmetries and Level Effects

Next, we reestimate the model and allow for asymmetries as well as level effects, i.e. we estimate our model with the augmented variance specification given by equation (3). Note that choosing the exponential formulation for the level effects ensures that our non-negativity conditions are still sufficient for guaranteeing that  $h_{\pi,t}$  and  $h_{y,t}$  are positive for all  $t$ .

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<sup>13</sup>When Grier and Perry (1998) look for institutional reasons why the inflation response to increased uncertainty varies across countries, they note that countries associated with an opportunistic response have much lower central bank independence than the countries associated with a stabilizing response. However, Conrad and Karanasos (2005) pointed out that one cannot argue that the most independent central banks are in countries where inflation falls in response to increased uncertainty. Our finding (and the one in Fountas and Karanasos, 2007) of a positive effect in the US reinforces their argument.

<sup>14</sup>In the previous studies which employed GARCH-in-mean models the uncertainties were restricted to affecting the levels contemporaneously, often resulting in insignificant parameter estimates (see Section 6).

The results are presented in Table 3. Our conclusions regarding the volatility feedback and the impact of macroeconomic uncertainty on performance are qualitatively not affected. That is, i) nominal uncertainty has a positive impact on real variability, and ii) the own in-mean effects are positive whereas the cross in-mean effects are negative. In addition, inflation affects growth negatively whereas on the other hand, when we control for level effects the impact of growth on inflation disappears. This highlights the importance of modeling not only the in-mean effects but the level effects as well. Now only the indirect impact through macroeconomic uncertainty remains [see below, Section 5.2.1 (second row in Panel A, Table 5)].<sup>15</sup>

As can be seen from Table 3, while there is no evidence for asymmetries in nominal uncertainty, there is strong asymmetry in real variability. The parameter estimate in equation (10),  $g_{yy} < 0$ , shows that real variability is to a large extent driven by negative growth shocks.

Table 3: Bivariate Model with Asymmetry and Level Effects.

$$\pi_t = \dots + \underset{(0.011)}{0.011} y_{t-3} \dots + \underset{(0.019)}{0.033} h_{\pi,t-3} - \underset{(0.0019)}{0.0031} h_{y,t-1} + \varepsilon_{\pi,t} \quad (8)$$

$$y_t = \dots - \underset{(0.100)}{0.249} \pi_{t-2} \dots - \underset{(0.090)}{0.270} h_{\pi,t} + \underset{(0.009)}{0.033} h_{y,t} + \varepsilon_{y,t} \quad (9)$$

$$\begin{aligned} \begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} &= \begin{pmatrix} -0.265 \\ (0.253) \\ 28.939 \\ (3.644) \end{pmatrix} + \begin{pmatrix} 0.259 & - \\ (0.039) & \\ 0.626 & 0.467 \\ (0.282) & (0.119) \end{pmatrix} \begin{pmatrix} \varepsilon_{\pi,t-1}^2 \\ \varepsilon_{y,t-1}^2 \end{pmatrix} \\ &+ \begin{pmatrix} - & - \\ - & -0.322 \\ & (0.123) \end{pmatrix} \begin{pmatrix} \mathbf{1}_{\{\varepsilon_{\pi,t-1} > 0\}} \varepsilon_{\pi,t-1}^2 \\ \mathbf{1}_{\{\varepsilon_{y,t-1} > 0\}} \varepsilon_{y,t-1}^2 \end{pmatrix} + \begin{pmatrix} 0.508 & - \\ (0.072) & \\ 1.372 & - \\ (0.574) & \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} \exp(0.105 \pi_{t-1}) & -0.201 \mathbf{1}_{\{y_{t-1} < 0\}} y_{t-1} \\ (0.032) & (0.084) \\ \exp(0.205 \pi_{t-1}) & - \\ (0.081) & \end{pmatrix} \end{aligned} \quad (10)$$

$$h_{\pi y,t} = \underset{(0.043)}{0.031} \sqrt{h_{\pi,t} h_{y,t}} \quad (11)$$

Residual Diagnostics

	$Q(4)$	$Q^2(4)$	$Q(10)$	$Q^2(10)$
Inflation Eq.	6.66 [0.16]	2.83 [0.59]	15.65 [0.11]	10.84 [0.37]
Output Eq.	5.12 [0.27]	1.36 [0.85]	8.79 [0.55]	16.02 [0.10]

**Notes:** See Table 2.

The two expressions in equation (10) also present the estimates for the level coefficients  $\lambda_{ij}$ ,  $i, j = \pi, y$ . The coefficient estimate,  $\lambda_{\pi\pi} > 0$ , indicates that higher lagged inflation tends to increase nominal uncertainty, thus supporting the Ungar-Zilberfarb the-

<sup>15</sup>Note, that  $\omega_\pi$  is estimated to be negative but insignificant. In the UECCC GARCH model (without level effects) the non-negativity constraints can be satisfied if one or even both constants are negative. Since it is insignificant, we treat  $\omega_\pi$  as being zero when checking the constraints.



ory. This finding highlights the harmful effects of inflation which is found to lead to less predictability. Since this unpredictability can reduce economic activity and misallocate resources in the economy, the incentive for lowering inflation is clear. Since  $\lambda_{y\pi} > 0$ , we also provide strong evidence in support of the Dotsey-Sarte conjecture. Thus inflation breeds uncertainty in many forms. Our results suggest the importance of devoting greater explicit attention to the effects of inflation on the variability in output growth.

We now turn to the effects of growth on the two volatilities. We find that only negative growth rates affect nominal uncertainty. As predicted by Brunner, the coefficient estimate,  $\lambda_{\pi y} < 0$ , provides support for a negative impact of growth on inflation variability.<sup>16</sup> As long as we control for the asymmetric effects of growth shocks on real variability, we do not find a significant direct effect of growth on its uncertainty. However, there is a negative indirect impact [see below, Section 5.2.1 (second row in Panel B, Table 5)]. This result is consistent with the theoretical underpinnings that predict a negative indirect effect because of the interaction of the Brunner conjecture with the Logue-Sweeney theory.<sup>17</sup>

Finally, note that apart from the fact that lagged output is no longer significant in the inflation equation, also the direct effect of lagged inflation on growth is much weaker (and less significant), when we appropriately control for asymmetries and level effects. On the other hand, the cross in-mean effects become more important. Thus, the comparison between the results presented in Table 2 and 3 clearly reveals the importance of the correct modeling of the conditional variances for estimating the bidirectional effects between inflation and growth.

Figure 1 shows the output growth series and the real uncertainty measure as obtained from the estimation presented in Table 3. The figure clearly shows the strong relation between negative growth rates and high real uncertainty. It also shows the decline in output volatility from the 1980's onwards, i.e. the Great Moderation.

## 5 Discussion

### 5.1 Summary

Table 4 summarizes all twelve effects. As can be seen, the two bidirectional feedbacks between the levels ( $\pi \overset{-}{\rightleftarrows} y$ ) and the variances ( $h_y \overset{-}{\rightleftarrows} h_\pi$ ) are mixed. That is, inflation and real uncertainty have a negative impact on growth and nominal variability respectively, whereas the two effects in the opposite direction are positive. Moreover, the two own in-mean effects are positive ( $h_\pi \overset{+}{\rightarrow} \pi$ ;  $h_y \overset{+}{\rightarrow} y$ ) whereas the two cross in-mean effects are negative ( $h_\pi \overset{-}{\rightarrow} y$ ;  $h_y \overset{-}{\rightarrow} \pi$ ). Finally, higher inflation increases macroeconomic uncertainty ( $\pi \overset{+}{\rightarrow} h_\pi, h_y$ ), whereas the effect of growth is negative ( $y \overset{-}{\rightarrow} h_\pi, h_y$ ). The latter result is in line with Fountas and Karanasos (2008) who find that in four out of five European countries macroeconomic performance affects real variability negatively and that the effect

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<sup>16</sup>Since we found no significant effect for positive growth rates, we employed a specification with  $\mathbf{1}_{\{y_{t-1} > 0\}} y_{t-1}$ .

<sup>17</sup>Interestingly, when we ignore the asymmetry then growth has a negative impact on its variability (results not reported).

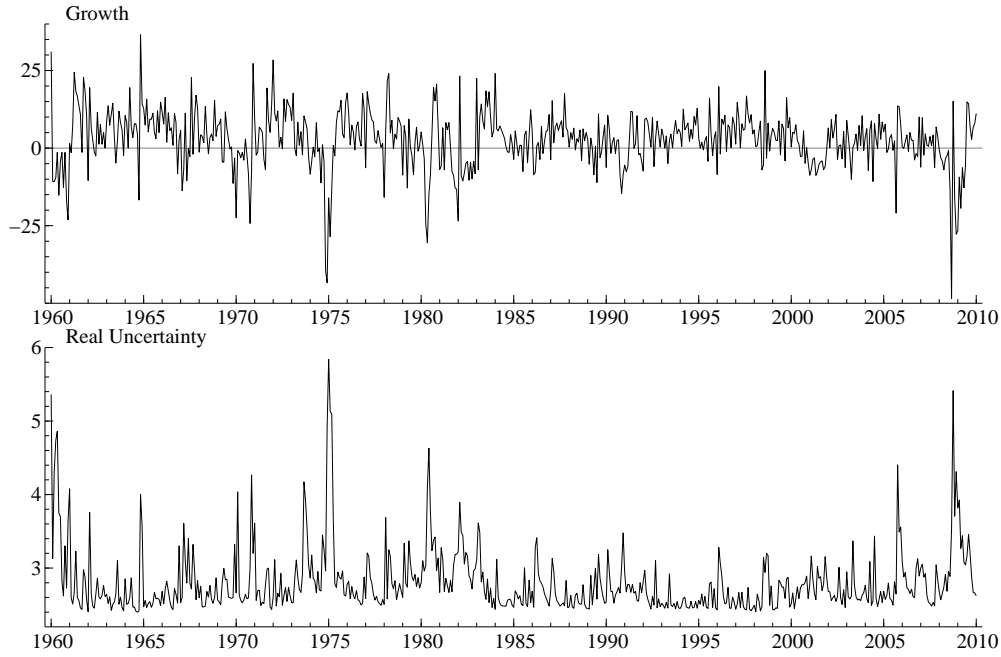


Figure 1: US output growth and real uncertainty (standard deviations) in the period 1960:01–2010:01.

of growth on its uncertainty works via the inflation channel. Interestingly the mixed bidirectional feedback between growth and its uncertainty ( $h_y \overset{+}{\rightleftarrows} y$ ) is in line with a number of economic theories (see Blackburn and Pelloni, 2005) which predict that the two variables could be either positively or negatively correlated.

Table 4: The Relationship between Inflation, Growth and their Uncertainties.

	$\pi$	$y$	$h_\pi$	$h_y$
$\pi$		+	+	-
$y$	-		-	+
$h_\pi$	+	-		- $I$
$h_y$	+	- $I$	+	

**Notes:** The effects of inflation (nominal uncertainty) on the other three variables are presented in the first (third) column. The effects of growth (real variability) on the other three variables are presented in the second (fourth) column. +/-: the impact is positive/negative.  $I$ : the effect is indirect.

## 5.2 Indirect influences

### 5.2.1 Inflation-growth link and level effects

The four variables are connected by a rich network of relations, which may be causal (direct effects), or reflect shared causal pathways (indirect effects). Direct and indirect impacts often occur together. Co-occurrence depends on the strength and number of these relations. However, in order to understand the mechanisms that are responsible for these effects sometimes it is necessary to consider them in isolation.

An empirically important issue is that it is difficult to separate the nominal variability from inflation as the source of the possible negative impact of the latter on growth. As a policy matter this distinction is important. Recall that inflation can affect growth either directly or indirectly (via the nominal uncertainty channel). As Judson and Orphanides (1999) point out: ‘If inflation volatility is the sole culprit, a high but predictably stable level of inflation achieved through indexation may be preferable to a lower, but more volatile, inflation resulting from an activist disinflation strategy. If on the other hand, the level of inflation per se negatively affects growth, an activist disinflation strategy may be the only sensible choice’. In our analysis, we find that the effect of nominal variability on growth is negative ( $\delta_{y\pi}^{(0)} < 0$ ; the second leg of the Friedman hypothesis). Most importantly, even when we control for the impact of inflation on its variability (the first leg of the Friedman hypothesis) and on growth (the direct effect), the evidence in support of the second leg remains. That is, as we can see from the first row of Panel A in Table 5, the likelihood ratio tests reject the null hypotheses:  $H_0 : \lambda_{\pi\pi} = \delta_{y\pi}^{(0)} = 0$ ,  $H_0 : \gamma_{y\pi}^{(2)} = 0$ .

Table 5: Likelihood Ratio Tests.

Panel A: Inflation-growth link		
$\pi_t \overset{+}{\rightarrow} h_{\pi,t+1} \overset{-}{\rightarrow} y_{t+1}$ (2 legs of the Friedman hypothesis)	$H_0 : \lambda_{\pi\pi} = \delta_{y\pi}^{(0)} = 0,$ $H_0 : \gamma_{y\pi}^{(2)} = 0$	10.55 [<0.01]
$y_t \overset{-}{\rightarrow} h_{\pi,t+1} \overset{+}{\rightarrow} h_{y,t+2} \overset{-}{\rightarrow} \pi_{t+3}$ (Brunner/Logue-Sweeney/★)	$H_0 : \lambda_{\pi y} = b_{y\pi} = \delta_{\pi y}^{(1)} = 0$	14.41 [<0.01]
Panel B: Level effects		
$\pi_t \overset{+}{\rightarrow} h_{\pi,t+1} \overset{+}{\rightarrow} h_{y,t+2}$ (2 legs of the Dotsey-Sarte conjecture)	$H_0 : \lambda_{\pi\pi} = b_{y\pi} = 0$	13.68 [<0.01]
$y_t \overset{-}{\rightarrow} h_{\pi,t+1} \overset{+}{\rightarrow} h_{y,t+2}$ (Brunner conjecture/Logue-Sweeney)	$H_0 : \lambda_{\pi y} = b_{y\pi} = 0$	9.10 [0.01]
$\pi_t \overset{-}{\rightarrow} y_{t+2} \overset{-}{\rightarrow} h_{\pi,t+3} \overset{+}{\rightarrow} h_{y,t+4}$ (Gillman-Kejak/Brunner/Logue-Sweeney)	$H_0 : \gamma_{y\pi}^{(2)} = \lambda_{\pi y} = b_{y\pi} = 0$	15.61 [<0.01]

**Notes:** The table reports the results of the likelihood ratio tests discussed in the text. The numbers in brackets are  $p$ -values. ★ : As yet, there is no theory regarding the effect of real uncertainty on inflation.

Next, the indirect positive impact of growth on inflation works via the volatility channel:  $y_t \overset{-}{\rightarrow} h_{\pi,t+1} \overset{+}{\rightarrow} h_{y,t+2} \overset{-}{\rightarrow} \pi_{t+3}$ . The likelihood ratio test rejects the null hypothesis:  $H_0 : \lambda_{\pi y} = b_{y\pi} = \delta_{\pi y}^{(1)} = 0$ , and confirms the indirect effect (see the second row of Panel A).

It is worth noting that the direct relation is qualitatively altered by the presence of the indirect influences. That is, when we include the level effects in the model and in particular the negative influence of growth on inflation uncertainty, as predicted by Brunner, the direct impact disappears, that is  $\gamma_{\pi y}^{(3)}$  becomes insignificant (see equation (8) in Table 3).

Moreover, inflation, via the nominal uncertainty channel, affects not only growth but real variability as well. That is, the indirect evidence regarding the positive impact of inflation on real uncertainty ( $\pi_t \xrightarrow{+} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2}$ ) agrees well with the direct evidence supporting the Dotsey and Sarte conjecture. In particular, the likelihood ratio test rejects the null hypotheses:  $H_0: \lambda_{\pi\pi} = b_{y\pi} = 0$  (see the first row in Panel B). Similarly to the Friedman hypothesis, the Dotsey and Sarte conjecture has two legs. The first one is identical to the first leg of the Friedman hypothesis ( $\pi_t \xrightarrow{+} h_{\pi,t+1}$ ) while the second one is the Logue and Sweeney theory ( $h_{\pi,t+1} \xrightarrow{+} h_{y,t+2}$ ).

Next, we hypothesize that the effects of growth on its variability could work through changes in inflation uncertainty as well. Theoretically speaking the impact is based on the interaction of two effects. A higher growth rate will reduce nominal uncertainty (the Brunner conjecture) and, therefore, real variability (the Logue-Sweeney theory):  $y_t \xrightarrow{-} h_{\pi,t+1} \xrightarrow{+} h_{y,t+2}$ . The evidence for both these influences confirms the negative indirect impact. In particular, the null hypothesis  $H_0: \lambda_{\pi y} = b_{y\pi} = 0$  is rejected (see the second row in Panel B). That is, both inflation and growth affect real uncertainty indirectly via the nominal variability channel. Whereas the former impact is positive (as predicted by Dotsey and Sarte) the latter one is negative. Interestingly inflation breeds uncertainty in many ways and forms. In particular, higher inflation increases both variabilities, nominal and real, directly (the first leg of the Friedman hypothesis, and the Dotsey-Sarte conjecture respectively) and indirectly via the growth channel:  $\pi_t \xrightarrow{-} y_{t+2} \xrightarrow{-} h_{\pi,t+3} \xrightarrow{+} h_{y,t+4}$  since the null hypothesis  $H_0: \gamma_{y\pi}^{(2)} = \lambda_{\pi y} = b_{y\pi} = 0$  is rejected (see the last row in Panel B). These results suggest the importance of devoting explicit attention to the effects of macroeconomic performance on its uncertainty.

### 5.2.2 Volatility feedback and in-mean effects

For our purposes it helped to distinguish between direct and indirect impacts. Our analysis has highlighted reciprocal interactions in which two or more variables influence each other, either directly or indirectly. As we have already seen these kinds of interactions can be very important.

Recall again that the two legs of the Friedman hypothesis imply that growth is negatively affected by inflation via the nominal uncertainty channel. Our results also suggest that real variability is related indirectly to nominal uncertainty through inflation:  $h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{+} h_{y,t+4}$ . As we can see from the first row of panel A in Table 6, the null hypothesis  $H_0: \delta_{\pi\pi}^{(3)} = \lambda_{y\pi} = 0$  is rejected. Similarly, the indirect negative influence of real variability on nominal uncertainty through its (first lag) impact on inflation,  $h_{y,t} \xrightarrow{-} \pi_{t+1} \xrightarrow{+} h_{\pi,t+2}$ , tells essentially the same story with the indirect evidence which is consistent with the Blackburn-Pelloni theory and supports the Brunner conjecture:  $h_{y,t} \xrightarrow{+} y_t \xrightarrow{-} h_{\pi,t+1}$ . That is, the likelihood ratio tests reject the null hypotheses  $H_0: \delta_{\pi y}^{(1)} = \lambda_{\pi\pi} = 0$ ,  $H_0: \delta_{yy}^{(0)} = \lambda_{\pi y} = 0$  and confirm the two indirect effects (see the last

Table 6: Likelihood Ratio Tests.

Panel A: Volatility feedback		
$h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{+} h_{y,t+4}$ (Cukierman-Meltzer/Dotsey-Sarte)	$H_0 : \delta_{\pi\pi}^{(3)} = \lambda_{y\pi} = 0$	4.29 [0.12]
$h_{y,t} \xrightarrow{-} \pi_{t+1} \xrightarrow{+} h_{\pi,t+2}$ (*/Ungar-Zilberfarb)	$H_0 : \delta_{\pi y}^{(1)} = \lambda_{\pi\pi} = 0$	16.50 [<0.01]
$h_{y,t} \xrightarrow{+} y_t \xrightarrow{-} h_{\pi,t+1}$ (Blackburn-Pelloni/Brunner conjecture)	$H_0 : \delta_{yy}^{(0)} = \lambda_{\pi y} = 0$	11.44 [<0.01]
Panel B: In-mean effects		
$h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{-} y_{t+5}$ (Cukierman-Meltzer/Gillman-Kejak)	$H_0 : \delta_{\pi\pi}^{(3)} = \gamma_{y\pi}^{(2)} = 0$	7.92 [0.02]
$h_{y,t} \xrightarrow{-} \pi_{t+1} \xrightarrow{-} y_{t+3}$ (*/Pindyck)	$H_0 : \delta_{\pi y}^{(1)} = \gamma_{y\pi}^{(2)} = 0$	10.34 [<0.01]
$h_{y,t} \xrightarrow{+} y_t \xrightarrow{-} h_{\pi,t+1} \xrightarrow{+} \pi_{t+4}$ (Blackburn-Pelloni/Brunner/Cukierman-Meltzer)	$H_0 : \delta_{yy}^{(0)} = \lambda_{\pi y} = \delta_{\pi\pi}^{(3)} = 0$	22.84 [<0.01]

**Notes:** See Table 5.

two rows of panel A). In sharp contrast, there is a lack of a direct impact. As mentioned earlier this indirect evidence is in line with the Fuhrer theory.

Interestingly, the direct effect of nominal uncertainty on growth (the second leg of the Friedman hypothesis:  $h_{\pi,t} \xrightarrow{-} y_t$ ) is in agreement with the indirect impact that works via the inflation channel:  $h_{\pi,t} \xrightarrow{+} \pi_{t+3} \xrightarrow{-} y_{t+5}$ . That is, the likelihood ratio test rejects the null hypothesis:  $H_0 : \delta_{\pi\pi}^{(3)} = \gamma_{y\pi}^{(2)} = 0$  and confirms the indirect effect (see the first row of panel B). Thus nominal uncertainty, via the inflation channel, affects not only real variability but growth as well. In particular, higher nominal uncertainty leads to an increase in inflation, as predicted by Cukierman and Meltzer, which in turn increases real variability (the Dotsey and Sarte conjecture) and reduces growth (the Gillman-Kejak theory).

Moreover, both nominal and real variability affect growth indirectly via the inflation channel. Whereas the former impact is negative (as we have shown in the previous paragraph) the latter one is positive ( $h_{y,t} \xrightarrow{-} \pi_{t+1} \xrightarrow{-} y_{t+3}$ ) which is in line with the Blackburn-Pelloni theory. That is the null hypothesis:  $H_0 : \delta_{\pi y}^{(1)} = \gamma_{y\pi}^{(2)} = 0$  is rejected (see the second row of Panel B). Therefore, real variability, via the inflation channel, affects not only nominal uncertainty but growth as well (see the second rows of Panels A and B).

Finally, both types of evidence, direct and indirect, point unequivocally to a negative effect of growth variability on inflation. That is, the direct evidence supporting the negative effect is in line with the evidence which is consistent with the Blackburn-Pelloni theory, and supports the Brunner conjecture and the Cukierman-Meltzer theory:  $h_{y,t} \xrightarrow{+} y_t \xrightarrow{-} h_{\pi,t+1} \xrightarrow{+} \pi_{t+4}$ . In other words the likelihood ratio test rejects the null hypothesis  $H_0 : \delta_{yy}^{(0)} = \lambda_{\pi y} = \delta_{\pi\pi}^{(3)} = 0$  (see the last row of Panel B). Whereas real uncertainty affects inflation directly after one month, the indirect effect takes four months to show up.

## 6 Sensitivity Analysis

In this section we analyze the robustness of our findings with respect to changes in our baseline specification.

### *Specification in standard deviations*

As a first robustness check we replace  $\mathbf{h}_{t-r}$  by  $\mathbf{h}_{t-r}^{\wedge 1/2}$  in equation (1), i.e. we express the in-mean effects in terms of standard deviations instead of conditional variances. As can be seen from Table 7, our main conclusions remain unchanged. The in-mean effects are significant ( $\delta_{\pi y}$  at the 10% level, the other coefficients at the 1% or 5% level) and of the same signs as before. While the impact of inflation on real uncertainty is no longer significant, we now find a significantly negative level effect of growth on real uncertainty. This direct negative impact is line with the indirect influence via the Brunner conjecture and the Logue-Sweeney theory discussed above.

Table 7: Bivariate Model with Standard Deviations in-mean.

$$\pi_t = \dots + \underset{(0.011)}{0.012} y_{t-3} \dots + \underset{(0.139)}{0.292} \sqrt{h_{\pi,t-3}} - \underset{(0.044)}{0.071} \sqrt{h_{y,t-1}} + \varepsilon_{\pi,t} \quad (12)$$

$$y_t = \dots - \underset{(0.107)}{0.177} \pi_{t-2} \dots - \underset{(0.703)}{2.498} \sqrt{h_{\pi,t}} + \underset{(0.263)}{1.090} \sqrt{h_{y,t}} + \varepsilon_{y,t} \quad (13)$$

$$\begin{aligned} \begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} &= \begin{pmatrix} -0.224 \\ (0.256) \\ 28.065 \\ (3.595) \end{pmatrix} + \begin{pmatrix} 0.266 & - \\ (0.040) & \\ 0.713 & 0.459 \\ (0.284) & (0.125) \end{pmatrix} \begin{pmatrix} \varepsilon_{\pi,t-1}^2 \\ \varepsilon_{y,t-1}^2 \end{pmatrix} \\ &+ \begin{pmatrix} - & - \\ - & -0.353 \\ & (0.129) \end{pmatrix} \begin{pmatrix} \mathbf{1}_{\{\varepsilon_{\pi,t-1}>0\}} \varepsilon_{\pi,t-1}^2 \\ \mathbf{1}_{\{\varepsilon_{y,t-1}>0\}} \varepsilon_{y,t-1}^2 \end{pmatrix} + \begin{pmatrix} 0.496 & - \\ (0.072) & \\ 1.504 & - \\ (0.574) & \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} \exp(0.107 \pi_{t-1}) & -0.187 \mathbf{1}_{\{y_{t-1}<0\}} y_{t-1} \\ (0.031) & (0.078) \\ \exp(0.159 \pi_{t-1}) & \exp(-0.125 y_{t-1}) \\ (0.156) & (0.077) \end{pmatrix} \end{aligned} \quad (14)$$

$$h_{\pi y,t} = \underset{(0.044)}{0.033} \sqrt{h_{\pi,t} h_{y,t}} \quad (15)$$

**Notes:** See Table 2.

### *Linear level effects*

In the estimation presented in Table 8, we replace the exponential by linear level effects. Again, our results remain unchanged, i.e. the level effect of inflation on nominal and real variability is significant and positive, while growth has a negative impact on nominal uncertainty. The sign and the significance of all four in-mean effects is as before.

### *Lagged level effects*

We also investigate the robustness of our findings with respect to the lag order of the level variables (results not reported). Recall that in the baseline specification we employ only the first lag of inflation and output growth. However, we find the level effects of higher order lags to be either of the same sign as before or insignificant. In particular, the negative effect of growth on nominal uncertainty is confirmed at lag two for negative

Table 8: Bivariate Model with Linear Level Effects.

$$\pi_t = \dots + \underset{(0.011)}{0.010} y_{t-3} \dots + \underset{(0.018)}{0.033} h_{\pi,t-3} - \underset{(0.0017)}{0.003} h_{y,t-1} + \varepsilon_{\pi,t} \quad (16)$$

$$y_t = \dots - \underset{(0.093)}{0.265} \pi_{t-2} \dots - \underset{(0.081)}{0.250} h_{\pi,t} + \underset{(0.009)}{0.032} h_{y,t} + \varepsilon_{y,t} \quad (17)$$

$$\begin{aligned} \begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} &= \begin{pmatrix} \underset{(0.248)}{0.491} \\ \underset{(3.660)}{26.814} \end{pmatrix} + \begin{pmatrix} \underset{(0.041)}{0.275} & - \\ \underset{(0.280)}{0.718} & \underset{(0.121)}{0.473} \end{pmatrix} \begin{pmatrix} \varepsilon_{\pi,t-1}^2 \\ \varepsilon_{y,t-1}^2 \end{pmatrix} \\ &+ \begin{pmatrix} - & - \\ - & \underset{(0.127)}{-0.321} \end{pmatrix} \begin{pmatrix} \mathbf{1}_{\{\varepsilon_{\pi,t-1}>0\}} \varepsilon_{\pi,t-1}^2 \\ \mathbf{1}_{\{\varepsilon_{y,t-1}>0\}} \varepsilon_{y,t-1}^2 \end{pmatrix} + \begin{pmatrix} \underset{(0.071)}{0.512} & - \\ \underset{(0.576)}{0.881} & - \end{pmatrix} \begin{pmatrix} h_{\pi,t-1} \\ h_{y,t-1} \end{pmatrix} \\ &+ \begin{pmatrix} \underset{(0.076)}{0.200} \pi_{t-1} & \underset{(0.081)}{-0.184} \mathbf{1}_{\{y_{t-1}<0\}} y_{t-1} \\ \underset{(0.971)}{1.981} \pi_{t-1} & - \end{pmatrix} \end{aligned} \quad (18)$$

$$h_{\pi y,t} = \underset{(0.043)}{0.032} \sqrt{h_{\pi,t} h_{y,t}} \quad (19)$$

**Notes:** See Table 2.

as well as positive growth rates. (Recall that in Table 3 only negative growth rates have a significant level effect on inflation uncertainty.)

#### *Sub-periods*

In order to control for possible changes in the conduct of monetary policy, we reestimate our favored specification by interacting the main variables of interest with dummy variables for the period 1980-2010 (results not reported). While our conclusions regarding the link between the two variabilities remain unchanged, we find some changes in the in-mean effects. Among the four interaction terms only those on the own in-mean effects are significant. That is, the one on nominal uncertainty in the inflation equation and the one on real variability in the growth equation. Both own in-mean effects tail off from the 1980's onwards. The fact that the positive effect of nominal uncertainty on inflation becomes weaker is line with the observation that the FED became more inflation focused during that time and, hence, supports the Holland argument. A damped negative effect of real variability on growth is expected from the literature on the Great Moderation, i.e. the observation that the volatility of growth has considerably declined since the early 1980's.<sup>18</sup>

#### *Quarterly data*

As a final robustness check, we reestimate our model using quarterly data (see Table 9). Again, we find that inflation has a direct and highly significant negative effect on output growth. To the contrary, the direct effect of growth on inflation is positive but significant at the 12% level only. Three out of the four in-mean effects are significant. Inflation

<sup>18</sup>The recent studies by McConnell and Perez-Quiros (2000) and Stock and Watson (2002) highlight the importance of the reduction in US GDP growth volatility in the last two decades and its implications for growth theory.

uncertainty increases inflation while it affects growth negatively (both significant at the five percent level). Real uncertainty reduces inflation (significant at the 12% level), but has no significant influence on growth. All four in-mean effects have the same signs as for the monthly data, but are now contemporaneous. Similarly, the level effects of inflation on the two uncertainties are positive and significant. Although, there is no evidence for volatility interaction in the quarterly data, we still find evidence for strong asymmetry in real uncertainty ( $\hat{g}_{yy} = -0.255$  (0.118)).

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Table 9: AR-UECCC-GARCH(1, 1)-in-mean Model for Quarterly Data.

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$$\pi_t = \dots + \frac{0.032}{(0.020)} y_{t-1} \dots + \frac{0.457}{(0.200)} h_{\pi,t} - \frac{0.012}{(0.007)} h_{y,t} + \varepsilon_{\pi,t} \quad (20)$$

$$y_t = \dots - \frac{0.318}{(0.116)} \pi_{t-1} \dots - \frac{0.540}{(0.241)} h_{\pi,t} + \frac{0.016}{(0.063)} h_{y,t} + \varepsilon_{y,t} \quad (21)$$

$$\Lambda = \begin{pmatrix} \frac{0.052}{(0.023)} & - \\ \frac{0.153}{(0.041)} & - \end{pmatrix} \quad (22)$$


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**Notes:** See Table 2.

## 7 Comparison with other work

The previous GARCH time series studies that examine the inflation-growth link in the US use various sample periods, data frequencies and empirical methodologies. Some GARCH studies of this issue utilize the simultaneous estimation approach. For example, Baillie et al. (1996) and Fountas and Karanasos (2006) employ univariate GARCH models that allow for simultaneous feedback between the conditional mean and variance of inflation and growth respectively. Other recent studies use bivariate GARCH-in-mean models – either the CCC (Grier and Perry, 2000) or the BEKK specification (Grier et al., 2004, Bredin and Fountas, 2005, Shields et al., 2005) – to examine the impact of macroeconomic uncertainty on performance. Some researchers employ the two-step Granger-causality approach. For example, Grier and Perry (1998), Conrad and Karanasos (2005) and Fountas and Karanasos (2007) estimate univariate GARCH models, while Karanasos and Kim (2005) and Fountas et al. (2006) use bivariate BEKK and CCC GARCH formulations respectively. In the first step, the estimated models are used to generate conditional variances of inflation and output growth as proxies of nominal and real uncertainty and, in the second step, Granger-causality tests are performed. Table 10 presents a summary of the aforementioned studies and their findings.

The results presented in our paper can be related to those obtained from three previous recent studies that have made use of the GARCH approach. First, a comparison can be made with the studies by Grier et al. (2004) and Shields et al. (2005) which find evidence supporting eight out of the twelve links. However, these authors do not consider level effects in their analysis, impose a positive volatility feedback and they only test for contemporaneous in-mean effects. Second, the recent study by Fountas et al. (2006) uses



the two-step approach and finds evidence supporting only six out of the twelve links. No evidence for cross level effects is found. They also fail to find any significant effect of either growth or real uncertainty on inflation. Finally, another difference is the lack of evidence for volatility feedback. Below we discuss the results and the related recent literature in more detail.

#### *Inflation-growth link*

As pointed out earlier we find a mixed bidirectional feedback between inflation and growth:  $\pi \overset{-}{\rightleftarrows} y$ . Recall that when we include the level effects in the model and in particular the  $\overset{+}{\rightleftarrows}$  negative influence of growth on inflation uncertainty (as predicted by Brunner) the direct positive impact of growth on inflation disappears. From the bivariate studies only Grier et al. (2004), Shields et al. (2005) and Fountas and Karanasos (2007) find that growth affects inflation positively. However, none of these three studies has taken into account level effects.

#### *Volatility feedback*

We find that nominal variability has a positive impact on real uncertainty whereas the (indirect) effect in the opposite direction is negative. The studies that employ bivariate GARCH-in-mean models use either a CCC or a BEKK GARCH specification, and hence, impose either no feedback or a positive one. Only Karanasos and Kim (2005) find evidence for a mixed volatility feedback but they employ the two-step approach.

#### *In-mean effects*

Recall that according to our results with a time delay of one month increasing output volatility appears to lower the average inflation rate ( $h_{y,t} \overset{-}{\rightarrow} \pi_{t+1}$ ). In Bredin and Fountas (2005) the two uncertainties have no impact on inflation. From the five studies that employ bivariate GARCH-in-mean models only two, Grier et al. (2004) and Shields et al. (2005) find that real uncertainty affects inflation negatively. Nevertheless, the results of these five studies must be considered with caution. Doubts arise as to whether in-mean effects were sufficiently accounted for by not including lags of the conditional variances in the two mean equations. We also find that, with a lag of three months, higher nominal uncertainty leads to more inflation ( $h_{\pi,t} \overset{+}{\rightarrow} \pi_{t+3}$ ). In sharp contrast, the two aforementioned studies find that nominal variability affects inflation negatively. However, they only test for a contemporaneous effect. As mentioned above, since the theoretical rationale for this in-mean effect suggests that it takes some time for it to materialize (see Section 3), it appears more appropriate to investigate such effects within a specification that includes several lags of its conditional variance in the mean equation of inflation.

#### *Level effects*

The level effects that are shown as being important have not been accounted for in the previous studies which employed bivariate GARCH-in-mean models. From the studies that employ the two-step approach only Fountas et al. (2006) test and find, as the present study does, a negative/positive effect of growth/inflation on its uncertainty. However, in Fountas et al. (2006) the cross level effects are found to be insignificant. As pointed out earlier we provide evidence regarding the negative impact of growth on nominal uncertainty and in support of the Dotsey-Sarte theory as well. Thus our study is the only one that provides quite strong evidence supporting all four level effects. These results carry noteworthy implications for macroeconomic modeling and policymaking.

Table 10: The inflation-growth link in the US. Summary of previous studies.

Papers	Sample; Data	Cross (mean and variance) effects				In mean effects				Level Effects				Signific. Effects
		$\pi$	$y$	$h_\pi$	$h_y$	$h_\pi$	$h_\pi$	$h_y$	$h_y$	$\pi$	$\pi$	$y$	$y$	
		$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	
		$y$	$\pi$	$h_y$	$h_\pi$	$\pi$	$y$	$\pi$	$y$	$h_\pi$	$h_y$	$h_\pi$	$h_y$	
<i>In-mean-level models</i>														
Univariate specifications														
BCT, 96		x	x	x	x	0	x	x	x	0	x	x	x	0
FK, 06	1860-1999; IPI	x	+	x	x	x	x	x	0	x	-	x	-	3
Bivariate specifications														
GP, 00	48-96; PPI, IPI	-	x	x	x	0	$-R_c$	0	0	x	x	x	x	$2;1^{R_c}$
E*, 04	66-00; CPI, IPI	-	x	x	x	x	-	x	x	x	x	x	x	2
GHOS, 04	47-00; PPI, IPI	-	+	$+^{R+}$	$+^{R+}$	$-R_c$	$-R_c$	$-R_c$	$+^{R_c}$	x	x	x	x	$8;2^{R+},4^{R_c}$
BF, 05	57-03; PPI,IPI	0	0	$+^{R+}$	$+^{R+}$	0	$-R_c$	0	$+^{R_c}$	x	x	x	x	$4;2^{R+},2^{R_c}$
SOHB, 05	47-00; PPI,IPI	-	+	$+^{R+}$	$+^{R+}$	$-R_c$	$-R_c$	$-R_c$	$+^{R_c}$	x	x	x	x	$8;2^{R+},4^{R_c}$
CK <sup>o</sup> , 10	60-07; CPI	-	+	+	$-I$	+	-	-	+	+	+	-	$-I$	12
<i>Granger-causality tests</i>														
Univariate specifications														
GP,98	48-93; CPI	+	x	x	x	-	x	x	x	+	x	x	x	3
CK, 05	62-00; CPI	x	x	x	x	0	x	x	x	+	x	x	x	1
Bivariate specifications														
KK, 05	57-00; PPI,IPI	x	x	+	-	x	x	x	x	x	x	x	x	2
FKK, 06	57-00; WPI, IPI	-	0	0	0	$\pm$	-	0	+	+	0	0	-	6
FK, 07	57-00; PPI, IPI	-	+	x	x	+	-	-	0	+	x	x	x	6
Overall effect:		-	+	+	$-I$	$\pm$	-	-	+	+	+	-	$-I$	12

**Notes:** The  $\rightarrow$  column presents the effect of  $x_t$  on  $z_t$ . +/ -: The effect is positive/negative. 0: The effect is zero; x: The link is not examined. \*: This is a trivariate model. <sup>o</sup>: This is the present study.  $R_+$ : The effect is restricted to be positive.  $R_c$ : The effect is restricted to being contemporaneous.  $I$ : This is an indirect effect. BCT: Baillie, Chung and Tieslau; BF: Bredin and Fountas; CK: Conrad and Karanasos; E: Elder; GHOS: Grier, Henry, Olekalns and Shields; GP: Grier and Perry; FKK: Fountas, Karanasos and Kim; FK: Fountas and Karanasos; KK: Karanasos and Kim; SOHB: Shields, Olekalns, Henry and Brooks. Overall effect: this is sign which appears most frequently in each column.

## 8 Future Work

### *Econometric methods*

As a contribution to econometric methods, this study opens up several possible areas for future investigation. Kumar and Okimoto (2007) point out that several studies have analyzed inflation persistence and reached diverged conclusions. Conrad et al. (2010a) show that in a mean-level framework the persistence from the variance is transmitted to the mean and vice versa, and, hence, by studying the conditional mean/variance independently one will tend to overestimate the degree of persistence. In particular, unit root tests for the level will not be able to reject the null hypothesis of the process being integrated of order one. One can extend their analysis to our bivariate framework.

### *Multivariate extensions*

The results in Campos et al. (2010) show that financial development and political instability exhibit the most robust first-order effects on growth and its volatility in Argentina since 1890, and therefore are the main causes for the long-run relative decline of the Argentinean economy. They also find that trade openness and international financial integration play important yet secondary roles. In the context of our analysis, incorporating macroeconomic variables either in the conditional mean or in the conditional variances or in both could be at work. We look forward to sorting this out in future work. Grier and Perry (2000) in their bivariate formulation include the 6-month commercial paper-3-month treasury bill spread as a predictor of growth. In addition, it would be instructive to examine the relation between inflation, growth and their variabilities in a trivariate framework that accommodates the interactions of interest in an internally consistent fashion (see, for example, Elder, 2004).

### *Impulse response functions*

The growing popularity of the multivariate GARCH models and their application to the study of inflation, growth and their volatilities has led to the requirement of calculating the magnitude and persistence of macroeconomic uncertainty following a shock. By taking into account the complex interaction between the first and second moments of our UECCC-AGARCH-ML model we can derive and analyze such measures in the context of our dynamic specification. Following Fiorentini and Sentana (1998), Conrad et al. (2010a) develop analytical expressions for an impulse-response function for univariate GARCH-ML models. We can apply the methodology in Conrad et al. (2010a) and investigate the dynamics implied by the conditional variance-covariance structure of our bivariate model by perturbing the system with innovations to inflation and output growth. Alternatively, we can also use the variance/volatility impulse response functions (VIRF) as defined in Shields et al. (2005) to uncover volatility dynamics operating between the two variables.

### *Structural Breaks*

We consider the effect of structural changes from the perspective of a robustness check. But testing and modelling the structural change more formally in the bivariate UECCC-AGARCH-ML model is an important future work. Campos et al. (2010), who take into account structural breaks in the Argentinian growth by incorporating dummy variables both in the conditional mean and variance, provide good starting points for this problem (see also Caporale and Kontonikas, 2009). Further, Baillie and Morana (2009) introduce a new long-memory volatility specification, denoted by Adaptive FIGARCH, which is

designed to account for both long-memory and structural change in the conditional variance process. One could provide an enrichment of the bivariate in-mean level models by allowing the intercepts of the two means and variances to follow a slowly varying function as in Baillie and Morana (2009). This is undoubtedly a challenging yet worthwhile task. Pesaran et al. (2006) provide a new approach to forecasting time series that are subject to discrete structural breaks. Their results suggest several avenues for further research.

#### *G7/Euro countries*

It would be instructive to examine whether the link between macroeconomic performance and uncertainty in the US is considerably different from those in other G7 or European countries. For example, Caporale and Kontonikas (2009) examine the relation between inflation and its uncertainty (both short-run and steady-state) in 12 European monetary union (EMU) countries and find a considerable degree of heterogeneity across the EMU countries (see also, Conrad and Karanasos, 2005a, and Bredin and Fountas, 2009).

## 9 Conclusions

We employ an augmented version of the UECCC GARCH model to investigate the relationship between inflation, nominal uncertainty, output growth and real variability using US data. The main advantage of this new specification is that it allows for i) lagged in-mean effects, ii) level effects and iii) volatility feedback of either sign. Thus, we can test the economic theories which imply causal relationships between the four variables in a unified framework. Our results highlight the importance of modeling all possible interactions simultaneously. In particular, we find that most effects work indirectly via the uncertainty channel. For example, we find strong support for the two legs of the Friedman (1977) hypothesis, that is higher inflation increases nominal uncertainty which then negatively affects output growth. Maybe even more importantly, we show that the positive direct effect of output growth on inflation disappears, once we appropriately for asymmetries and level effect. To the contrary, the indirect effect via real variability becomes stronger. Interestingly, in all cases the signs of the direct and the indirect effects are the same. Thus, our results suggest that the methodologies employed in previous studies which only allowed to look for the direct effects have masked the existence of the potentially even more important indirect effects.

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