University of Heidelberg

Department of Economics

Discussion Paper Series No. 480

CROSS-COUNTRY INCOME DIFFERENCES AND TECHNOLOGY DIFFUSION IN A COMPETITIVE WORLD

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December 2008

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Abstract: This paper develops a new open-economy endogenous growth model where technology diffusion allows for a stable and non-degenerate world income distribution. In accordance with the empirical literature, I find that country characteristics such as the social infrastructure, the degree of openness, the investment rate, population growth, the level of human capital, or growth policies such as subsidies to innovation investments explain a country's position in the eventual world income distribution. Club convergence in growth rates can be traced back to a country's openness and to a minimum required level of human capital.

Keywords: Capital Accumulation, Technology Diffusion, Neoclassical Growth Model.

JEL-Classification: O11, O33, O41.

This Version: December 9, 2008.

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I would like to thank Burkhard Heer and participants of the workshop "Innovation and Growth", Free University of Bolzano, May 2006, for helpful comments. Financial assistance from Deutsche Forschungsgemeinschaft, research grant IR 44 1-1, is gratefully acknowledged.

1 Introduction

Technological knowledge diffuses across the boundaries of open economies.[1] As a consequence, backward countries with access to the knowledge contained in the world's technological frontier, may adopt this knowledge and, thereby, grow faster than advanced countries (Gerschenkron (1962), Abramovitz (1986)). This paper develops a new open-economy endogenous growth model where this mechanism allows for a stable and non-degenerate world income distribution. The purpose is to detect both country characteristics and properties of the growth process that explain a country's position in the eventual world income distribution.

From a macroeconomic point of view, international technology diffusion is the process by which domestic firms incorporate new ideas and techniques from abroad into their production technology and, thereby, raise the productivity of the available domestic factors of production. At the microeconomic level, I motivate this process by what Griffith, Redding, and Reenen (2004) call *the second face of R&D*, i. e., the fact that firms engaged in innovation activity acquire external knowledge and assimilate discoveries of others.[2] From this point of view, the intensity of domestic innovation activity becomes a key determinant of a country's capacity to absorb previously unknown technological knowledge from abroad.

Following Nelson and Phelps (1966), the second component of the diffusion process is the gap between the state of the world's technological frontier and a country's current state of technological knowledge. This gap represents the pool of ideas and techniques from which the second face of R&D can draw. I take the view that no country has access to the entire knowledge embodied in the world's technological frontier. Hence, over time, the technology gap may rise or fall, however, it remains positive throughout. In a steady state, each country absorbs a constant fraction of the world's technological frontier. The remaining steady-state gap turns out to be a key determinant of a country's relative position in the steady-state world income distribution.

I refer to international technology diffusion as the *foreign* contribution to the advancement of a country's accessible level of technological knowledge. The *domestic* contribution reflects the first face of R&D, i. e., research and development of new

[1] Recent empirical studies in support of this include Coe and Helpman (1995), Eaton and Kortum (1996), Nadiri and Kim (1996), and Coe, Helpman, and Hoffmaister (2008).

[2] Besides Griffith, Redding, and Reenen (2004), there is considerable support for this motive in the empirical literature (see, e. g., Tilton (1971), Allen (1977), Mowery (1983)). Cohen and Levinthal (1989) study the implications of such activity for partial industry equilibria.

technological knowledge undertaken by domestic firms. Overall, the evolution of a country's accessible level of technological knowledge is given by the sum of the domestic and the foreign contribution.

The set-up of the domestic economy adds a competitive intermediate-good sector to an otherwise neoclassical economy to incorporate endogenous economic growth.[3] Innovation investments are undertaken by intermediate-good firms in an attempt to gain an advantage over rivals. These investments are endogenously determined, raise the productivity of domestic labor, and, as a byproduct, bring about a knowledge inflow from abroad.

For this set-up, I establish the intertemporal general equilibrium and the existence of a unique steady state that pins down a country's capital intensity and its relative position with respect to the world's technological frontier. Similar to other theoretical studies of technology diffusion, including Parente and Prescott (1994), Barro and Sala-í-Martin (1997), or Howitt (2000), all economies share the same steady-state growth rate of per-capita magnitudes which coincides with the exogenous growth rate of the world's technological frontier. However, differences in the level of technological knowledge survive even in the steady state and cause cross-country income differences, a feature consistent with empirical findings of, e. g., Hall and Jones (1999).

A particular focus of the analysis is on human capital and on growth policies based on subsidies for innovation investments. Besides its static labor-augmenting effect in the spirit of Becker (1993) and Mincer (1974), I argue that human capital is favorable to innovation because it reduces the amount of resources necessary to adapt to or to invent something new. This follows ideas expressed in, e. g., Nelson and Phelps (1966), Schultz (1975), or Galor and Moav (2000). However, in my context where economic growth is endogenous, the positive effect of human capital on domestic innovation and technology diffusion may be offset by general equilibrium effects. The latter implies that the rate of diffusion, i. e., the rate at which the gap between the technological frontier and the current state of the domestic technology closes, does not necessarily increase in human capital. Thus, the Nelson-Phelps hypothesis (Nelson and Phelps (1966), p. 70) fails in general. In a similar vein, I find that the partial equilibrium effect of a subsidy on innovation investments is positive but may be outweighed by general equilibrium effects.

What determines steady-state income differences across countries? I find that an economy's size, its available research technology, properties of the diffusion pro-

[3] The analytical framework extends and complements the endogenous growth model developed in Irmen (2005). The latter, in turn, builds on Hellwig and Irmen (2001) and Bester and Petrakis (2003).

cess, and the behavioral assumption on savings determine both the set of country characteristics that matter and the sign of the predicted effect. However, for open economies that engage in innovation activity the analysis shows that independent of these categories a) a social infrastructure that fosters the efficiency of an economy's domestic production technology, and b) institutions that facilitate the inflow of technological knowledge from abroad increase a country's position in the steady-state world income distribution. Such countries benefit more from their own R&D and from international R&D spillovers. This is in line with the empirical evidence provided by, e. g., Hall and Jones (1999), Sachs and Warner (1995), or Coe, Helpman, and Hoffmaister (2008). Moreover, a high savings rate and a small population growth rate imply a high steady-state per-capita income, a finding consistent with the correlations that appear in the data provided by Heston, Summers, and Aten (2002) (see, e. g., Weil (2005), p. 70 and 84).

The role of human capital and of subsidies for innovation investments as determinants of cross-country income differences is strengthened if the savings rate is endogenous à la Ramsey (1928), Cass (1965), Koopmans (1965). In this setting the steady-state capital intensity is pinned down by a first-order condition rather than by a market equilibrium condition. As a result, the general equilibrium effects in the comparative statics disappear. Both variables tend to raise domestic research activity and, thereby, increase a country's capacity to absorb knowledge embodied in the technological frontier. Through both channels the steady-state per-capita income rises.

Several studies of the evolution of the world's income distribution question the view according to which *all* countries converge to parallel growth paths (see, e. g., Quah (1997), Durlauf and Johnson (1995), or Pritchet (1997)). My framework highlights two mechanisms in support of this view. Both are consistent with the observation of a growing divergence between the world's richest and poorest countries and with the presence for convergence clubs in growth rates.

First, countries may be closed, i. e., cut-off from the evolution of the world's technological frontier. Historical examples include China's isolationism starting in the 15th century AD or Japan's isolationism ending in the mid 19th century AD. I show that closing an economy means that it falls behind forever because growth relies solely on domestic innovation efforts. The example of China is a case in point (see, e. g. Mokyr (1990), Chapter 9, or Landes (1998), p. 93-97). I find that the country characteristics that generate level effects in the open economy induce also growth effects in the closed economy. Hence, country characteristics determine whether closed economies converge to parallel growth paths or not.

Second, an open economy may be caught up in a *no-innovation trap* if country characteristics prevent profit-maximizing domestic firms from engaging in innovation investments. As a consequence, the second face of R&D is mute. The country does not absorb technological knowledge from abroad and converges to a stationary steady state as in Solow (1956). For such a setting, I show that a minimum level of human capital is necessary to induce innovation activity in equilibrium. This is consistent Benhabib and Spiegel (2005) who claim for a sample of 84 countries that a minimum level of human capital corresponding to an average 1.78 years of schooling in 1960 was necessary to catch up with US total factor productivity growth over the following 35 years.

The paper is organized as follows. I present the details of the model in Section 2. Section 3 studies the intertemporal general equilibrium and characterizes the dynamical system. Section 4 extends the basic model in three directions. First, I consider an endogenous savings rate generated by infinitely lived dynasties in Section 4.1. The closed economy and the implications for club convergence are analyzed in Section 4.2. Section 4.3 studies the possibility no-innovation traps. Section 5 concludes. All proves are relegated to the Appendix.

2 The Basic Model

The economy has a household sector, a final-good sector, and an intermediate-good sector in an infinite sequence of periods $t = 1, 2, ...$ There are four objects of exchange, a manufactured final good, a manufactured intermediate good, labor, and bonds. I call 'final good' a commodity that serves for consumption as well as for investment. If invested, this commodity is either used as future capital in the final-good sector or as an immediate input into innovation undertaken by firms of the intermediate-good sector.

In each period t, there are markets for all four objects of exchange. Treating the final good as the numéraire, p_t denotes the real price of the intermediate good, w_t the real hourly wage. A bond at t is a claim on one unit of the final good at $t+1$. Accordingly, the price of a bond at t is $1/(1 + r_{t+1})$, where r_{t+1} is the real interest rate from t to $t+1$.

2.1 The Household Sector

The household sector has an initial endowment of B_1 bonds coming due at $t = 1$ and owns the shares of all firms in the economy. In each period it is equipped with a labor endowment of L_t hours of time that coincides with the aggregate supply of labor. Due to population growth, this endowment grows at a constant rate $\lambda > (-1)$

such that $L_t = (1+\lambda)^{t-1}$ for $t \geq 1$ with $L_1 = 1$ given as an initial condition. Let $h \geq 1$ denote the level of human capital that augments each hour worked.

The allocation of per-period income to consumption and savings is subject to the budget constraint

$$C_t + \frac{B_{t+1}}{1+r_{t+1}} = w_t\, L_t + B_t + \Pi_t - T_t, \tag{2.1}$$

where C_t is consumption of the final good, B_{t+1} is bond demand in t, $w_t\, L_t$ is wage income, B_t is capital income from the repayment of bonds due in t, Π_t is the aggregate dividend distribution, and T_t denotes the lump-sum tax levied by the government to finance possible subsidies for innovation investments.

As to the consumption-savings decision of the household sector I assume that real aggregate savings in t is a fixed fraction of aggregate income in t, i. e.,[4]

$$\frac{B_{t+1}}{1+r_{t+1}} = s\left(w_t\, L_t + B_t + \Pi_t - T_t\right), \tag{2.2}$$

with $s \in (0,1)$ denoting the marginal and average propensity to save.

2.2 The Final-Good Sector

The final-good sector produces according to the production function $Y_t = \Gamma F\left(K_t, X_t\right)$, where $\Gamma > 0$ and F is a neoclassical production function with the usual properties (see, e. g., Barro and Sala-í-Martin (2004), pp. 26 - 28). Here, Y_t is aggregate output, Γ is meant to capture what Hall and Jones (1999) call social infrastructure, K_t is capital input in t, and X_t denotes the amount of the intermediate good used in period-t production. I assume F to be Cobb-Douglas, i. e.,

$$Y_t = \Gamma\, K_t^{\alpha}\, X_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{2.3}$$

Capital in t must be installed one period before its use in production and, without loss of generality, fully depreciates after being used. A capital investment of K_t units undertaken in period $t-1$ is financed by an issue of $(1+r_t)\, K_t$ bonds.

In terms of the final good of period t as numéraire the profit in t of the final-good sector is

$$Y_t - (1+r_t)\; K_t - p_t\, X_t, \tag{2.4}$$

[4]Similar findings are obtained when I represent the household sector by two-period lived overlapping generations with log utility. Since the savings hypothesis of (2.2) avoids expectations over a possibly infinite horizon to play a role it proves particularly plausible in the presence of growth stages to which I turn in Section 4.3. I study the case of an endogenous savings rate along the lines of Ramsey (1928), Cass (1965), and Koopmans (1965) in Section 4.1.

where $(1 + r_t)\ K_t$ is capital service payments and $p_t\, X_t$ is the cost of the intermediate-good input.

The final-good sector takes the sequence $\{p_t, r_t\}$ of prices and interest rates as given and maximizes the sum of the present discounted values of profits in all periods. Since it simply buys capital and intermediate goods for each period, its maximization problem is equivalent to a series of one-period maximization problems. Define the period-t capital intensity in the final good-sector as

$$k_t \equiv \frac{K_t}{X_t}. \tag{2.5}$$

Using $f(k_t) \equiv F(k_t, 1) = k_t^{\alpha}$ the respective first-order conditions for $t = 1, 2, ...$ are

$$K_t \quad : \quad \alpha\, \Gamma\, k_t^{\alpha-1} = 1 + r_t \tag{2.6}$$

$$X_t \quad : \quad (1-\alpha)\, \Gamma\, k_t^{\alpha} = p_t. \tag{2.7}$$

Initially, the final-good sector has K_1 units of capital at its disposal. It stems from investment decisions prior to period $t = 1$ and causes outstanding debt obligations equal to $(1 + r_1)\, K_1$.

2.3 The Intermediate-Good Sector

The set of all intermediate-good firms is represented by the set $\Re_+$ of nonnegative real numbers with Lebesgue measure.

2.3.1 Technology

At any date, t, all firms have access to the same technology with production function

$$x_t = \min\{1, a_t\, h\, l_t\}, \tag{2.8}$$

where x_t is output, 1 a capacity limit,[5] a_t the firm's labor productivity in period t, $h\, l_t$ human capital augmented labor input. The index $h \geq 1$ reflects the Becker-Mincer

[5] The analysis is easily generalized to allow for an endogenous capacity choice requiring prior capacity investments, with investment outlays a strictly convex function of capacity. In such a setting profit-maximizing behavior implies that a large innovation investment is accompanied by a large capacity investment (see, Hellwig and Irmen (2001) for details). Thus, the simpler specification treated here abstracts from effects on firm size in an environment with changing levels of innovation investments.

view that human capital increases the productivity of the labor hours employed by firms (Mincer (1974), Becker (1993)). The firm's labor productivity is equal to

$$a_t = A_{t-1}(1 + q_t); \tag{2.9}$$

here A_{t-1} is an economy-wide indicator of the level of technological knowledge accumulated up to period $t-1$, and q_t is an indicator of productivity growth at the firm.

To achieve a productivity growth rate $q_t > 0$ from period $t-1$ to period t, the firm must invest $i(q_t, h)$ units of the final good in period $t-1$. The function i is time invariant and satisfies for $h \geq 1$

$$i(0,h) = i_q(0,h) = 0, \quad i_q(q,h) > 0, \quad i, i_q \to \infty \text{ as } q \to \infty, \quad i_{qq} > 0 \text{ for } q \geq 0. \tag{2.10}$$

and for $q > 0$

$$i(q,h) > 0, \quad i_h(q,h) < 0, \quad i_{qh}(q,h) < 0. \tag{2.11}$$

Hence, a higher rate of productivity growth requires a larger investment, and more human capital enhances the effect of a given investment volume on productivity growth. The latter captures the idea enunciated by, e. g., Nelson and Phelps (1966), Schultz (1975), or Galor and Moav (2000) that human capital is favorable to change, for instance, because it speeds up the process of learning how to work with a new technology.[6] A functional form that fulfills these conditions is

$$i(q,h) = q^v\, h^{-z}, \quad \text{with} \quad 1 < v \leq 2 \quad \text{and} \quad z > 0. \tag{2.12}$$

It also complies with the following regularity condition that I impose on the convexity of i. For all $q > 0$ let

$$\left(\frac{i_{qq}}{i_q} - \frac{i_{qqq}}{i_{qq}}\right) > \frac{1}{1+q}. \tag{2.13}$$

If the firm innovates the assumption is that an innovation in period t is proprietary knowledge of the firm only in t, i. e., in the period when it materializes. Subsequently, the innovation becomes embodied in the economy-wide productivity indicators A_t, A_{t+1}, ..., with no further scope for proprietary exploitation. The evolution of these indicators will be specified below. If firms decide not to undertake an innovation investment in period $t-1$ then, for production in t, they have access to the production technique represented by A_{t-1} such that $a_t = A_{t-1}$. This will matter when we discuss no-innovation traps in Section 4.3.

[6]Empirical evidence supporting this idea provide, e. g., Welch (1970) and Bartel and Lichtenberg (1987).

2.3.2 Profit Maximization and Zero-Profits

To finance an innovation investment $i(q_t, h)$ the firm issues $(1 + r_t)\, i(q_t, h)$ bonds in period $t-1$. In period t the government grants a subsidy on such investment equal to $\sigma\, (1 + r_t)\, i(q_t, h)$, where $\sigma \in (0, 1)$ is the time-invariant subsidy rate. In terms of the final good of period t as numéraire, a production plan (q_t, l_t, x_t) for period t thus yields the profit

$$\pi_t = p_t\, x_t - w_t\, l_t - (1 + r_t)\, (1 - \sigma)\, i(q_t, h), \tag{2.14}$$

where $p_t\, x_t = p_t \min\{1,\ A_{t-1}(1 + q_t)\, h\, l_t\}$ is the firm's revenue from output sales, $w_t\, l_t$ its wage bill at the real wage rate w_t, and $(1+r_t)\, (1-\sigma)\, i(q_t, h)$ its debt service net of subsidies.

Competitive firms take the sequence $\{p_t, w_t, r_t\}$ of real prices, the sequence $\{A_t\}$ of aggregate productivity indicators, the subsidy rate, σ, and the level of human capital, h, as given and choose their production plan so as to maximize the sum of the present discounted values of profits in all periods. Because production choices for different periods are independent of each other, for each period t, they choose the plan (q_t, l_t, x_t) to maximize the profit π_t from this plan in period t.

If the firm innovates, it incurs an investment cost $(1 + r_t)\, (1 - \sigma)\, i(q_t, h)$ that is associated with a given innovation rate $q_t > 0$ and is independent of the output x_t. This introduces a positive scale effect, namely if the firm innovates, then it wants to apply the innovation to as large an output as possible and produces at the capacity limit $x_t = 1$. The choice of (q_t, l_t) must then minimize the costs of producing the capacity output.

Suppose $w_t > 0$ and $r_t > (-1)$, then an input combination (q_t, l_t) that minimizes unit costs must satisfy

$$l_t = \frac{1}{A_{t-1}(1 + q_t)\, h}, \tag{2.15}$$

and

$$q_t \in \arg\min_{q \geq 0} \left[\frac{w_t}{A_{t-1}(1 + q)\, h} + (1 + r_t)\, (1 - \sigma)\, i(q, h) \right]. \tag{2.16}$$

Given the convexity of the innovation cost function and the fact that $i_q(0, h) = 0$, (2.16) determines a unique level $q_t^* > 0$ as the solution to the first-order condition

$$\frac{w_t}{A_{t-1}(1 + q_t^*)^2\, h} = (1 + r_t)\, (1 - \sigma)\, i_q(q_t^*, h). \tag{2.17}$$

The latter relates the marginal reduction of the firm's wage bill to the marginal increase in its investment costs. As both marginal effects are proportional to the respective factor price, condition (2.17) implies a map q that assigns to each triple of $w_t/A_{t-1}\,h\,(1+r_t) \geq 0$, $h \geq 1$, and $\sigma \in (0,1)$ the cost-minimizing growth rate of labor productivity

$$q_t^* = q\left(\frac{w_t}{A_{t-1}\,h\,(1+r_t)}, h, \sigma\right). \tag{2.18}$$

Given A_{t-1} the chosen growth rate of labor productivity increases in the relative factor price ratio, and the properties of the input requirement function i imply that $q\,(0,h,\sigma) = 0$ and $q\,(\infty,h,\sigma) = \infty$. Moreover, q_t^* increases in the subsidy rate whereas the effect of an increase in the level of human capital has an ambiguous effect. Indeed, one readily verifies that $dq_t^*/dh \gtreqless 0 \quad \Leftrightarrow \quad -\left(\partial i_q/\partial h\right) h/i_q \gtreqless 1$. This condition reflects the countervailing effect of the Becker-Mincer versus the Nelson-Phelps logic on innovation incentives. As a result, we find that $dq_t^*/dh > 0$ obtains only if the impact of h on the reduction of the marginal investment requirement is stronger than the disincentive through the labor-augmenting effect of human capital. This is the case if the elasticity of the marginal investment requirement with respect to human capital at q^* is sufficiently strong. This intuition is confirmed for the specification of i given in (2.12), where $dq_t^*/dh > 0$ holds if and only if $z > 1$.

2.4 Consolidating the Production Sector

Turning to implications for the general equilibrium, recall that the set of intermediate-good firms is $\Re_+$ with Lebesgue measure. Therefore, maximum profits that producing intermediate-good firms attain in equilibrium for any t must be zero. Indeed, since the labor supply in each period is bounded, the set of intermediate-good firms employing more than some $\varepsilon > 0$ units of labor must have bounded measure and hence must be smaller than the set of all intermediate-good firms. Given that inactive intermediate-good firms must be maximizing profits just like the active ones, we need that maximum profits of intermediate-good firms at equilibrium prices are equal to zero, i. e.,

$$\pi_t = \pi\left(q_t^*; p_t, w_t, r_t, A_{t-1}, h, \sigma\right) = 0. \tag{2.19}$$

Since all intermediate-good firms face the same input and output prices, they all choose the same growth rate of labor productivity, q^*. Moreover, the following lemma establishes that the conditions for profit-maximization and zero-profits in the final-good and the intermediate-good sector relate this rate of productivity growth to the capital intensity in the final-good sector, k, to the level of human capital, h, and to the subsidy rate, σ, according to a well-behaved function $g\,(k_t, h, \sigma)$.

Lemma 1 *If (2.6), (2.7), (2.17) and (2.19) hold for all firms in t, then there is a map g such that for $k_t \geq 0$ and $h \geq 1$, $q_t^* = g(k_t, h, \sigma)$, with $g(0, \cdot, \cdot) = 0$, $g(\infty, \cdot, \cdot) = \infty$,*

$$g_k > 0, \quad g_h > 0, \quad \textit{and} \quad g_\sigma > 0. \tag{2.20}$$

The fact that $g_k > 0$ can be traced back to the properties of the neoclassical production function of the final-good sector. They imply that the marginal productivity of capital falls in k_t whereas the marginal productivity of the intermediate good rises. Accordingly, r_t falls and p_t rises in k_t. Through the zero-profit condition, these price movements feed back onto the wage, w_t, which must also rise. As a result, a higher k_t increases the relative wage in (2.18) and, therewith, the incentives that foster labor productivity growth. Moreover, the function g captures the effect of human capital in a changing environment, i. e., as human capital reduces total and marginal investment outlays, we find $g_h > 0$. Similarly, we obtain $g_\sigma > 0$ since a subsidy rate reduces marginal investment outlays.

2.5 Evolution of Technological Knowledge

As I set out in the Introduction, the evolution of the economy's level of technological knowledge comprises a *domestic* and a *foreign* contribution. These channels correspond to the two faces of R&D that Griffith, Redding, and Reenen (2004) identify empirically.

The domestic contribution at $t-1$ reflects productivity growth achieved at the level of those domestic intermediate good firms that produce at t. Denoting the measure of these firms by n_t, their contribution is equal to the highest level of labor productivity attained by one of them, i. e.,

$$\max\{a_t(n) = A_{t-1}\,(1 + q_t^*(n))\,|\,n \in [0, n_t]\}.$$

Since in equilibrium $q_t^*(n) = q_t^*$, the domestic contribution boils down to

$$a_t = A_{t-1}\,(1 + q_t^*). \tag{2.21}$$

The foreign contribution is an inflow of currently unavailable technological knowledge from abroad. It begs the notions of the *world's technological frontier* and of a *laggard country*. Let A_t^{max} denote the world's leading-edge productivity indicator at t which grows at the constant rate $\gamma > 0$, i. e., $A_t^{max} = (1 + \gamma)\,A_{t-1}^{max}$ with $A_0^{max} > 0$ as an initial condition. A country is called a laggard at t if $A_t^{max} > A_t$.

The strength of the foreign contribution at $t-1$ depends positively on three factors. First, it relies on the average investment activity of intermediate-good firms between $t-1$ and t, $i(q_t^*, h)$. Second, it hinges on the technological backwardness of the laggard country measured by the gap $A_{t-1}^{max} - A_{t-1}$.[7] Third, the country's openness to the rest of the world matters. The parameter θ is meant to capture institutional or technological factors that facilitate the inflow and implementation of new knowledge. It may be associated with the presence of restrictions on foreign trade or migration, to country-specific barriers to technology adoption as emphasized by Parente and Prescott (1994), to patent protection of new foreign technologies or to their appropriateness in the sense of, e. g., Atkinson and Stiglitz (1969) and Basu and Weil (1998). The economy is said to be *open* if $\theta > 0$ and *closed* if $\theta = 0$.

For simplicity, I stipulate the foreign contribution as the product of these three factors, i. e.,

$$\theta\, i(q_t^*, h)(A_{t-1}^{max} - A_{t-1}). \tag{2.22}$$

Thus, the second face of R&D measured by $i(q_t^*, h)$ determines the rate of diffusion. This specification provides a possible micro-foundation for the assumption introduced by Nelson and Phelps (1966) that the ability of a laggard country to close the technological gap depends positively on the average level of human capital in its population. Here, however, the link between the level of human capital and the strength of the inflow is endogenous.

Adding (2.21) to (2.22), we obtain the updating condition for the level of technological knowledge to which innovating domestic intermediate-good firms have access at t,

$$A_t = A_{t-1}(1 + q_t^*) + \theta\, i(q_t^*, h)(A_{t-1}^{max} - A_{t-1}). \tag{2.23}$$

This condition is a discrete time analogue of the confined exponential diffusion process studied in Benhabib and Spiegel (2005). To see this more clearly, consider the growth rate of A[8]

$$\frac{A_t - A_{t-1}}{A_{t-1}} = q_t^* + \theta\, i(q_t^*, h)\left(\frac{A_{t-1}^{max}}{A_{t-1}} - 1\right). \tag{2.24}$$

[7]The representation of technological knowledge by the real line reduces a complicated, multifaceted object to a one-dimensional entity. Therefore, one may argue that any domestic innovation investment of a laggard country creates knowledge that already exists. Then, it is not $A_{t-1}^{max} - A_{t-1}$ that matters as a component of the foreign contribution but rather the gap net of duplication $A_{t-1}^{max} - A_{t-1}(1 + q_t^*)$. It turns out that duplication introduced in this way adds a complication to the picture that does not affect most of my results. Details for this case are available upon request.

[8]This may be compared to equation 2.1 in Benhabib and Spiegel (2005) where the functions corresponding to q_t^* and $\theta\, i(q_t^*, h)$ are assumed to increase in a country's level of education and are not linked to microeconomic magnitudes.

According to Lemma 1, both components of this growth rate are determined in equilibrium. They will directly depend on the level of human capital and the subsidy rate, and indirectly on the variables that determine the equilibrium capital intensity in the final-good sector.

Observe that (2.24) can be linked to the idea that economic backwardness facilitates convergence (see, e. g., Gerschenkron (1962) and Abramovitz (1986)). Indeed, ceteris paribus, the growth rate of A increases in the gap A^{max}/A. A backward country may therefore experience what Gerschenkron called "spurts", i. e., periods of exceptional growth rates that even exceed γ.

For further reference we note that the updating condition (2.23) can be expressed in terms of the laggard country's relative position with respect to the leading-edge technology $\Delta_t \equiv A_t/A_t^{max}$. Indeed, one readily verifies the implication that

$$\Delta_t = \frac{\theta\, i(q_t^*, h)}{1+\gamma} + \frac{1 + q_t^* - \theta\, i(q_t^*, h)}{1+\gamma}\, \Delta_{t-1}. \tag{2.25}$$

3 Intertemporal General Equilibrium

I focus on a laggard country that remains throughout its evolution behind the leading-edge technology.

3.1 Definition

I refer to a sequence $\{p_t, w_t, r_t\}$ as a *price system.* By an *allocation* I understand a sequence $\{C_t, L_t, B_t, Y_t, K_t, X_t, n_t, q_t, l_t, T_t\}$ that comprises a strategy $\{C_t, L_t, B_t\}$ for the household sector, a strategy $\{Y_t, K_t, X_t\}$ for the final-good sector, a measure n_t of intermediate-good firms active at t producing the capacity output $x_t = 1$ with input choices (q_t, l_t), and the government's lump-sum tax, T_t.

An *equilibrium* will correspond to a price system, an allocation, and a sequence $\{\Pi_t, A_t, A_t^{max}, \Delta_t\}$ of distributed aggregate profits, indicators for the domestic level of technological knowledge, for the leading-edge, and the ensuing relative position Δ_t that satisfy the following conditions: First, given the initial bond endowment B_1 and the sequence $\{w_t, r_t, \Pi_t\}$, the household sector saves according to (2.2) and supplies L_t units of labor in all periods. Second, the production sector satisfies the assumptions underlying Lemma 1. Due to constant returns to scale in final-good production, $\Pi_t = 0$ in all periods. Third, in all periods markets clear. Forth, the domestic productivity indicator A_t evolves according to (2.23) and A_t^{max} grows at rate $\gamma > 0$. Fifth, the government balances its budget, i. e., $T_t = n_t \, \sigma \, (1 + r_t) \, i(q_t^*, h)$ for all t.

In specifying a consistent circular flow of income, one readily verifies that in equilibrium $w_t \, L_t + B_t + \Pi_t - T_t = Y_t$, i. e., for all periods the household sector's income stream is equal to final-good production. Accordingly, the equilibrium condition requiring savings to equal investment is

$$K_{t+1} + n_{t+1} i(q_{t+1}, h) = s \, \Gamma \, K_t^{\alpha} X_t^{1-\alpha} \text{ for } t = 1, 2, ... \tag{3.1}$$

3.2 The Dynamical System

I choose the capital intensity in the final-good sector, $k_t \equiv K_t / X_t$, and the relative position of the domestic technology, $\Delta_t \equiv A_t / A_t^{max}$, as the state variables of the dynamical system. To express (3.1) in terms of k and Δ, note first that the equilibrium in the market for intermediates and full employment in all periods imply

$$X_t = n_t = A_{t-1}(1 + q_t^*) \, h \, L_t, \tag{3.2}$$

i. e., aggregate output of the intermediate-good is equal to labor in efficiency units. Then, X_{t+1}/X_t is the growth factor of efficient labor. Using the updating condition (2.23), one finds

$$\begin{aligned} \frac{X_{t+1}}{X_t} &= \frac{A_t}{A_{t-1}} \frac{1 + q_{t+1}^*}{1 + q_t^*} \frac{L_{t+1}}{L_t} \\ &= (1 + q_{t+1}^*) \left(1 + \theta \, \frac{i(q_t^*, h)}{1 + q_t^*} \left(\frac{1}{\Delta_{t-1}} - 1 \right) \right) (1 + \lambda). \end{aligned} \tag{3.3}$$

Hence, for an open economy with $\theta > 0$ both the domestic and the foreign contribution matter for the growth of efficient labor.

From (3.1), the first equality in (3.2), (3.3), and Lemma 1, we find the equation of motion for k_t. Rearranging terms that depend on k_t or k_{t+1} gives

$$(1 + g(k_{t+1}, h, \sigma)) \, (k_{t+1} + i(g(k_{t+1}, h, \sigma), h)) = \frac{\tilde{s} k_t^{\alpha}}{1 + \theta \, \frac{i(g(k_t, h, \sigma), h)}{1 + g(k_t, h, \sigma)} \left(\frac{1}{\Delta_{t-1}} - 1 \right)}. \tag{3.4}$$

where $\tilde{s} \equiv s \, \Gamma / (1 + \lambda)$.

The equation of motion for Δ_t obtains from (2.25) and Lemma 1,

$$\Delta_t = \frac{\theta \, i(g\,(k_t, h, \sigma)\,, h)}{1 + \gamma} + \frac{1 + g\,(k_t, h, \sigma) - \theta \, i(g\,(k_t, h, \sigma)\,, h)}{1 + \gamma} \, \Delta_{t-1}. \tag{3.5}$$

An application of the implicit function theorem to (3.4) shows that the latter two equations constitute a two-dimensional system of first-order, autonomous, non-linear difference equations. This system may be stated as

$$(k_{t+1}, \Delta_t) = \phi(k_t, \Delta_{t-1}) \equiv \left(\phi^k(k_t, \Delta_{t-1}), \phi^\Delta(k_t, \Delta_{t-1})\right) \tag{3.6}$$

for given initial values k_1 and Δ_0. To assure a trajectory of $\Delta_{t-1} \in (0,1)$ for $t = 1, 2, ...$ we have to impose constraints on the parameters of the model. The following lemma makes this more precise.

Lemma 2 *There is a unique $\bar{k} > 0$ such that $g(\bar{k}, h, \sigma) = \gamma$. Let $\bar{\theta} \equiv (1+\gamma)/i(\gamma, h)$. The function $\phi(k_t, \Delta_{t-1})$ maps $\left[0, \bar{k}\right] \times (0,1)$ onto itself if*

$$\theta < \bar{\theta} \quad \textit{and} \quad \tilde{s}^{\frac{1}{1-\alpha}} < \bar{k}. \tag{3.7}$$

Lemma 2 states conditions on parameters such that a country remains behind the world's technological frontier throughout its evolution. Intuitively, $\theta < \bar{\theta}$ imposes an upper bound on the rate of diffusion in the updating condition (2.23). If $k \leq \bar{k}$ then, in equilibrium, domestic innovation incentives are not too strong and $g \leq \gamma$. Moreover, $\tilde{s}^{\frac{1}{1-\alpha}} < \bar{k}$ assures that $\bar{k}$ is indeed an upper bound on the attainable level of k through the process of capital accumulation. if not indicated otherwise, I shall assume henceforth initial values $k_1 \in (0, \bar{k})$ and $\Delta_0 \in (0,1)$ and that the parameters of the model satisfy the restrictions stated in (3.7).

Proposition 1 *There is a unique steady state (k^*, Δ^*) with $k^* \in (0, \bar{k})$ and $\Delta^* \in (0,1)$ that satisfy*

$$k^* + i(g(k^*, h, \sigma), h) = \frac{\tilde{s}}{1+\gamma} (k^*)^\alpha, \tag{3.8}$$

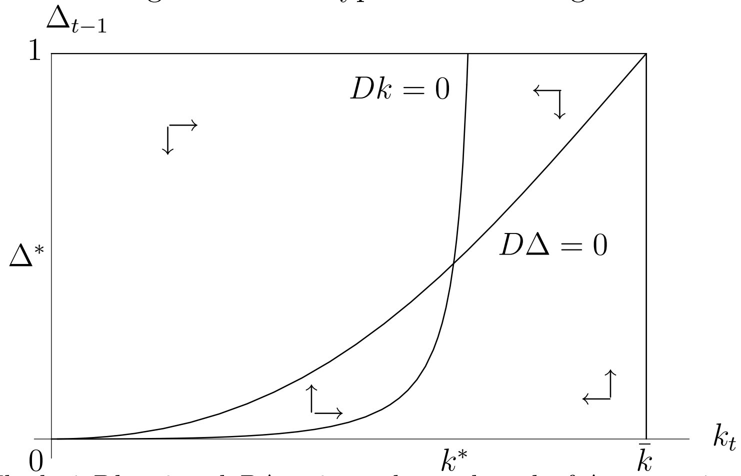
and

$$\Delta^* = \frac{\theta \, i(g(k^*, h, \sigma), h)}{\theta \, i(g(k^*, h, \sigma), h) + \gamma - g(k^*, h, \sigma)}. \tag{3.9}$$

Proposition 1 states and proves the existence of a unique steady state for a laggard country. Since the country's relative position with respect to the leading-edge technological knowledge, Δ^*, is constant, A_t grows at rate γ, which is also the growth rate of all domestic per-capita magnitudes such as income and consumption.

The equation for k^* is similar to the one of the neoclassical growth model with exogenous labor-augmenting technical change. The difference occurs on the left-hand

Figure 1: The Typical Phase Diagram.



The loci $Dk = 0$ and $D\Delta = 0$ are those where k of Δ are stationary.

side of (3.8), where the resources necessary to feed domestic innovation investments are added. With two investment opportunities the role of decreasing returns in the process of capital accumulation is more pronounced. As a consequence, the level of k^* is lower than in a Solow economy with costless exogenous technical change.

The analysis of the local and the global dynamics of the dynamical system is algebraically involved. Figure 1 shows some qualitative features in a typical phase diagram. Numerical results suggest that the steady state can be locally stable and a global attractor (see, Appendix 7 for details).

Since the steady-state growth rate is exogenous, comparative statics induce level effects. To develop an understanding for why steady-state per-capita income differs across countries we study first the effect of parameter changes on k^*.

Corollary 1 *It holds that*

$$\frac{dk^*}{d\tilde{s}} > 0, \quad \frac{dk^*}{d\gamma} < 0, \quad \frac{dk^*}{d\sigma} < 0. \tag{3.10}$$

Moreover,

$$\frac{dk^*}{dh} \gtreqless 0 \quad \Leftrightarrow \quad \left.\frac{di\,(g(k^*, h, \sigma), h)}{dh}\right|_{k=k^*} \lesseqgtr 0. \tag{3.11}$$

Similar to the neoclassical growth model with exogenous technical change, k^* increases both in s and Γ, i.e., in the investment rate and with a better social infrastructure. Moreover, k^* falls with the steady-state growth rate of labor λ, and with the growth rate of the leading-edge productivity indicator γ. These parameters directly affect the impact of diminishing returns on the accumulation of final-good sector capital.

Moreover, k^* falls in the subsidy rate. Intuitively, the subsidy rate increases $g(k^*, h, \sigma)$ and the equilibrium amount of innovation investments, $i(g(k^*, h, \sigma), h)$ increases, too. Accordingly, the level of k^* has to fall to reestablish the validity of condition (3.8).

The impact of human capital on k^* is indeterminate in general. This is the result of two opposing effects of h on the investment activity of intermediate-good firms. On the one hand, more human capital increases the incentive to engage in innovation investments, thus raising the productivity growth rate, g, and the investment requirements. On the other hand, given g, more human capital lowers investment requirements. While the former effect alone induces a lower level of k^*, the latter implies a higher level. I show in the proof of Corollary 1 that the indeterminacy vanishes if we impose more structure and assume an input requirement function i with constant elasticity like $i = q^v h^{-z}$ of (2.12). Then, the former effect dominates and $dk^*/dh < 0$.

Finally, observe that, k^* is independent of θ. The impact of the evolution of Δ on the evolution of k is a transitory phenomenon.

Next, I establish three results related to the steady-state rate of diffusion. The first questions the validity of the Nelson-Phelps hypothesis according to which this rate rises in human capital.

Proposition 2 *Denote*

$$k^i_{max} = \arg\max_k \frac{\tilde{s}}{1+\gamma} k^\alpha - k. \tag{3.12}$$

1) (Nelson-Phelps Hypothesis) The Nelson-Phelps hypothesis holds, i. e.,

$$\frac{di\left(g(k^*, h, \sigma), h\right)}{dh} > 0, \tag{3.13}$$

if and only if either

$$\left.\frac{di\left(g(k^*, h, \sigma), h\right)}{dh}\right|_{k=k^*} > 0 \quad \text{and} \quad k^* > k^i_{max} \tag{3.14}$$

or

$$\left.\frac{di\left(g(k^*, h, \sigma), h\right)}{dh}\right|_{k=k^*} < 0 \quad \text{and} \quad k^* < k^i_{max}. \tag{3.15}$$

2) (Growth Policy) It holds that

$$\frac{di\left(g(k^{*},h,\sigma),h\right)}{d\sigma} \gtreqless 0 \quad \Leftrightarrow \quad k^{*} \gtreqless k^{i}_{max}. \tag{3.16}$$

3) (Domestic Innovation versus Diffusion) Consider the steady-state growth rate of technological knowledge, $A_t/A_{t-1} - 1 = g + \theta\, i\left((\Delta^{*})^{-1} - 1\right)$*. Technology diffusion is the more important source of steady-state technological progress whenever*

$$g < \theta\, i\left((\Delta^{*})^{-1} - 1\right) \quad \Leftrightarrow \quad g < \frac{\gamma}{2}. \tag{3.17}$$

Statement 1 of Proposition 2 claims that the steady-state rate of diffusion may but need not rise with human capital. Thus, the Nelson-Phelps hypothesis fails in general. To gain an intuition for this result note that k^{i}_{max} is the level of the steady-state capital intensity that maximizes the steady-state rate of diffusion. Generically, the steady state consistent with (3.8) delivers a value $k^{*} \neq k^{i}_{max}$. For instance, if $k^{*} > k^{i}_{max}$ as suggested by (3.14), a higher h that also raises the steady-state rate of diffusion must reduce k^{*}. According to (3.11) of Corollary 1, such a general equilibrium effect occurs only if $di\left(g(k^{*},h,\sigma),h\right)/dh|_{k=k^{*}} > 0$. If the latter does not hold, k^{*} increases and, contrary to the Nelson-Phelps hypothesis, the steady-state rate of diffusion declines in h. With obvious changes, the same interpretation applies to the case shown in (3.15).

Statement 2 claims that a rise in the subsidy rate may lower the rate of diffusion. Intuitively, a higher subsidy increases the incentives to innovate. Hence, given k^{*} innovation investments increase. Then, however, condition (3.8) requires a smaller k^{*}. This general equilibrium effect increases (decreases) the steady-state rate of diffusion if $k^{*} > k^{i}_{max}$ ($k^{*} < k^{i}_{max}$).

Statement 3 gives the condition under which a country's share of steady-state productivity growth that stems from foreign innovations exceeds the share of productivity growth due to domestic innovations. According to the estimates of Eaton and Kortum (1996), all OECD countries but the US satisfy this condition. In view of Corollary 1, it is straightforward to see that countries with a higher savings rate, a better social infrastructure, and a lower population growth rate have a higher share of steady-state productivity growth that derives from domestic innovations. The presence of partial and general equilibrium effects, possibly of opposite sign, render the comparative static prediction about the innovation subsidy and human capital more involved. However, one readily verifies that $k^{*} > k^{i}_{max}$ is sufficient for both, σ and h, to have a positive effect on g.

Next, I turn to the country characteristics that determine Δ^{*} of (3.9).

Corollary 2 *It holds that*

$$\frac{d\Delta^*}{d\tilde{s}} > 0, \quad \frac{d\Delta^*}{d\gamma} < 0, \quad \frac{d\Delta^*}{d\sigma} \gtreqless 0 \quad \Leftrightarrow \quad k^* \gtreqless k^i_{max},$$
$$\frac{d\Delta^*}{dh} \gtreqless 0, \quad \textit{and} \quad \frac{d\Delta^*}{dh} > 0 \quad \textit{if (3.14) holds,} \quad \frac{d\Delta^*}{d\theta} > 0. \tag{3.18}$$

The intuition for these results is straightforward. A higher k^* increases the growth rate of labor productivity, g, as well as the investment outlays, i. Therefore, Δ^* is higher the higher k^*. Then, from Corollary 1 a larger investment rate, a better social infrastructure, a lower population growth rate, and a slower pace of the technological frontier increase Δ^*. Again, because of partial and general equilibrium effects, the impact of a subsidy and of human capital is in general ambiguous. Finally, a country with better access to the world's technological frontier ends up closer to it, i. e., a higher θ implies a higher Δ^*.

From the final-good production function (2.3), Lemma 1, the market-clearing condition (3.2), the definition of Δ, and assuming that each worker has one unit of labor per period we find per-capita income in the steady state as

$$\tilde{y}_t^* \equiv \left(\frac{Y_t}{L_t}\right)^* = \Gamma\,(k^*)^\alpha\; A_{t-1}^{max}\,\Delta^*\,(1+g(k^*,h,\sigma))\,h. \tag{3.19}$$

Roughly, $\tilde{y}_t^*$ consists of three components. First, $\Gamma\,(k^*)^\alpha$, reflects the economy's overall efficiency and the final-good production function. The second component, $A_{t-1}^{max}\,\Delta^*\,(1+g(k^*,h,\sigma))$ represents technical change. Third, there is the Becker-Mincer effect of human capital.

The presence of A_{t-1}^{max} assures growth of $\tilde{y}_t^*$ at rate γ. The level of Δ^* determines the fraction of the leading-edge knowledge at $t-1$ that the country is able to absorb within this period. The presence of the growth factor of domestic labor productivity recalls the fact that intermediate-good firms investing in $t-1$ can build on the level of knowledge $A_{t-1} = A_{t-1}^{max}\,\Delta^*$ and that the achieved level of labor productivity at t is $A_{t-1}\,(1+g(k^*,h,\sigma))$. Thus, a country's domestic innovation effort does not determine its steady-state growth rate but exerts a positive level effect on its steady-state per-capita income. This is the key difference between the steady-state predictions of the present model and the neoclassical growth model with exogenous technical change.[9]

[9] To see this more clearly, replace the intermediate-good sector by the assumption of exogenous technical change at rate γ, the final-good production function (2.3) by $Y_t = \Gamma\,K_t^\alpha\,(A_{t-1}(1+\gamma)L_t)^{1-\alpha}$, and set $\theta = 0$. Then, the neoclassical equivalent to (3.19) is $\tilde{y}_t^* = \Gamma\,(k^*)^\alpha\; A_{t-1}(1+\gamma)\,h$.

To establish the implications of (3.19) for cross-country income differences we first note that $\tilde{y}^*$ increases in k^* since final-good output, Δ^*, and g increase in k^*. In view of Corollaries 1 and 2, a prediction is then that countries with a higher investment rate, a better social infrastructure, and a lower population growth rate have a higher steady-state per-capita income. Neither the impact of the subsidy nor of human capital is clear cut. Both magnitudes increase g, however, from Corollaries 1 and 2, we know that the effect on k^* and Δ^* may be negative or positive. Quite intuitively, an economy that is more open than others is predicted to have a higher per-capita income since they are able to absorb a larger fraction of the leading-edge technology.

These results are summarized in the following proposition.

Proposition 3 *It holds that*

$$\frac{d\tilde{y}^*}{d\tilde{s}} > 0, \quad \frac{d\tilde{y}^*}{d\sigma} \gtreqless 0, \quad \frac{d\tilde{y}^*}{dh} \gtreqless 0, \quad \frac{d\tilde{y}^*}{d\theta} > 0. \tag{3.20}$$

4 Extensions and Discussion

4.1 Saving à la Ramsey-Cass-Koopmans

Consider a closed economy comprising many identical and infinitely lived households. I normalize the number of households to unity such that individual and aggregate variables coincide. In each period households supply the same amount of labor, $(1+\lambda)^{t-1}$, inelastically to the labor market, and, initially, own the same amount of bonds coming due in $t = 1$.

Households choose the sequence of consumption and bond holdings per household member $\{\tilde{c}_t, \tilde{b}_{t+1}\}_{t=1}^{t=\infty}$ that solves

$$\max_{\{\tilde{c}_t, \tilde{b}_{t+1}\}_{t=1}^{t=\infty}} \quad \sum_{t=1}^{\infty} \beta^{t-1} \frac{\tilde{c}_t^{1-\eta} - 1}{1-\eta} (1+\lambda)^{t-1}, \quad 0 < \beta(1+\lambda) < 1, \eta > 0, \tag{4.1}$$

subject to the budget constraint (2.1) and a Ponzi condition, which requires the present value of a household's bond holdings to be asymptotically non-negative.[10] As usual, β is the discount factor, and η the elasticity of marginal utility of consumption. With $c_t \equiv C_t/X_t$ and the market-clearing condition (3.2), we obtain the Euler condition for all $t = 1, 2, ...$

$$c_{t+1} A_t (1 + g(k_{t+1}, h, \sigma)) \quad = \quad [\beta(1 + r_{t+1})]^{\frac{1}{\eta}} c_t A_{t-1} (1 + g(k_t, h, \sigma)). \tag{4.2}$$

[10]See the Appendix 6.8.2 for details concerning the household's optimization problem.

Similarly, with $b_{t+1} \equiv B_{t+1}/X_{t+1}$ and (3.2) the transversality condition is

$$\lim_{t\to\infty} b_{t+1}\, A_t\, (1 + g(k_{t+1}, h, \sigma)) \left(\frac{1+\lambda}{1+\bar{r}}\right)^t = 0, \tag{4.3}$$

where $\bar{r} \equiv \left(\prod_{j=1}^{j=t}(1+r_{j+1})\right)^{1/t} - 1$ is the average real interest rate.

Proposition 4 *There is $\gamma > 0$ such that a unique balanced growth path for a laggard country exists. It involves*

$$k^*_{RCK} = \left[\frac{\alpha\,\beta\,\Gamma}{(1+\gamma)^\eta}\right]^{\frac{1}{1-\alpha}} \tag{4.4}$$

and, in view of (3.9), $\Delta^*_{RCK} \equiv \Delta^*\left(k^*_{RCK}\right) < 1$.

Proposition 4 establishes the existence of a steady-state equilibrium for a laggard economy. This requires $g\left(k^*_{RCK}, h, \sigma\right) < \gamma$, i. e., k^*_{RCK} must not be too large. The proof shows this to be the case if γ is sufficiently large. As for a constant savings rate, all per-capita magnitudes grow at the exogenous rate γ.

To understand the implications for the predicted differences in per-capita income I first note that the effect of preference and technology parameters on k^*_{RCK} is as in the neoclassical growth model with exogenous technical change: a higher valuation of future utility and an increased willingness to accept deviations from a smooth consumption profile, i. e., a higher β or a lower η, a better infrastructure, i. e., a higher Γ increase k^*_{RCK}, and faster growth of the technological frontier accentuates the role of diminishing returns and leads to a lower k^*_{RCK}.

Observe that neither the growth rate of the labor force, λ, nor human capital, h, or the subsidy, σ, affect k^*_{RCK}. This reflects the fact that here consumption growth is pegged to intertemporal prices rather than the result of a market equilibrium condition. This has direct implications both for the validity of the Nelson-Phelps hypothesis.

Proposition 5 *Consider the steady state characterized in Proposition 4.*

1) (Nelson-Phelps Hypothesis) The Nelson-Phelps hypothesis holds if

$$\frac{di\left(g(k^*_{RCK}, h, \sigma), h\right)}{dh} > 0. \tag{4.5}$$

2) (Growth Policy) It holds that

$$\frac{di\left(g(k^*_{RCK}, h, \sigma), h\right)}{d\sigma} > 0. \tag{4.6}$$

3) (Domestic Innovation versus Diffusion) Technology diffusion is the more important source of steady-state technological progress, i. e., $g < \gamma/2$, in countries with a small h and/or a small σ.

Absent of general equilibrium effects, the rate of diffusion increases if i increases in h or for a higher subsidy. Similarly, the condition $g < \gamma/2$ is easier satisfied the smaller h and/or σ. Next, we turn to the comparative statics of Δ^*_{RCK}.

Corollary 3 *It holds that*

$$\frac{d\Delta^*_{RCK}}{d\beta} > 0, \quad \frac{d\Delta^*_{RCK}}{d\eta} < 0, \quad \frac{d\Delta_{RCK}}{d\Gamma} > 0, \quad \frac{d\Delta^*_{RCK}}{d\gamma} < 0, \quad \frac{d\Delta^*_{RCK}}{d\sigma} > 0, \quad (4.7)$$

$$\frac{d\Delta^*_{RCK}}{dh} > 0 \textit{ if (4.5) holds}, \quad \frac{d\Delta^*_{RCK}}{d\theta} > 0. \quad (4.8)$$

Compared to Corollary 2 the elimination of general equilibrium effects gives rise to three differences. First, the impact of a subsidy is unequivocal. An increase in σ raises Δ^*_{RCK} since domestic innovation incentives become more pronounced and, as a consequence, the rate of diffusion rises. Second, human capital raises Δ^*_{RCK} if it raises the rate of diffusion such that the Nelson-Phelps hypothesis holds at k^*_{RCK}. Moreover, Δ^*_{RCK} becomes independent of population growth. These findings have implications for the steady-state level of per-capita income, which is still given by (3.19).

Proposition 6 *It holds that*

$$\frac{d\tilde{y}^*_{RCK}}{d\beta} > 0, \quad \frac{d\tilde{y}^*_{RCK}}{d\eta} < 0, \quad \frac{d\tilde{y}^*_{RCK}}{d\Gamma} > 0, \quad \frac{d\tilde{y}^*_{RCK}}{d\gamma} < 0, \quad (4.9)$$

$$\frac{d\tilde{y}^*_{RCK}}{d\sigma} > 0, \quad \frac{d\tilde{y}^*_{RCK}}{dh} > 0, \textit{ if (4.5) holds}, \quad \frac{d\tilde{y}^*_{RCK}}{d\theta} > 0. \quad (4.10)$$

The comparison with Proposition 3 reveals that the effect of σ and h is positive if these parameters have a positive impact on Δ^*_{RCK}. Moreover, the level of steady-state per-capita income is independent of population growth.

4.2 The Closed Economy and Club Convergence

Consider a laggard economy as described in Sections 2 and 3 that is cut off from the evolution of the world's technological frontier. Then, $\theta = 0$ and the evolution of k is independent of Δ. The equations of motion for these variables, (3.4) and (3.5), become

$$(1 + g(k_{t+1}, h, \sigma))\,(k_{t+1} + i(g(k_{t+1}, h, \sigma), h)) = \tilde{s}k_t^{\alpha}, \tag{4.11}$$

and

$$\Delta_{c,t} = \frac{1 + g\,(k_t, h, \sigma)}{1 + \gamma}\,\Delta_{c,t-1}. \tag{4.12}$$

To simplify I assume that the input requirement function i satisfies

$$i_{qq} \leq \frac{2\,i_q^2}{i} \quad \text{for all } q > 0\ . \tag{4.13}$$

Again, a functional form that fulfills this condition is $i = q^v\,h^{-z}$, $1 < v \leq 2$.

Proposition 7 *Let* $(\tilde{s})^{1/(1-\alpha)} < \bar{k}$ *and assume that (4.13) holds. For any initial value* $k_1 \in (0, \bar{k})$*, the evolution of* k_t *according to (4.11) gives rise to a unique, globally stable steady state,* $k_c^* > 0$*, that solves*

$$(1 + g(k_c^*, h, \sigma))\,(k_c^* + i(g(k_c^*, h, \sigma), h)) = \tilde{s}(k_c^*)^{\alpha}. \tag{4.14}$$

The steady state satisfies

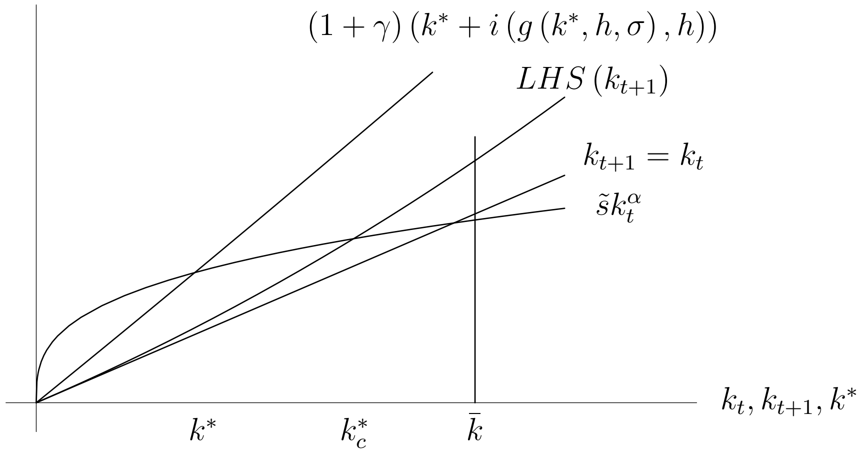
$$g(k^*, h, \sigma) < g(k_c^*, h, \sigma) < \gamma \tag{4.15}$$

and

$$\left(\frac{\Delta_{c,t}}{\Delta_{c,t-1}}\right)^* = \frac{1 + g\,(k_c^*, h, \sigma)}{1 + \gamma} < 1. \tag{4.16}$$

The intuition behind Proposition 7 can be learned from Figure 2, which depicts the right-hand side, $\tilde{s}(k_t)^{\alpha}$, and the left-hand side, $LHS(k_{t+1})$, of (4.11). Condition (4.13) assures that the left-hand side is a convex function in k_{t+1}. Thus, there is a unique and globally stable steady state, $k_c^* > 0$. The steady-state growth rate of all per-capita magnitudes must be smaller than γ since $k_c^* < \bar{k}$. As a consequence, $\Delta_{c,t}$ declines at a constant rate and the distance to the technological frontier becomes larger over time. The latter result obtains in spite of the fact that the domestic steady-state innovation activity in the closed economy is greater than in the open

Figure 2: The Evolution of k in the Closed Economy.



Here, $LHS(k_{t+1}) \equiv (1 + g(k_{t+1}, h, \sigma))(k_{t+1} + i(g(k_{t+1}, h, \sigma), h))$ of equation (4.11).

economy. To see why, I multiply the steady-state condition (3.8) by $1+\gamma$ and show the left-hand side of the resulting equation in Figure 2. Since $\gamma > g$, we have $k^* < k_c^*$.

Steady-state per-capita income in the closed economy is $\tilde{y}_{c,t}^* = \Gamma (k_c^*)^{\alpha} A_{t-1} (1 + g(k_c^*, h, \sigma))\, h$. It grows at rate $g(k_c^*, h, \sigma)$ such that changes in country characteristics generate level *and* growth effects. Nevertheless, an implication of the global stability is that a country starting at k^* converges to k_c^* following a cut-off from the technological frontier. As a consequence, the growth rate of per-capita income declines below γ and the country falls behind forever. This mechanism suggests that China's self-imposed isolationism in the 15th century AD is a cause for the subsequent relative decline of its economy.

Corollary 4 *Consider a steady state of Proposition 7. It holds that*

$$\frac{dk_c^*}{d\tilde{s}} > 0, \quad \frac{dk_c^*}{d\sigma} < 0 \tag{4.17}$$

$$\frac{dk_c^*}{dh} \gtreqless 0 \quad \Leftrightarrow -g_h\,(k_c^* + i) - (1+g)\, \left.\frac{di\,(g(k_c^*, h, \sigma), h)}{dh}\right|_{k=k_c^*} \gtreqless 0. \tag{4.18}$$

The qualitative predictions of Corollary 3 mimic those for the open economy (see, Corollary 1). The effects of s, Γ, λ, and σ are of the same sign. The effect of h is indeterminate in general. However, due to the direct effect of h on g, which appears in (4.18), it is more likely to be negative. Intuitively, a rise in g increases next period's amount of efficient labor and, therefore, the amount of final-good capital necessary to keep k constant. Hence, capital accumulation grinds to a halt at a lower level of k_c^*. Denote $k_c^* \equiv k_c^*(\tilde{s}, \sigma, h)$ the function resulting from Corollary 3.

Proposition 8 *The steady-state growth rate of the closed economy is*

$$q_c^* = g\left(k_c^*\left(\tilde{s}, h, \sigma\right), h, \sigma\right) \tag{4.19}$$

with

$$\frac{dq_c^*}{d\tilde{s}} > 0, \quad \left.\frac{dq_c^*}{dh}\right|_{\sigma=0} > 0, \quad \left.\frac{dq_c^*}{d\sigma}\right|_{\sigma=0} > 0. \tag{4.20}$$

Hence, steady-state growth rates differ across closed economies and reflect country characteristics. A higher investment rate, a better social infrastructure, and a lower population growth rate raise the steady-state growth rate of the economy. The effects of the subsidy rate and of human capital involve partial and general equilibrium effects of opposite sign. Low values of the subsidy rate weaken the general equilibrium effect such that both h and σ raise the steady-state growth rate of the closed economy. Since over time level effects are dominated by growth effects, these comparative statics also determine $\tilde{y}_{c,t}^*$. In a world with closed and open economies, club convergence in growth rates occurs among open economies that eventually grow at rate γ and groups of closed economies with country characteristics such that q_c^* is the same.

4.3 No-Innovation Traps and Club Convergence

In many countries profit-maximizing agents do not undertake innovation investments. When technology transfer is a byproduct of domestic innovation activities, these open economies do not benefit from foreign innovations. Club convergence results with some countries approaching a stationary steady state.[11]

Unprofitability of innovation investments arises if investment requirements are too high. Suppose that $i_q(0, h) > 0$, i.e., the first marginal unit of q is no longer costless. Without loss of generality as to the upcoming qualitative results, we rely on the functional form of i as given in (2.12) with $v = 1$ such that $i_q(0, h) = h^{-z} > 0$.

[11] The analysis of this section complements and extends the analysis in Irmen (2005).

An immediate implication is that the equilibrium does not necessarily involve $q_t^* > 0$. To see this consider the first-order condition (2.17). Since $i_q(0,h) > 0$, there are parameter constellations such that

$$\frac{w_t}{A_{t-1}\,h} \leq (1+r_t)\,(1-\sigma)\,i_q(0,h), \tag{4.21}$$

and the cost-minimizing choice is $q_t^* = 0$. It follows that the consolidated production sector gives rise to a function $g(k,h,\sigma)$ that is piecewise defined. In view of Lemma 1 we have

$$g(k,h,\sigma) = \max\left\{0, \frac{1}{2}\left(\frac{1-\alpha}{\alpha}\,\frac{h^z}{1-\sigma}\,k - 1\right)\right\}. \tag{4.22}$$

Consequently, an equilibrium at $t+1$ involves $g > 0$ if and only if

$$k_{t+1} > h^{-z}\,(1-\sigma)\,\frac{\alpha}{1-\alpha} \equiv \hat{k}, \tag{4.23}$$

and $g = 0$ otherwise. Intuitively, if at t intermediate-good firms expect $k_{t+1} < \hat{k}$, then they expect an equilibrium factor price ratio $w_{t+1}/(1+r_{t+1})$ too small to justify an investment in labor-saving technical change. Without an investment at t, firms produce in $t+1$ with the technology of period t. Moreover, there is no technology transfer between period t and $t+1$ either, and the updating condition (2.23) simplifies to $A_{t+1} = A_t$. Then, the equations of motion (3.4) and (3.5) for k_t and Δ_t become

$$k_{t+1} = \tilde{s}\,k_t^{\alpha}, \tag{4.24}$$

and

$$\Delta_{t+1} = \frac{\Delta_t}{1+\gamma}. \tag{4.25}$$

Next, we determine the conditions under which intermediate-good firms innovate. From (4.23), an equilibrium at $t+1$ without innovation ceases to exist if and only if

$$k_{t+1} = \tilde{s}\,k_t^{\alpha} > \hat{k} \quad \Leftrightarrow \quad k_t > \left(\frac{\hat{k}}{\tilde{s}}\right)^{\frac{1}{\alpha}} \equiv \underline{\hat{k}}. \tag{4.26}$$

It is replaced by an equilibrium with innovation. Indeed, the equation of motion for k at t takes innovation investments into account and becomes

$$(1+g(k_{t+1},h,\sigma))\,(k_{t+1} + i(g(k_{t+1},h,\sigma),h)) = \tilde{s}\,k_t^{\alpha}. \tag{4.27}$$

The equilibrium with innovation exists and is unique since the left-hand side of (4.27) satisfies $LHS(\hat{k}) = k_{t+1}$ and $LHS'(k_{t+1}) > 0$.

The evolution of k between t and $t+1$ as given in (4.27) is not affected by an inflow of technological knowledge from abroad since such inflow requires previous domestic innovation investments. Only firms that innovate at $t+1$ benefit from the inflow of foreign knowledge. Thus, the evolution of k between periods $t+1$ and $t+2$ is again governed by equation (3.4).[12]

The following Proposition shows what country characteristics determine whether an evolution that is initially driven by capital accumulation alone leads to domestic innovation and an inflow of technological knowledge from abroad.

Proposition 9 *Let $k_1 < \underline{\hat{k}}$ such that the economy experiences at least one period without innovation investments.*

1. *If $\tilde{s}^{\frac{1}{1-\alpha}} \leq \hat{k}$, then the economy evolves without innovation and converges towards a stationary steady state with $k^* = \tilde{s}^{\frac{1}{1-\alpha}}$. At any time, the country's relative position with respect to the leading-edge technology declines at rate $\gamma/(1+\gamma)$.*
2. *If $\bar{k} > \tilde{s}^{\frac{1}{1-\alpha}} > \hat{k}$, then the economy reaches a level of k in finite time and switches into a regime with domestic innovation in the following period. The innovation regime has a unique steady state (k^*, Δ^*) given by (3.8) and (3.9).*

Proposition 9 emphasizes that economies starting out with the same initial conditions may evolve in quite different ways. Using Statement 2 and the definition of $\hat{k}$ we obtain the requirement for economies to reach the regime with domestic innovation investments and technology transfer as

$$\left(\frac{s}{1+\lambda}\Gamma\right)^{\frac{1}{1-\alpha}} > h^{-z}\left(1-\sigma\right)\frac{\alpha}{1-\alpha}. \tag{4.28}$$

This condition is more likely to be fulfilled the more thrifty the economy is, the lower its growth rate of the labor force, the better its social infrastructure, the higher its level of human capital, and the higher the subsidy rate for innovation investments.

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[12] The evolution of the economy may well involve cycles since the inflow of technological knowledge in $t+1$ necessarily reduces the capital intensity k_{t+2}. If this effect is sufficiently strong, firms in $t+1$ rationally expect $k_{t+2} \leq \hat{k}$ and no innovation investment occurs. However, if the condition stated in Proposition 9 is fulfilled, such economy must again reach a period with a regime switch as characterized above. Calibration exercises show that there are paths that converge from the stationary regime to the steady state of Proposition 1. The complete characterization of the dynamics involved is beyond the scope of this paper and left for future research.

Solving (4.28) for h gives a minimum requirement of human capital for innovation and catch-up with the technological frontier. This is consistent the empirical findings of Benhabib and Spiegel (2005).[13]

Hence, the world income distribution may exhibit club convergence with some countries trapped in a stationary steady state while others experience steady growth.

5 Concluding Remarks

Arguably, the differential evolution of productivity across countries is the main force behind cross-country income differences. To understand these income differences one must understand what causes productivity growth. I take the view that productivity growth is due to the growth of a country's level of accessible technological knowledge. In turn, growth of this knowledge is the result of the interaction between a domestic and a foreign contribution via technology transfer. I show that the magnitudes that affect this interaction also account for steady-state cross-country income differences.

The analysis suggests several routes for future research. First, one may want to generalize the diffusion process and separate institutional from technological factors that foster technology diffusion. To accomplish this, I rely on a variant of a logistic process for which Benhabib and Spiegel (2005) find evidence. Preliminary results suggest the emergence of multiple steady states in the basic model, thus allowing for club convergence.

Second, one may argue that the degree of openness is not constant over time. On the one hand, historical evidence suggests waves of globalization that are correlated with rapid growth of the world economy (O'Rourke and Williamson (1999), Helpman (2004)). On the other hand, technical progress per se is likely to have increased the rate of diffusion. Finally, one may want to endogenize the growth rate of the world's technological frontier, to account for possible feedback effects from worldwide innovation efforts to the evolution of domestic productivity.

[13] The role of skill levels for the occurrence of club convergence is also stressed in Howitt and Mayer-Foulkes (2005).

6 Appendix I: Proofs

6.1 Proof of Lemma 1

Without loss of generality, suppress time subscripts.

Zero-profit implies $w = A_{-1}\,(1+q^*)\,h\,[p-(1+r)\,(1-\sigma)\,i(q^*,h)]$. With (2.6) and (2.7), this can be written as

$$\frac{w}{A_{-1}\,h\,(1+r)} = (1+q^*)\left[\frac{1-\alpha}{\alpha}\,k-(1-\sigma)\,i(q^*,h)\right].$$

Using the latter in (2.17), we obtain

$$\frac{1-\alpha}{\alpha}\,k = (1-\sigma)\,\left((1+q^*)\,i_q(q^*,h)+i(q^*,h)\right). \tag{6.1}$$

The derivatives stated in (2.20) follow from the implicit function theorem applied to (6.1) and the properties of the input requirement function i as stated in (2.10). For further reference, we note that

$$g_k = \frac{\frac{1-\alpha}{\alpha}}{(1-\sigma)(2\,i_q+(1+g)\,i_{qq})} > 0, \tag{6.2}$$

$$g_h = -\frac{(1+g)\,i_{qh}+i_h}{2\,i_q+(1+g)\,i_{qq}} > 0, \tag{6.3}$$

$$g_\sigma = \frac{(1+g)\,i_q+i}{(1-\sigma)(2\,i_q+(1+g)\,i_{qq})} > 0, \tag{6.4}$$

where the argument of g is (k,h,σ), and the argument of i is (g,h). ■

6.2 Proof of Lemma 2

The existence of a unique $\bar{k}$ follows from the properties of the function $g(k,h,\sigma)$, which satisfies $g(0,h,\sigma)=0$, $g(\infty,h,\sigma)=\infty$, and $g_k(k,h,\sigma)>0$ for all $k>0$ (see Lemma 1).

The remaining part of the proof of Lemma 2 proceeds with the statement and proof of four claims.

Claim 1 There is a unique $\bar{\bar{k}}>0$ that solves

$$\theta\,i(g(\bar{\bar{k}},h,\sigma),h) = 1+\gamma. \tag{6.5}$$

Moreover, there is a function

$$\bar{\bar{k}} = \bar{\bar{k}}(\theta),\text{ with }\bar{\bar{k}}'(\theta)<0,\ \lim_{\theta\to 0}\bar{\bar{k}}(\theta)=\infty,\ \lim_{\theta\to\infty}\bar{\bar{k}}(\theta)=0. \tag{6.6}$$

Proof of Claim 1 The existence of $\bar{\bar{k}}>0$ follows from the properties of the function g and those of the function i as stated in (2.10). An application of the implicit function theorem to (6.5) reveals that there is a function $\bar{\bar{k}}=\bar{\bar{k}}(\theta)$, with $\bar{\bar{k}}'(\theta)<0$. To study its asymptotic properties write (6.5) as $i(g(\bar{\bar{k}}, h, \sigma), h) = (1+\gamma)/\theta$. Since $i \to \infty$ as $q \to \infty$ and $g \to \infty$ as $k \to \infty$, it follows that $\lim_{\theta \to 0} \bar{\bar{k}}(\theta) = \infty$. Since $i(0, h) = 0$ and $g(0, h, \sigma) = 0$, it follows that $\lim_{\theta \to \infty} \bar{\bar{k}}(\theta) = 0$. □

Claim 2 There is $\bar{\theta} \equiv (1+\gamma)/i(\gamma, h)$ that solves $\bar{\bar{k}}(\bar{\theta}) = \bar{k}$. Then

$$\bar{\bar{k}} \gtreqless \bar{k} \quad \Leftrightarrow \quad \theta \lesseqgtr \bar{\theta}. \tag{6.7}$$

Proof of Claim 2 The existence of a unique value $\bar{\theta}$ and inequality (6.7) follow from the properties of the function $\bar{\bar{k}}(\theta)$ as set out in Claim 1. By construction, $\bar{\theta}$ satisfies $\bar{\theta}\, i(g(\bar{\bar{k}}, h, \sigma), h) = 1 + g(\bar{k}, h, \sigma) = 1 + \gamma$. Hence, $\bar{\theta} = (1+\gamma)/i(\gamma, h)$. □

Claim 3 The function $\phi^{\Delta}(k_t, \Delta_{t-1})$ maps $\Delta_{t-1} \in (0,1)$ onto itself if and only if $\theta < \bar{\theta}$ and $k_t \in [0, \bar{k}]$.

Proof of Claim 3 From (3.5) it is obvious that a trajectory with $\Delta_{t-1} \in (0,1)$ for all $t \geq 1$ requires $1 > \phi^{\Delta}(k, \Delta_{t-1}) > 0$ for all $t = 1, 2, \ldots$ or

$$1 > \frac{\theta\, i(g(k,h,\sigma), h)}{1+\gamma} + \frac{1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma), h)}{1+\gamma} \Delta_{t-1} > 0. \tag{6.8}$$

The following cases must be distinguished:

- $\theta < \bar{\theta}$, thus $\bar{\bar{k}} > \bar{k}$:
  - if $k \in [0, \bar{k}]$ and $1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma), h) > 0$, then from the definition of $\bar{k}$ both the left-hand inequality and the right-hand inequality of (6.8) hold for all $\Delta_{t-1} \in (0,1)$.

    To see that $k \in [0, \bar{k}]$ is necessary consider values $k > \bar{k}$. Since $\bar{\bar{k}} > \bar{k}$, there is a bound, $\bar{\Delta} \in (0,1)$, for all $k \in (\bar{k}, \bar{\bar{k}})$ such that the left-hand inequality is only satisfied for $\Delta_{t-1} < \bar{\Delta}$. To compute $\bar{\Delta}$, solve the left-hand inequality of (6.8) for Δ_{t-1}. This gives

    $$\Delta_{t-1} < \frac{1 + \gamma - \theta\, i(g(k,h,\sigma), h)}{1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma), h)} \equiv \bar{\Delta}. \tag{6.9}$$

    Clearly, $\bar{\Delta} \in (0,1)$ as long as $1 + \gamma - \theta\, i(g(k,h,\sigma), h) > 0$, $\partial\bar{\Delta}/\partial k < 0$, and $\bar{\Delta} = 0$ for $k = \bar{\bar{k}}$. Hence, for $k \in (\bar{k}, \bar{\bar{k}})$ only values of Δ_{t-1} that satisfy $\Delta_{t-1} < \bar{\Delta}$ imply $\Delta_t \in (0,1)$. The set of admissible values for Δ_{t-1} is therefore smaller than the set $(0,1)$. For $k \geq \bar{\bar{k}}$, it is empty. Hence, for $1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma), h) > 0$ Claim 3 holds.
  - The case $1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma), h) \leq 0$ can only arise if there is $k = \tilde{k} < \infty$ that satisfies the latter inequality as an equality. However, since $\bar{\bar{k}} > \bar{k}$ and $1 + g(\bar{\bar{k}}, h, \sigma) - \theta\, i(g(\bar{\bar{k}}, h, \sigma), h) > 0$ it follows that $\tilde{k} > \bar{\bar{k}}$. In turn, from Claim 1, it follows for $k > \bar{\bar{k}}$ that $\theta\, i(g(k,h,\sigma), h) > 1 + \gamma$. Hence, there is no $\Delta_{t-1} \in (0,1)$ that satisfies the left-hand inequality of (6.8). Hence, if $k \in [0, \bar{k}]$, this case cannot arise.
- $\theta > \bar{\theta}$, thus $\bar{\bar{k}} < \bar{k}$:
  - The inequality $1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma), h) > 0$ requires $k \in (0, \tilde{k})$. Since $\tilde{k} < \bar{\bar{k}}$ both the left-hand and the right-hand inequality of (6.8) hold for all $k \in [0, \tilde{k}]$ and $\Delta_{t-1} \in (0,1)$.

- Consider $1 + g(k,h,\sigma) - \theta\, i(g(k,h,\sigma),h) \leq 0$, which requires $k \geq \tilde{k}$. Then, the left-hand side inequality of (6.8) is satisfied for all $\Delta_{t-1} \in (0,1)$ as long as $k \in (\tilde{k}, \bar{\bar{k}})$. For $k \geq \bar{\bar{k}}$, $\bar{\Delta}$ serves as a lower bound such that the left-hand inequality of (6.8) is satisfied whenever $\Delta_{t-1} > \bar{\Delta}$. As $\partial\bar{\Delta}/\partial k > 0$ and $\bar{\Delta} = 1$ for $k = \bar{k}$, there is no $\Delta_{t-1} \in (0,1)$ that satisfies this inequality for $k \geq \bar{k}$. Hence, there is $k \in [0, \bar{k}]$ and $\Delta_{t-1} \in (0,1)$ such that (6.8) cannot be satisfied.

- $\theta = \bar{\theta}$, thus $\bar{\bar{k}} = \bar{k}$:
This constellation violates the left-hand inequality of (6.8) since $1+g(\bar{k},h,\sigma)-\theta\, i(g(\bar{\bar{k}},h,\sigma),h) = 0$ and $\bar{\theta}\, i(g(\bar{\bar{k}},h,\sigma),h)/(1+\gamma) = 1$. □

Claim 4 The function $\phi\,(k_t, \Delta_{t-1})$ maps $\left[0, \bar{k}\right] \times (0,1)$ onto itself if the conditions in (3.7) hold.

Proof of Claim 4 We have to show that $\phi^k(k, \Delta_{t-1}) \in [0, \bar{k}]$ for all $k \in \left[0, \bar{k}\right]$ and $\Delta_{t-1} \in (0,1)$. From (3.4), $\phi^k(k, \Delta_{t-1}) \geq 0$ is trivially satisfied, however $\bar{k} \geq \phi^k(k, \Delta_{t-1})$ may not. To make sure that the latter holds, we first note that the left-hand side of (3.4) is increasing in k_{t+1}. Hence, $\phi^k(\bar{k}, 1) = \tilde{s}\,\bar{k}^\alpha$ is an upper bound on k_{t+1} since the right-hand side of (3.4) increases in Δ_{t-1} and k_t. Moreover, since the slope of the left-hand side of (3.4) with respect to k_{t+1} is strictly greater than one, a sufficient condition for $k_{t+1} \leq \bar{k}$ is $\tilde{s}\,\bar{k}^\alpha < \bar{k}$, or $(\tilde{s}\,)^{1/(1-\alpha)} < \bar{k}$. Then, Claim 4 follows from Claim 3. ■

6.3 Proof of Proposition 1

Set $\Delta_t = \Delta_{t-1} = \Delta^*$ and $k_t = k^* \in (0, \bar{k})$ in (3.5) and obtain (3.9). Using $k_t = k_{t-1} = k^*$ and (3.9) in (3.4) gives (3.8). It remains to be shown that (3.8) gives rise to a unique solution $k^* \in (0, \bar{k})$.

First, I show that (3.8) has a unique solution $k^* > 0$. Define a function $LHS(k) \equiv k + i(g(k,h,\sigma),h)$. The properties of the functions g and i (see Lemma 1, (2.10), and (2.13)) imply that $LHS(k)$ is continuous in k with $LHS(0) = 0 + i(g(0,h,\sigma),h) = 0$, $LHS'(k) = 1 + i_q(g(k,h,\sigma),h)\, g_k(k,h,\sigma) > 1$ for $k > 0$, and $\lim_{k\to 0} LHS'(k) = 1$. Moreover, $LHS''(k) = i_{qq}(g(k,h,\sigma),h)\, g_k^2(k,h,\sigma) + i_q(g(k,h,\sigma),h)\, g_{kk}(k,h,\sigma) > 0$ for $k > 0$ since (2.13) holds. To verify this, we start from (6.2) and find for $k > 0$ that

$$g_{kk} = -g_k^2\, \frac{3 i_{qq} + (1+g) i_{qqq}}{2 i_q + (1+g) i_{qq}}, \tag{6.10}$$

where the argument of g is (k,h,σ), and the argument of i is (g,h). Then,

$$LHS''(k) > 0 \quad \Leftrightarrow \quad \frac{i_{qq}}{i_q} + \frac{g_{kk}}{g_k^2} > 0. \tag{6.11}$$

In view of (6.10) this comes down to

$$LHS''(k) > 0 \quad \Leftrightarrow \quad \frac{i_{qq}}{i_q} - \frac{3 i_{qq} + (1+g) i_{qqq}}{2 i_q + (1+g) i_{qq}} > 0. \tag{6.12}$$

The latter inequality is satisfied whenever the regularity requirement (2.13) holds.

Define $RHS(k) \equiv \tilde{s}\, k^\alpha/(1+\gamma)$. This function satisfies $RHS(0) = 0$ and $RHS'(k) > 0$ for all $k \geq 0$ with $RHS'(0) = \infty$ and $\lim_{k\to\infty} RHS'(k) = 0$. Hence, there is one and only one strictly positive value k^* that satisfies $LHS(k^*) = RHS(k^*)$.

To see that the the intersection $LHS(k^*) = RHS(k^*)$ occurs for some $k < \bar{k}$ recall from Lemma 2 that $\bar{k}$ is independent of $\tilde{s}$. Moreover, $RHS(k)$ becomes arbitrarily small as $\tilde{s} \to 0$. Hence, there are parameter constellations, $(\theta, \gamma, h, \sigma, \alpha)$ such that $k^* \in (0, \bar{k})$. ■

6.4 Proof of Corollary 1

Consider the total differential of (3.8)

$$\begin{aligned} 0 &= \left[1 + i_q g_k - \frac{\tilde{s}}{1+\gamma}\,\alpha\,(k^*)^{\alpha-1}\right] dk^* \\ &+ \;[i_q g_h + i_h]\, dh \\ &+ \; i_q g_\sigma\, d\sigma \\ &- \;(k^*)^\alpha\, d\left(\frac{\tilde{s}}{1+\gamma}\right). \end{aligned} \tag{6.13}$$

In equation (6.13) the first term in brackets is positive. To see this recall the functions $LHS(k)$ and $RHS(k)$ as defined in the proof of Proposition 1. Obviously, the term is brackets corresponds to $LHS'(k) - RHS'(k)$. The proof of Proposition 1 implies that the function $LHS(k)$ intersects the function $RHS(k)$ from below at k^*. Therefore, we must have $LHS(k^*) > RHS(k^*)$.

The comparative statics stated in (3.10) and (3.11) follow from the definition of $\tilde{s}$ and the properties of the functions i and g as stated in (2.10), (2.11), and Lemma 1.

To strengthen the result in (3.11) we express the critical inequality $-\,(i_q\, g_h + i_h) \gtreqless 0$ in terms of the following elasticities

$$\varepsilon_{i,h} \equiv -\frac{\partial i}{\partial h}\,\frac{h}{i} > 0, \quad \varepsilon_{i_q,h} \equiv -\frac{\partial i_q}{\partial h}\,\frac{h}{i_q} > 0, \quad \varepsilon_{i,q} \equiv \frac{\partial i}{\partial q}\,\frac{q}{i} > 0, \quad \varepsilon_{i_q,q} \equiv \frac{\partial i_q}{\partial q}\,\frac{q}{i_q} > 0.$$

Using the latter and Lemma 1, we have

$$g_h = -\frac{(1+g)\,i_{qh} + i_h}{2\,i_q + (1+g)\,i_{qq}} = \frac{(1+g)\,\varepsilon_{i_q,h} + \frac{i}{i_q}\,\varepsilon_{i,h}}{2\,h + (1+g)\frac{h}{q}\,\varepsilon_{i_q,q}}.$$

It follows that

$$-\,(i_q\, g_h + i_h) \gtreqless 0 \quad \Leftrightarrow \quad \frac{\varepsilon_{i_q,q}}{\varepsilon_{i,q}} + \frac{g}{1+g}\,\frac{1}{\varepsilon_{i,q}} \gtreqless \frac{\varepsilon_{i_q,h}}{\varepsilon_{i,h}}. \tag{6.14}$$

If $i = q^v\, h^{-z}$, we have $\varepsilon_{i,h} = \varepsilon_{i_q,h} = z$, $\varepsilon_{i,q} = v$, $\varepsilon_{i_q,q} = v - 1$, such that (6.14) becomes

$$\frac{v-1}{v} + \frac{g}{1+g}\,\frac{1}{v} \gtreqless 1 \quad \Leftrightarrow \quad 0 \gtreqless 1. \tag{6.15}$$

Hence, $-\,(i_q\, g_h + i_h) < 0$ and $dk^*/dh < 0$. ■

6.5 Proof of Proposition 2

First, observe that the steady-state condition (3.8) implies

$$k^i_{max} = \left(\frac{\alpha\,\tilde{s}}{1+\gamma}\right)^{\frac{1}{1-\alpha}}. \tag{6.16}$$

As to Statement 1, we have to study under what conditions

$$\frac{di(g(k^*,h,\sigma),h)}{dh} = i_q\, g_k \frac{dk^*}{dh} + i_q g_h + i_h \gtreqless 0. \tag{6.17}$$

From Corollary 1, we have

$$\frac{dk^*}{dh} = -\frac{i_q g_h + i_h}{1 + i_q\, g_k - \frac{\tilde{s}\,\alpha}{1+\gamma}\,(k^*)^{\alpha-1}}, \tag{6.18}$$

where the denominator is strictly positive. Therefore, inequality (6.17) is equivalent to

$$\frac{di(g(k^*,h,\sigma),h)}{dh} \gtreqless 0 \quad \Leftrightarrow \quad [i_q g_h + i_h]\left[1 - \frac{\alpha\,\tilde{s}}{1+\gamma}\,(k^*)^{\alpha-1}\right] \gtreqless 0. \tag{6.19}$$

Hence, $di(.)/dh > 0$ if either (3.14) or (3.15) hold.

As to Statement 2, we have to study under what conditions

$$\frac{di(g(k^*,h,\sigma),h)}{d\sigma} = i_q\, g_k \frac{dk^*}{d\sigma} + i_q g_\sigma \gtreqless 0. \tag{6.20}$$

From Corollary 1, we have

$$\frac{dk^*}{d\sigma} = -\frac{i_q g_\sigma}{1 + i_q\, g_k - \frac{\tilde{s}\,\alpha}{1+\gamma}\,(k^*)^{\alpha-1}}. \tag{6.21}$$

Then, inequality (6.20) is equivalent to

$$\frac{di(g(k^*,h,\sigma),h)}{d\sigma} \gtreqless 0 \quad \Leftrightarrow \quad 1 - \frac{\tilde{s}\,\alpha}{1+\gamma}\,(k^*)^{\alpha-1} \gtreqless 0. \tag{6.22}$$

Hence, (3.16) follows.

Statement 3 follows from (2.24), steady-state condition (3.9), and the fact that in the steady state $A_t/A_{t-1} - 1 = \gamma$. ■

6.6 Proof of Corollary 2

Consider Δ^* of (3.9). A change in one of the parameters $j = s, \Gamma, \lambda$ affects Δ^* only indirectly through k^*. Hence,

$$\frac{d\Delta^*}{dj} = \frac{\partial \Delta^*}{\partial k^*}\frac{dk^*}{dj}, \quad j = s, \Gamma, \lambda. \tag{6.23}$$

Since

$$\frac{\partial \Delta^*}{\partial k^*} = \frac{\theta\, i_q\, g_k(\gamma - g) + g_k\,\theta\, i}{(\theta\, i + \gamma - g)^2} > 0 \tag{6.24}$$

the first result of (3.18) follows from Corollary 1.

The total effect of a change of γ involves a direct and an indirect effect. Starting with the effect of γ, we find

$$\frac{\partial \Delta^*}{\partial \gamma} = -\frac{\Delta^*}{\theta\, i + \gamma - g} < 0, \quad \text{and} \quad \frac{d\Delta^*}{d\gamma} = \frac{\partial \Delta^*}{\partial \gamma} + \frac{\partial \Delta^*}{\partial k}\frac{dk^*}{d\gamma} < 0 \tag{6.25}$$

since $dk^*/d\gamma < 0$ (see Corollary 1).

The total effect of a change of σ and h involve direct and indirect effects through g. As to σ, we find

$$\frac{\partial \Delta^*}{\partial \sigma} \quad = \quad \frac{\theta\, i_q g_\sigma\, (\gamma - g) + \theta\, i\, g_\sigma}{(\theta i + \gamma - g)^2} > 0 \tag{6.26}$$

and

$$\frac{d\Delta^*}{d\sigma} = \frac{\partial \Delta^*}{\partial \sigma} + \frac{\partial \Delta^*}{\partial k}\frac{dk^*}{d\sigma} \gtreqless 0. \tag{6.27}$$

Since $dk^*/d\sigma < 0$ (Corollary 1), the sign of $d\Delta^*/d\sigma$ is indeterminate. However, using (6.24), (6.26), and (6.13) one verifies that inequality (6.27) is equivalent to

$$\frac{d\Delta^*}{d\sigma} \gtreqless 0 \quad \Leftrightarrow \quad 1 - \frac{\tilde{s}\,\alpha}{1+\gamma}\,(k^*)^{\alpha-1} \gtreqless 0. \tag{6.28}$$

Then, (3.18) follows with the proof of Proposition 2.

As to the effect of h, we have

$$\frac{\partial \Delta^*}{\partial h} \quad = \quad \frac{\theta(i_q\, g_h + i_h)(\gamma - g) + \theta\, i\, g_h}{(\theta\, i + \gamma - g)^2} \gtreqless 0, \tag{6.29}$$

and

$$\frac{d\Delta^*}{dh} = \frac{\partial \Delta^*}{\partial h} + \frac{\partial \Delta^*}{\partial k}\frac{dk^*}{dh} \gtreqless 0. \tag{6.30}$$

Using (6.29), (6.24), (3.8), we find that

$$\frac{d\Delta^*}{dh} \gtreqless 0 \quad \Leftrightarrow \quad \left(1 - \frac{\tilde{s}\,\alpha}{1+\gamma}\,(k^*)^{\alpha-1}\right)\,[(i_q\, g_h + i_h)\,(\gamma - g) + i\, g_h] \gtreqless i_h\, g_k\, i \tag{6.31}$$

Assume $i_q\, g_h + i_h > 0$ and $k^* > k^i_{max}$. Then, (3.14) holds and $d\Delta^*/dh > 0$.

Finally,

$$\frac{d\Delta^*}{d\theta} \quad = \quad \frac{i(\gamma - g)}{(\theta\, i + \gamma - g)^2} > 0. \tag{6.32}$$

■

6.7 Proof of Proposition 3

Denote $k^* = k^*(s, \Gamma, \lambda, \gamma, \sigma, h)$ and $\Delta^* = \Delta^*(k^*, \sigma, h, \gamma, \theta)$ the functions defined by Corollary 1 and 2 and recall $q^* = g\,(k^*, h, \sigma)$. Given A^{max}_{-1} we have per-capita income of the next period as

$$\tilde{y}^* \equiv \tilde{y}^*\left(\Gamma, k^*, \Delta^*(k^*, \gamma, \sigma, h, \theta), g\,(k^*, h, \sigma)\,, h\right). \tag{6.33}$$

From (3.19), Corollary 2 and Lemma 1 we have

$$\frac{d\tilde{y}^*}{dk^*} = \frac{\partial \tilde{y}^*}{\partial k^*} + \frac{\partial \tilde{y}^*}{\partial \Delta^*}\frac{\partial \Delta^*}{\partial k^*} + \frac{\partial \tilde{y}^*}{\partial q^*}\frac{\partial g}{\partial k^*} > 0. \tag{6.34}$$

Using Corollary 1 in addition gives

$$\frac{d\tilde{y}^*}{ds} = \frac{d\tilde{y}^*}{dk^*}\frac{dk^*}{ds} > 0, \quad \frac{d\tilde{y}^*}{d\Gamma} = \frac{\partial \tilde{y}^*}{\partial \Gamma} + \frac{d\tilde{y}^*}{dk^*}\frac{dk^*}{d\Gamma} > 0, \quad \frac{d\tilde{y}^*}{d\lambda} = \frac{d\tilde{y}^*}{dk^*}\frac{dk^*}{d\lambda} < 0. \tag{6.35}$$

Similarly, we obtain

$$\frac{d\tilde{y}^*}{d\gamma} = \frac{d\tilde{y}^*}{dk^*}\frac{dk^*}{d\gamma} + \frac{\partial \tilde{y}^*}{\partial \Delta^*}\frac{\partial \Delta^*}{\partial \gamma} < 0, \tag{6.36}$$

which proves the first two results in (3.20). Invoking Corollary 1, 2, and Lemma 1, we find

$$\frac{d\tilde{y}^*}{d\sigma} = \frac{d\tilde{y}^*}{dk^*}\frac{dk^*}{d\sigma} + \frac{\partial \tilde{y}^*}{\partial \Delta^*}\frac{d\Delta^*}{d\sigma} + \frac{\partial \tilde{y}^*}{\partial q^*} g_\sigma \gtreqless 0 \tag{6.37}$$

and

$$\frac{d\tilde{y}^*}{dh} = \frac{d\tilde{y}^*}{dk^*}\frac{dk^*}{dh} + \frac{\partial \tilde{y}^*}{\partial \Delta^*}\frac{d\Delta^*}{dh} + \frac{\partial \tilde{y}^*}{\partial q^*} g_h + \frac{\partial \tilde{y}^*}{\partial h} \gtreqless 0. \tag{6.38}$$

These two comparative statics involve terms of opposite sign such that the sum cannot be signed in general. Finally, Corollary 2 implies

$$\frac{d\tilde{y}^*}{d\theta} = \frac{\partial \tilde{y}^*}{\partial \Delta^*}\frac{\partial \Delta^*}{\partial \theta} > 0 \tag{6.39}$$

which proves the remaining terms in (3.20). ■

6.8 Proposition 4

This section comprises three parts. Subsection 6.8.1 has the details concerning the representative household's optimization problem. Subsection 6.8.2 has the proof of Proposition 4. Section **??** proves the property of the steady-state savings rate stated in the main text.

6.8.1 The Problem of the Representative Household

Denote per-capita magnitudes with a tilde, e. g., $\tilde{\tau}_t \equiv T_t/L_t$. The household solves

$$\max_{\{\tilde{c}_t, \tilde{b}_{t+1}\}_{t=1}^{\infty}} \quad \sum_{t=1}^{\infty} (\beta(1+\lambda))^{t-1} \frac{\tilde{c}_t^{1-\eta} - 1}{1-\eta}, \quad 0 < \beta(1+\lambda) < 1, \eta > 0, \tag{6.40}$$

subject to the flow budget constraint

$$\tilde{c}_t + \tilde{b}_{t+1} \frac{1+\lambda}{1+r_{t+1}} \quad \leq \quad w_t + \tilde{b}_t - \tilde{\tau}_t, \quad t = 1, 2, ... \tag{6.41}$$

and the Ponzi condition

$$\lim_{t\to\infty} \tilde{b}_{t+1} \left(\frac{1+\lambda}{1+\bar{r}}\right)^t \quad \geq \quad 0, \tag{6.42}$$

where $\bar{r} \equiv \left(\prod_{j=1}^{j=t}(1+r_{j+1})\right)^{1/t} - 1$ is the average real interest rate. In (6.41) we use the fact that dividends are zero in equilibrium, i. e., $\tilde{\pi}_t = 0$. Since $\lim_{\tilde{c}\to 0} (\tilde{c})^{-\eta} = \infty$, the flow budget constraint is binding at all t, and optimal plan involves $\tilde{c} > 0$ at all t.

Setting up the Lagrangian gives

$$\mathfrak{L} \quad = \quad \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{\tilde{c}_t^{1-\eta} - 1}{1-\eta} (1+\lambda)^{t-1} + \mu_t \left(w_t + \tilde{b}_t - \tilde{\tau}_t - \tilde{c}_t - \tilde{b}_{t+1} \frac{1+\lambda}{1+r_{t+1}} \right) \right],$$

and the following first-order conditions

$$(\beta\,(1+\lambda))^{t-1}\,(\tilde{c}_t)^{-\eta} - \mu_t \quad = \quad 0, \quad t = 1,2,... \tag{6.43}$$

$$-\mu_t\,\frac{1+\lambda}{1+r_{t+1}} + \beta\,\mu_{t+1} \quad = \quad 0, \quad t = 1,2,... \tag{6.44}$$

$$w_t + \tilde{b}_t - \tilde{\tau}_t - \tilde{c}_t - \tilde{b}_{t+1}\,\frac{1+\lambda}{1+r_{t+1}} \quad = \quad 0, \quad t = 1,2,... \tag{6.45}$$

$$\lim_{t\to\infty} \beta^{t-1}\,\mu_t\,\tilde{b}_{t+1}\,\frac{1+\lambda}{1+r_{t+1}} \quad = \quad 0. \tag{6.46}$$

From (6.43) and (6.44), we obtain the Euler condition

$$\tilde{c}_{t+1} \quad = \quad [\beta\,(1+r_{t+1})]^{\frac{1}{\eta}}\;\tilde{c}_t. \tag{6.47}$$

To express the latter in terms of efficient labor we use the definition $c_t \equiv C_t/X_t = \tilde{c}_{t+1}\,L_t/X_t$ and the market-clearing condition (3.2). This gives (4.2).

Condition (6.44) implies the following evolution of the multiplier μ_t,

$$\mu_t \quad = \quad \mu_1\,\left(\frac{1+\lambda}{\beta}\right)^{t-1}\,\frac{1}{\prod_{j=1}^{j=t-1}(1+r_{j+1})}, \quad t = 2,3,.... \tag{6.48}$$

Using the latter, the transverslity condition becomes

$$\begin{aligned}
0 \quad &= \quad \lim_{t\to\infty} \beta^{t-1}\,\mu_1\,\left(\frac{1+\lambda}{\beta}\right)^{t-1}\,\frac{1}{\prod_{j=1}^{j=t-1}(1+r_{j+1})}\,\tilde{b}_{t+1}\frac{1+\lambda}{1+r_{t+1}} \\
&= \quad \lim_{t\to\infty} \mu_1\,\frac{(1+\lambda)^t}{\prod_{j=1}^{t}(1+r_{j+1})}\,\tilde{b}_{t+1} \\
&= \quad \lim_{t\to\infty} \left(\frac{1+\lambda}{1+\bar{r}}\right)^t\,\tilde{b}_{t+1},
\end{aligned} \tag{6.49}$$

where the last step uses (6.43) to conclude that $\mu_1 > 0$ and the definition of $\bar{r}$. Invoking the definition $b_{t+1} \equiv B_{t+1}/X_{t+1} = \tilde{b}_{t+1}\,L_{t+1}/X_{t+1}$ and the market-clearing condition (3.2) gives (4.3).

6.8.2 Proof of Proposition 4

To describe the evolution of the economy we use, as before, k_t and Δ_{t-1} as the state variables of the dynamical system. Since aggregate consumption equals output minus investment, we obtain with Lemma 1 and the equilibrium conditions (3.2)

$$c_t = \Gamma\,k_t^{\alpha} - (1+\lambda)\,\frac{A_t}{A_{t-1}}\,\frac{1+g(k_{t+1},h,\sigma)}{1+g(k_t,h,\sigma)}\,\left(k_{t+1} + i(g(k_{t+1},h,\sigma),h)\right). \tag{6.50}$$

In equilibrium r_{t+1} is a function of k_{t+1} (see condition (2.6)) and, from (2.23), the growth factor A_t/A_{t-1} is a function of k_t and Δ_{t-1}. Therefore, the Euler condition becomes a difference equation in k_t, k_{t+1}, k_{t+2}, Δ_{t-1}, and Δ_t.

The bond market equilibrium assures that $b_{t+1} = (1 + r_{t+1})\,(k_{t+1} + i(g(k_{t+1}, h, \sigma), h))$ for all $t = 1, 2, ...$ Invoking (2.6) and the definition $A_t \equiv \Delta_t\, A_t^{max}$, the transversality condition can be expressed in terms of the state variables of the system. As a result, the dynamical system comprises the Euler condition, the equation of motion for Δ as stated in (3.5), initial values k_1, Δ_0, and Δ_1, and the transversality condition.[14]

In the steady state all magnitudes in efficiency units are constant, i. e., $c_{t+1} = c_t = c$ and $k_{t+1} = k_t = k$. With (2.6) it follows from (4.2) that the steady-state level, k^*_{RCK}, must satisfy

$$\frac{A_t}{A_{t-1}} = \left[\alpha\,\beta\,\Gamma\;(k^*_{RCK})^{\alpha-1}\right]^{\frac{1}{\eta}}. \tag{6.51}$$

Since $A_t/A_{t-1} = 1 + \gamma$, the finding (4.4) of Proposition 4 is immediate.

Next we have to show that k^*_{RCK} can be part of an equilibrium allocation of a laggard country.

First, consider the transversality condition. Since $\tilde{b}_{t+1} \equiv A_{t+1}\, b_{t+1} > 0$, (6.49) at the steady state can be stated as

$$\lim_{t\to\infty} \left(\frac{1+\lambda}{\alpha\,\Gamma\;(k^*_{RCK})^{\alpha-1}}\right)^t A_{t+1} = \lim_{t\to\infty} \left(\frac{(1+\lambda)(1+\gamma)}{(1+\gamma)^{\eta}/\beta}\right)^t = 0, \tag{6.52}$$

where the last step uses uses (4.4). To satisfy (6.52) we need

$$\beta(1+\lambda) < (1+\gamma)^{\eta-1}. \tag{6.53}$$

Since $\beta(1+\lambda) < 1$, the latter condition is only binding if $0 < \eta < 1$. In this case, it is satisfied whenever

$$\gamma < [\beta(1+\lambda)]^{\frac{-1}{1-\eta}} - 1 \equiv \bar{\gamma}. \tag{6.54}$$

Second, we have to make sure that $k^*_{RCK} < \bar{k}$. This requirement imposes a lower bound on γ. Consider the function $k^*_{RCK}(\gamma) \equiv [\alpha\,\beta\,\Gamma/\,(1+\gamma)^{\eta}]^{1/(1-\alpha)}$. It satisfies

$$k^*(0) = (\alpha\,\beta\,\Gamma)^{\frac{1}{1-\alpha}}, \quad \frac{\partial k^*(\gamma)}{\partial\gamma} < 0, \quad \text{and} \quad \lim_{\gamma\to\infty} k^*(\gamma) = 0. \tag{6.55}$$

Next, consider the properties of the function g and $g(\bar{k}, h, \sigma) = \gamma$. The latter equation implicitly defines a function $\bar{k}(\gamma)$ with the following properties

$$\bar{k}(0) = 0, \quad \frac{\partial \bar{k}(\gamma)}{\partial\gamma} = \frac{1}{g_k} > 0, \quad \text{and} \quad \lim_{\gamma\to\infty} \bar{k}(\gamma) > 0. \tag{6.56}$$

Hence, the functions $k^*_{RCK}(\gamma)$ and $\bar{k}(\gamma)$ intersect once and only once at some $\gamma > 0$. Let $k^*_{RCK}(\underline{\gamma}) = \bar{k}(\underline{\gamma})$. Then it holds that

$$k^*_{RCK}(\gamma) < \bar{k}(\gamma) \quad \Leftrightarrow \quad \gamma > \underline{\gamma}. \tag{6.57}$$

Accordingly, if $\eta \geq 1$ the balanced growth path exists for any $\gamma > \underline{\gamma}$, if $\eta \in (0,1)$ it exists for any $\gamma \in (\underline{\gamma}, \bar{\gamma})$. There are parameter constellations, (β, λ, η) such that $(\underline{\gamma}, \bar{\gamma})$ is non-empty. ■

[14] In fact, given k_1 and Δ_0, Δ_1 is fully determined by (3.5).

6.9 Proof of Proposition 5

Proposition 5 follows immediately from Lemma 1, the definition of the rate of diffusion, and the observation that k^*_{RCK} is independent of h and σ. ■

6.10 Proof of Corollary 3

A change in one of the parameters $j = \beta, \eta, \Gamma$ affects Δ^*_{RCK} only indirectly through k^*_{RCK}. Hence,

$$\frac{d\Delta^*_{RCK}}{dj} = \frac{\partial \Delta^*_{RCK}}{\partial k^*_{RCK}} \frac{dk^*_{RCK}}{dj}, \quad j = \beta, \epsilon, \Gamma. \tag{6.58}$$

From Corollary 2, we know that $d\Delta^*_{RCK}/dk^*_{RCK} > 0$. Hence, the first three results of (4.7) follow immediately from (4.4).

The effect of a change of γ involves a direct and an indirect effect through k^*_{RCK}. From (4.4) and Corollary 2 both are negative. The parameters σ, h, and θ induce effects that are given in equations (6.26), (6.29), and (6.32) in the proof of Corollary 2. ■

6.11 Proof of Proposition 6

Denote $k^*_{RCK} = k^*_{RCK}(\beta, \eta, \Gamma, \gamma)$ the function defined by the steady state (4.4), $\Delta^* = \Delta^*(k^*_{RCK}, \sigma, h, \gamma, \theta)$ the function defined by Corollary 3, and recall $q^* = g\left(k^*_{RCK}, h, \sigma\right)$. Given A^{max}_{-1} we have per-capita income of the next period as

$$\tilde{y}^*_{RCK} \equiv \tilde{y}^*_{RCK}\left(\Gamma, k^*_{RCK}, \Delta^*_{RCK}(k^*_{RCK}, \gamma, \sigma, h, \theta), g\left(k^*_{RCK}, h, \sigma\right), h\right). \tag{6.59}$$

From (3.19), Corollary 2 and Lemma 1 we have

$$\frac{d\tilde{y}^*_{RCK}}{dk^*_{RCK}} > 0 \tag{6.60}$$

for the same reason as in equation (6.34) in the proof of Proposition 3. Then, the first three results stated in (4.9) result from (6.60) and the properties of the function $k^*_{RCK}(.)$. The comparative static with respect to γ follows from the analogue of equation (6.36).

Invoking Corollary 3 and Lemma 1, we find

$$\frac{d\tilde{y}^*_{RCK}}{d\sigma} = \frac{\partial \tilde{y}^*_{RCK}}{\partial \Delta^*} \frac{\partial \Delta^*_{RCK}}{\partial \sigma} + \frac{\partial \tilde{y}^*_{RCK}}{\partial q^*} g_\sigma > 0 \tag{6.61}$$

and

$$\frac{d\tilde{y}^*_{RCK}}{dh} = \frac{\partial \tilde{y}^*_{RCK}}{\partial \Delta^*} \frac{\partial \Delta^*_{RCK}}{\partial h} + \frac{\partial \tilde{y}^*_{RCK}}{\partial q^*} g_h + \frac{\partial \tilde{y}^*_{RCK}}{\partial h} \gtreqless 0. \tag{6.62}$$

Invoking Corollary 3 reveals that the latter is strictly positive if $i_q g_h + i_h > 0$. Finally, Corollary 3 also implies

$$\frac{d\tilde{y}^*_{RCK}}{d\theta} = \frac{\partial \tilde{y}^*_{RCK}}{\partial \Delta^*} \frac{\partial \Delta^*_{RCK}}{\partial \theta} > 0. \tag{6.63}$$

■

6.12 Proof of Proposition 7

Consider the left-hand side of (4.14) and define

$$LHS(k) \equiv (1 + g(k, h, \sigma)) \left(k + i(g(k, h, \sigma), h)\right).$$

The properties of the functions g and i (see Lemma 1, (2.10), and (2.13)) imply that $LHS(k)$ is a continuous function with $LHS(0) = 0$, $LHS'(k) = g_k \left(k + i(g, h)\right) + (1 + g) \left(1 + i_q g_k\right) > 1$ for $k > 0$, and $\lim_{k\to 0} LHS'(k) = 1$. Moreover, $LHS''(k) = g_{kk}(k + i) + 2g_k \left(1 + i_q g_k\right) + (1 + g) \left(i_{qq} g_k^2 + i_q g_{kk}\right) > 0$ for $k > 0$ if condition (4.13) is satisfied. To verify the latter, recall from the proof of Proposition 1 that (2.13) implies $i_{qq} g_k^2 + i_q g_{kk} > 0$ for $k > 0$. As I show below, condition (4.13) is sufficient for $LHS''(k) > 0$ and $k > 0$ since it assures that $g_{kk}(k + i) + 2g_k \left(1 + i_q g_k\right) \geq 0$ for $k > 0$. Indeed, with (6.10) the latter can be written

$$\frac{2}{g_k} + 2i_q \quad \geq \quad \frac{3i_{qq} + (1 + g)i_{qqq}}{2i_q + (1 + g)i_{qq}} \; (k + i)\,. \tag{6.64}$$

From (6.12) in the proof of Proposition 1 we know that the function i is such that

$$\frac{i_{qq}}{i_q} > \frac{3i_{qq} + (1 + g)i_{qqq}}{2i_q + (1 + g)i_{qq}}. \tag{6.65}$$

Hence, (6.64) is satisfied whenever

$$\frac{2}{g_k} + 2i_q \quad \geq \quad \frac{i_{qq}}{i_q} \; (k + i)\,. \tag{6.66}$$

Next, we use (6.2) and the fact that (6.1) relates k to i, α, and σ. We obtain successively

$$\begin{aligned}\frac{2(1-\sigma)\alpha}{1-\alpha} \left(2i_q + (1+g)i_{qq}\right) + 2i_q \quad &\geq \quad \frac{i_{qq}}{i_q} \left(\frac{(1-\sigma)\alpha}{1-\alpha} \left((1+g)i_q + i\right) + i\right) \\ &= \quad \frac{(1-\sigma)\alpha}{1-\alpha} \left((1+g)i_{qq} + i\frac{i_{qq}}{i_q}\right) + i\frac{i_{qq}}{i_q}.\end{aligned}$$

Rearranging terms gives

$$\frac{(1-\sigma)\alpha}{1-\alpha} (1+g)i_{qq} + \frac{(1-\sigma)\alpha}{1-\alpha} \left(4i_q - i\frac{i_{qq}}{i_q}\right) + \left(2i_q - i\frac{i_{qq}}{i_q}\right) \quad \geq \quad 0. \tag{6.67}$$

Since $i_q > 0$ whenever $q > 0$, the latter is satisfied if $2i_q^2 \geq i\, i_{qq}$, which coincides with (4.13).

The right-hand side of (4.14) defines $RHS(k) \equiv \tilde{s}k^\alpha$, a strictly concave function with $RHS(0) = 0$, $RHS'(0) = \infty$, and $RHS'(\infty) = 0$. Hence, there is one and only one value $k_c^* > 0$ that satisfies $LHS(k_c^*) = RHS(k_c^*)$.[15] A simple graphical argument shows that any sequence $\{k_t\}$ that starts below or above k_c^* converges monotonically.

---

[15] If $LHS(k)$ is not convex on $k \in (0, \bar{k})$ there may be multiple steady states. To see this observe that $RHS'(k) = 1$ at $k = (\alpha\tilde{s})^{1/(1-\alpha)}$. If the functions $LHS(k)$ and $RHS(k)$ intersect for the first time at some $k_c^* \geq (\alpha\tilde{s})^{1/(1-\alpha)}$, then, since $LHS'(k) > 1$, the steady state is unique and globally stable. If the functions $RHS(k)$ and $LHS(k)$ intersect for the first time at some $k_c^* < (\alpha\tilde{s})^{1/(1-\alpha)}$, they may intersect more than once if $LHS(k)$ is concave with sufficient curvature. In any case, the argument that proves the existence of a unique $k_c^* > 0$ implies that the total number of steady states must be odd. Moreover, the first, third, fifth,... intersection of $RHS(k)$ and $LHS(k)$ is locally stable since it satisfies $RHS'(k_c^*) < LHS'(k_c^*)$. Those associated with an even number must be locally unstable.

Since, $(\tilde{s})^{1/(1-\alpha)}$ is the steady state if $LHS'(k) = 1$ for all k, we have $k_c^* < (\tilde{s})^{1/(1-\alpha)} < \bar{k}$. Hence, $g(k_c^*, h, \sigma) < \gamma$ and (4.16) holds. Moreover, a comparison of (4.14) with (3.8) reveals readily the validity of result (4.15). Result (4.16) is immediate from (4.12).

■

6.13 Proof of Corollary 4

Consider the total differential of (4.14)

$$\begin{aligned}
0 &= \left[g_k\,(k_c^* + i) + (1+g)\,(1 + i_q\,g_k) - \tilde{s}\,\alpha\,(k_c^*)^{\alpha-1}\right] dk_c^* \\
&+ \left[g_h\,(k_c^* + i) + (1+g)(i_q\,g_h + i_h)\right] dh \\
&+ \left[g_\sigma\,(k_c^* + i) + (1+g) i_q\,g_\sigma\right] d\sigma \\
&- (k_c^*)^\alpha\,d\tilde{s}, \qquad (6.68)
\end{aligned}$$

where the argument of i is (g, h) and the argument of g is (k_c^*, h, σ).

The first term in brackets is positive since, at k_c^*, the slope of the left-hand side of (4.14) is greater than the slope of the right-hand side.

The comparative statics stated in (4.17) and (4.18) follow from (6.68), the definition of $\tilde{s}$, the properties of the functions i and g as stated in (2.10), (2.11), and Lemma 1. Moreover, the result for $i = q^v\,h^{-z}$ follows from $g_h > 0$ and Corollary 1. ■

6.14 Proof of Proposition 8

Throughout this proof the argument of g is (k_c^*, h, σ) and the argument of i is (g, h).

The parameters $j = s, \Gamma, \lambda$ affect q_c^* indirectly through their effect on k_c^*, i. e.,

$$\frac{dq_c^*}{dj} = g_k\,\frac{dk_c^*}{dj}.$$

The signs given in (4.20) follow directly from $g_k > 0$ and Corollary 3. As to the the comparative statics of σ and h there is a direct and an indirect effect, namely

$$\frac{dq_c^*}{dj} = g_k\,\frac{dk_c^*}{dj} + g_j, \quad j = \sigma, h. \qquad (6.69)$$

In view of Corollary 3 and the properties of the function g, these effects may be of opposite sign. As to σ, we obtain using (6.68)

$$\begin{aligned}
\frac{dk_c^*}{d\sigma} &= -\frac{g_\sigma\,(k_c^* + i) + (1+g)\,i_q\,g_\sigma}{g_k\,(k_c^* + i) + (1+g)\,(1 + i_q\,g_k) - \tilde{s}\,\alpha\,(k_c^*)^{\alpha-1}} \\
&= -\frac{g_\sigma}{g_k}\,\frac{k_c^* + i + (1+g)\,i_q}{k_c^* + i + \frac{1+g}{g_k}\,(1 + i_q\,g_k) - \frac{\tilde{s}\,\alpha}{g_k}\,(k_c^*)^{\alpha-1}} \\
&= -\frac{g_\sigma}{g_k}\,\frac{k_c^* + i + (1+g)\,i_q}{k_c^* + i + (1+g)\,i_q + \frac{1}{g_k}\left(1 + g - \tilde{s}\,\alpha\,(k_c^*)^{\alpha-1}\right)}. \qquad (6.70)
\end{aligned}$$

Hence, (6.69) and (6.70) imply

$$\frac{dq_c^*}{d\sigma} \gtreqless 0 \quad \Leftrightarrow \quad \frac{g_k}{g_\sigma}\frac{dk_c^*}{d\sigma} + 1 \gtreqless 0 \quad \Leftrightarrow \quad 1 + g - \tilde{s}\,\alpha\,(k_c^*)^{\alpha-1} \gtreqless 0.$$

Using the steady-state condition (4.14), the latter inequality can be expressed as

$$1 - \frac{\alpha}{k_c^*}\,(k_c^* + i) \gtreqless 0 \quad \Leftrightarrow \quad \frac{1-\alpha}{\alpha}\,k_c^* \gtreqless i.$$

From (6.1) in the proof of Lemma 1, the left-hand side of the latter inequality becomes

$$(1-\sigma)\,((1+g)\,i_q + i) \gtreqless i, \tag{6.71}$$

where the argument of i is (g, h) and the argument of g is (k_c^*, h, σ). Hence, $dq_c^*/d\sigma > 0$ at $\sigma = 0$.

Turning to the effect of h, we obtain with (6.68) and (4.14)

$$\begin{aligned}
\frac{dk_c^*}{dh} &= -\frac{g_h\,(k_c^*+i) + (1+g)(i_q\,g_h + i_h)}{g_k\,(k_c^*+i) + (1+g)\,(1+i_q\,g_k) - \tilde{s}\,\alpha\,(k_c^*)^{\alpha-1}} \\
&= -\frac{g_h}{g_k}\,\frac{k_c^* + i + \frac{1+g}{g_h}(i_q\,g_h + i_h)}{k_c^* + i + \frac{1+g}{g_k}\,(1+i_q\,g_k) - \frac{\alpha}{g_k\,k_c^*}\,(1+g)\,(k_c^*+i)} \\
&= -\frac{g_h}{g_k}\,\frac{k_c^* + i + (1+g)\,i_q + \frac{1+g}{g_h}\,i_h}{k_c^* + i + (1+g)\,i_q + \frac{1+g}{g_k}\left(1 - \frac{\alpha}{k_c^*}\,(k_c^*+i)\right)}
\end{aligned} \tag{6.72}$$

Here, (6.69) and (6.72) imply

$$\frac{dq_c^*}{dh} \gtreqless 0 \quad \Leftrightarrow \quad -\frac{g_k}{g_h}\frac{dk_c^*}{dh} + 1 \gtreqless 0.$$

The latter inequality is satisfied whenever

$$-\frac{i_h}{g_h} + \frac{1}{g_k}\left(1 - \frac{\alpha}{k_c^*}\,(k_c^*+i)\right) \gtreqless 0. \tag{6.73}$$

The same steps that lead to (6.71) reveal that $dq_c^*/dh > 0$ at $\sigma = 0$. ■

6.15 Proof of Proposition 9

From (4.26) we know that $k_1 < \underline{\hat{k}}$ induces intermediate-good firms not to undertake innovation investments. Hence, initially k evolves according to the equation of motion (4.24) that gives rise to a globally stable steady state equal to $\tilde{s}^{\frac{1}{1-\alpha}}$.

If $\tilde{s}^{\frac{1}{1-\alpha}} \leq \hat{k}$, then the economy never reaches the critical level of k necessary to switch into the regime with innovation. For $t = 1, 2, ..$ the evolution of Δ is given by (4.25) and $\Delta_{t+1}/\Delta_t - 1 = -\gamma/(1+\gamma)$.

If $\tilde{s}^{\frac{1}{1-\alpha}} > \hat{k}$, then the economy initially grows according to (4.24). However, before reaching the steady state associated with this equation of motion it arrives at the critical level given in (4.26). The switch into the regime with innovation investments is as described in the main text.

To prove the existence of a unique steady state we need to show that there is a unique $k^* \in \left(\hat{k}, \bar{k}\right)$ that solves $(1+\gamma)\left(k^* + i\left(g\left(k^*, h, \sigma\right), h\right)\right) = \tilde{s}\left(k^*\right)^{\alpha}$, which restates (3.8). First, we observe that (4.22) and the definitions of $\hat{k}$ and $\bar{k}$ imply $\bar{k} > \hat{k}$ for all $\gamma > 0$. Hence, there are parameter constellations such that $\bar{k} > \tilde{s}^{\frac{1}{1-\alpha}} > \hat{k}$. Next, consider the functions $LHS(k) \equiv (1+\gamma)\left(k + i\left(g\left(k, h, \sigma\right), h\right)\right)$ and $RHS(k) \equiv \tilde{s}\left(k^*\right)^{\alpha}$. The function $LHS(k)$ satisfies $LHS(\hat{k}) = \hat{k}$, $LHS'(k) = 1 + (1 - \alpha)/\left(2\alpha(1-\sigma)\right) > 1$ for $k \geq \hat{k}$, $LHS(\bar{k}) = \bar{k} + i\left(\gamma, h, \sigma\right) > \tilde{s}^{\frac{1}{1-\alpha}}$. The function $RHS(k)$ satisfies $RHS(\hat{k}) > \hat{k}$ because $(\tilde{s})^{\frac{1}{1-\alpha}} > \hat{k}$, and $RHS(\bar{k}) < LHS\left(\bar{k}\right)$. Since $LHS(k)$ is linear and $RHS(k)$ concave on $(\hat{k}, \bar{k})$ both functions intersect once and only once on this interval. Since $k^* < \bar{k}$ the steady state involves $\Delta^* \in (0, 1)$ as given by (3.9). ■

7 Appendix II: Phase Diagram and Local Stability

This section develops the phase diagram and the equations underlying the local stability analysis of the steady state characterized in Proposition 1.

7.1 Phase Diagram

We develop the phase diagram in the $(\Delta, k) -$ plane.

First, consider the locus $Dk \equiv k_{t+1} - k_t$. From (3.4) and omitting time subscripts, it follows that

$$Dk = 0 \quad \Leftrightarrow \quad \Delta_{t-1} \equiv \Delta^k(k) = (1 - \zeta(k))^{-1}, \tag{7.1}$$

where

$$\zeta(k) \equiv \frac{(1+g)\,(k+i) - \tilde{s}k^{\alpha}}{\theta\, i\,(k+i)}, \tag{7.2}$$

and the argument of i is (g, h) and the argument of g is (k, h, σ). We summarize important properties of (7.1) and (7.2) as Result 1.

Result 1
(a) Let $k > 0$, then $\zeta(k) = 0$ if and only if $k = k_c^*$.
(b) The function $\Delta^k(k)$ satisfies $\lim_{k \to 0} \Delta^k(k) = 0$, $\Delta^k(k_c^*) = 1$, and is continuous on $k \in [0, k_c^*]$.
(c) It holds that

$$\bar{k} > k_c^* > k^* > 0.$$

Proof
(a) For $k > 0$ the denominator of (7.2) is strictly positive. The numerator can be expressed as $LHS(k) - RHS(k)$, where the two functions $LHS(k)$ and $RHS(k)$ are those defined in the proof of Proposition 7. Then, Result 1 (a) follows from the properties of the functions $LHS(k)$ and $RHS(k)$ as indicated in the proof of Proposition 7.

(b) It holds that

$$\lim_{k \to 0} \Delta^k(k) = \frac{1}{1 - \lim_{k \to 0} \zeta(k)}. \tag{7.3}$$

Moreover, an application of l'Hôpital's rule reveals that

$$\lim_{k \to 0} \zeta(k) = \lim_{k \to 0} \frac{(1+g)(k+i) - \tilde{s}k^{\alpha}}{\theta\, i\,(k+i)} = -\infty. \tag{7.4}$$

Hence, $\lim_{k \to 0} \Delta^k(k) = 0$. Moreover, $\Delta^k(k_c^*) = 1$ is immediate from (a). Continuity of ζ follows from the continuity of the functions i and g and the fact that $(1+g)\,(k+i) \leq \tilde{s}k^{\alpha}$ for $k \in [0, k_c^*]$.

(c) Follows from the proof of inequality (4.15) of Proposition 7. ■

Next, we turn to the locus $D\Delta \equiv \Delta_t - \Delta_{t-1}$. Omitting time subscripts, one obtains from (3.5)

$$D\Delta = 0 \quad \Leftrightarrow \quad \Delta_{t-1} \equiv \Delta^{\Delta}(k) = \frac{\theta\, i(g\,(k,h,\sigma)\,,h)}{\theta\, i(g\,(k,h,\sigma)\,,h) + \gamma - g\,(k,h,\sigma)}. \tag{7.5}$$

Result 2 The function $\Delta^{\Delta}(k)$ has the following properties.

$$\Delta^{\Delta}(0) = 0, \quad \Delta^{\Delta}(\bar{k}) = 1, \quad \text{for } k > 0 \quad \partial\Delta^{\Delta}(k)/\partial k > 0, \quad \text{and} \tag{7.6}$$

$$\text{for } k \in (0, k_c^*) \quad \Delta^{\Delta}(k) \gtreqless \Delta^{k}(k) \quad \Leftrightarrow \quad k \lesseqgtr k^*. \tag{7.7}$$

Proof The properties under (7.6) follow immediately from the properties of the functions i and g, the definition of $\bar{k}$, and the fact that

$$\frac{\partial\Delta^{\Delta}(k)}{\partial k} = [i_q g_k(\gamma - g) + g_k\, i]\, \frac{\theta}{(\theta\, i + \gamma - g)^2} > 0 \quad \text{for } k > 0.$$

The property stated under (7.7) follows from the fact that on $k \in (0, k_c^*)$ the functions $\Delta^k(k)$ and $\Delta^{\Delta}(k)$ intersect only once at k^* (see Proposition 1), while both are continuous and $\Delta^k(k_c^*) = 1 > \Delta^{\Delta}(k_c^*)$ since $k_c^* < \bar{k}$. ■

To understand the forces that affect the evolution of both state variables, consider the $Dk = 0$ - locus first. Above this locus, we have $\Delta_{t-1} > \Delta^k(k)$ and the right-hand side of (3.4) is greater. Since the left-hand side is increasing in k_{t+1}, it holds that $Dk > 0$. An analogous argument shows that $Dk < 0$ below the $Dk = 0$ - locus.

Next, consider the $D\Delta = 0$ - locus. From (3.5) we obtain

$$D\Delta = \frac{\theta\, i(g\,(k,h,\sigma)\,,h)}{1+\gamma} - \Delta_{t-1}\left(\frac{\theta\, i(g\,(k,h,\sigma)\,,h) + \gamma - g\,(k,h,\sigma)}{1+\gamma}\right). \tag{7.8}$$

Since $\theta\, i(g\,(k_t,h,\sigma)\,,h) + \gamma - g\,(k_t,h,\sigma) > 0$ for all admissible values of k it holds that $D\Delta < 0$ for all $\Delta_{t-1} > \Delta^{\Delta}(k)$ and, similarly, $D\Delta > 0$ for all $\Delta_{t-1} < \Delta^{\Delta}(k)$. These qualitative features are depicted in Figure 1.

7.2 Local Stability

The steady state is a fixed point of the system (3.6). To study the local behavior of the system around the steady state, we have to know the eigenvalues of the Jacobian matrix

$$D\,\phi(k^*, \Delta^*) \quad \equiv \quad \begin{bmatrix} \frac{\partial\phi^k}{\partial k_t} & \frac{\partial\phi^k}{\partial\Delta_{t-1}} \\ \frac{\partial\phi^{\Delta}}{\partial k_t} & \frac{\partial\phi^{\Delta}}{\partial\Delta_{t-1}} \end{bmatrix}. \tag{7.9}$$

We study each of the four elements of the Jacobian in turn.

- An application of the implicit function theorem to (3.4) shows that $\partial\phi^k/\partial k_t = NUM^k/DEN$, where

$$NUM^k \equiv \frac{\tilde{s}\,\alpha\, k_t^{\alpha-1}\left(1 + \theta\,\frac{i(g,h)}{1+g}\,\left(\frac{1}{\Delta_{t-1}} - 1\right)\right) - \tilde{s}\,k_t^{\alpha}\,\theta\,\frac{i_q\,g_k\,(1+g) - g_k\,i}{(1+g)^2}\left(\frac{1}{\Delta_{t-1}} - 1\right)}{\left(1 + \theta\,\frac{i(g,h)}{1+g}\,\left(\frac{1}{\Delta_{t-1}} - 1\right)\right)^2}; \tag{7.10}$$

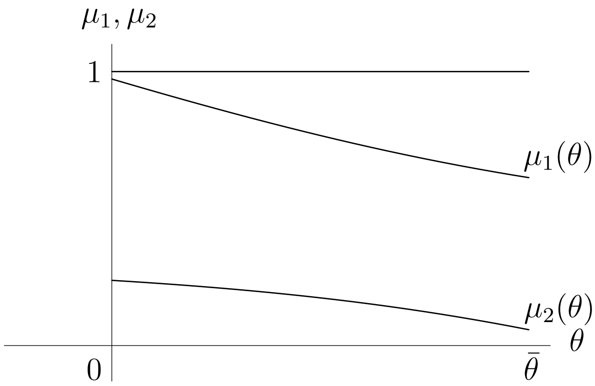

Figure 3: The Eigenvalues of the Jacobian (7.9) - A Typical Finding.

here the argument of g is (k_t, h, σ), and

$$DEN \equiv g_k\ (k_{t+1} + i) + (1+g)\,(1 + i_q\, g_k)\,, \tag{7.11}$$

where the argument of g is (k_{t+1}, h, σ) and the argument of i is (g, h). Evaluated at (k^*, Δ^*), NUM^k becomes

$$NUM^k = (1+g)\alpha\left(1 + \frac{i}{k^*}\right) - \left(\frac{k^*}{i} + 1\right)\left(\frac{1+g}{1+\gamma}\right) i_q\, g_k(\gamma - g) + (k^* + i)\,\frac{g_k(\gamma - g)}{(1+\gamma)} \tag{7.12}$$

It follows that $NUM^k/DEN < 1$ if and only if

$$(1+g)\left(\alpha\left(1 + \frac{i}{k^*}\right) - 1 - i_q\, g_k\right) - [+] - g_k\,(k^* + i)\left(1 - \frac{\gamma - g}{1+\gamma}\right) < 0. \tag{7.13}$$

In the steady state, we have from (3.8)

$$\alpha\left(1 + \frac{i}{k^*}\right) = \alpha\,\frac{k^* + i}{k^*} = \frac{\alpha\,\tilde{s}}{1+\gamma}\ (k^*)^{\alpha - 1}. \tag{7.14}$$

Hence, for the reason set out in the proof of Corollary 1, the first term in (7.13) is negative. Moreover, the last term is negative since $1 > (\gamma - g)/(1+\gamma)$. Hence $\partial\phi^k(k^*, \Delta^*)/\partial k_t < 1$.

- An application of the implicit function theorem to (3.4) also shows that $\partial\phi^k/\partial\Delta_{t-1} = NUM^\Delta/DEN$, where,

$$NUM^\Delta = \frac{s\,k_t^\alpha\left(\theta\,\frac{i(g,h)}{1+g}\,\frac{1}{\Delta_{t-1}^2}\right)}{\left(1 + \theta\,\frac{i(g,h)}{1+g}\,\left(\frac{1}{\Delta_{t-1}} - 1\right)\right)^2}, \tag{7.15}$$

where the argument of g is (k_t, h, σ). Evaluated at (k^*, Δ^*), NUM^Δ becomes

$$NUM^\Delta = (k^* + i)\left(\frac{1+g}{1+\gamma}\right)\frac{(\theta\, i + \gamma - g)^2}{\theta\, i}. \tag{7.16}$$

- From (3.5) we obtain

$$\frac{\partial\phi^\Delta}{\partial k_t} = \frac{\theta\, i_q\, g_k}{1+\gamma} + \frac{g_k - \theta\, i_q\, g_k}{1+\gamma}\,\Delta^*. \tag{7.17}$$

- From (3.5) we also have

$$\frac{\partial \phi^{\Delta}}{\partial \Delta_{t-1}} = \frac{1 + g - \theta\, i}{1 + \gamma}. \tag{7.18}$$

Figure 3 shows a typical result for both eigenvalues $\mu_1(\theta)$ and $\mu_2(\theta)$ which are strictly between zero and one and declining in θ. The calibration uses $h = 1$ and the investment requirement function $i = q^2$. Moreover, $\alpha = 1/3$, $\sigma = 0$, $\gamma = .14$, and $\tilde{s} = .3$. Hence, the steady state is locally stable.

References

Abramovitz, M. (1986): "Catching Up, Forging Ahead, and Falling Behind," *Journal of Economic History*, 46(2), 385–406.

Allen, T. J. (1977): *Managing the Flow of Technology.* MIT Press, Cambridge, MA.

Atkinson, A. B., and J. E. Stiglitz (1969): "A New View of Technological Change," *Economic Journal*, 79, 573–578.

Barro, R. J., and X. Sala-í-Martin (1997): "Technological Diffusion, Convergence and Growth," *Journal of Economic Growth*, 2(1), 1–26.

——— (2004): *Economic Growth.* MIT Press, Cambridge, MA, 2nd edn.

Bartel, A. P., and F. R. Lichtenberg (1987): "The Comparative Advantage of Educated Workers in Implementing New Technology," *The Review of Economics and Statistics*, 69, 1–11.

Basu, S., and D. Weil (1998): "Appropriate Technology and Growth," *Quarterly Journal of Economics*, 113, 1025–1054.

Becker, G. S. (1993): *Human Capital - A Theoretical and Empirical Analysis, with Special Reference to Education.* Chicago University Press, Chicago, 3rd edn.

Benhabib, J., and M. M. Spiegel (2005): "Human Capital and Technology Diffusion," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf, pp. 935–965. Elsevier, Amsterdam.

Bester, H., and E. Petrakis (2003): "Wage and Productivity Growth in a Competitive Industry," *Journal of Economic Theory*, 109, 52–69.

Cass, D. (1965): "Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, 32, 233–240.

Coe, D. T., and E. Helpman (1995): "International R&D Spillovers," *European Economic Review*, 39, 859–887.

Coe, D. T., E. Helpman, and A. Hoffmaister (2008): "International R&D Spillovers and Institutions," *CEPR Discussion Paper No. 6882.*

Cohen, W. M., and D. A. Levinthal (1989): "Innovation and Learning: The Two Faces of of R&D," *The Economic Journal*, 99.

Durlauf, S. N., and P. A. Johnson (1995): "Multiple Regimes and Cross Country Growth Behaviour," *Journal of Applied Econometrics*, 105(4), 365–384.

Eaton, J., and S. Kortum (1996): "Trade in Ideas: Patenting and Productivity in the OECD," *Journal of International Economics*, 40(3).

Galor, O., and O. Moav (2000): "Ability Biased Technological Transition, Wage Inequality, and Economic Growth," *Quarterly Journal of Economics*, 115, 469–498.

Gerschenkron, A. (1962): *Economic Backwardness in Historical Perspective: A Book of Essays.* Harvard University Press, Cambridge, MA.

Griffith, R., S. Redding, and J. V. Reenen (2004): "Mapping the Two Faces of R&D: Productivity Growth in a Panel of OECD Countries," *The Review of Economics and Statistics*, 86.

Hall, R. E., and C. I. Jones (1999): "Why do Some Countries Produce so Much More Output per Workers than Others?," *Quarterly Journal of Economics*, 114(1), 83–116.

Hellwig, M., and A. Irmen (2001): "Endogenous Technical Change in a Competitive Economy," *Journal of Economic Theory*, 101, 1–39.

Helpman, E. (2004): *The Mystery of Economic Growth*. Harvard University Press, Cambridge.

Heston, A., R. Summers, and B. Aten (2002): "Penn World Table Version 6.1," *Center for International Comparisons at the University of Pennsylvania*.

Howitt, P. (2000): "Endogenous Growth and Cross-Country Income Differences," *American Economic Review*, 90, 829–846.

Howitt, P., and D. Mayer-Foulkes (2005): "R&D, Implementation, and Stagnation: A Schumpeterian Theory of Convergence Clubs," *The Journal of Money, Credit and Banking*, 37, 147–177.

Irmen, A. (2005): "Extensive and Intensive Growth in a Neoclassical Framework," *Journal of Economic Dynamics and Control*, 29(8), 1427–1448.

Koopmans, T. (1965): "On the Concept of Optimal Economic Growth," in *The Economic Approach to Development Planning*. North Holland, Amsterdam.

Landes, D. S. (1998): *The Wealth and Poverty of Nations*. W. W. Norton and Company, New York.

Mincer, J. (1974): *Schooling, Experience, and Earnings*. Columbia University Press, New York.

Mokyr, J. (1990): *The Lever of Riches - Technological Creativity and Economic Progress*. Oxford University Press, Oxford.

Mowery, D. C. (1983): "The Relationship Between Intrafirm and Contractual Forms of Industrial Research in American Manufacturing, 1900-1940," *Explorations in Economic History*, 20, 351–374.

Nadiri, M. I., and S. Kim (1996): "International R&D Spillovers, Trade and Productivity in Major OECD Countries," *NBER Working Paper, No. 5801*.

Nelson, R. R., and E. S. Phelps (1966): "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, 56, 69–75.

O'Rourke, K. H., and J. G. Williamson (1999): *Globalization and History: The Evolution of a Nineteenth-Cenury Atlantic Economy*. MIT Press, Cambridge.

Parente, S. L., and E. C. Prescott (1994): "Barriers to Technology Adoption and Development," *Journal of Political Economy*, 102, 298–321.

Pritchet, L. (1997): "Divergence: Big Time," *Journal of Economic Perspectives*, 11(3), 3–17.

Quah, D. T. (1997): "Empirics for Growth and Distribution: Stratification, Polarization, and Convergence Clubs," *Journal of Economics Growth*, 2, 27–59.

Ramsey, F. P. (1928): "A Mathematical Theory of Savings," *Economic Journal*, 38, 543–559.

Sachs, J. D., and A. Warner (1995): "Economic reform and the process of Global Integration," *Brooking Papers on Economic Activity*, 195(1), 1–118.

Schultz, T. W. (1975): "The Value of the Ability to Deal with Disequilibria," *Journal of Economic Literature*, 13(3), 827–846.

Solow, R. M. (1956): "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70(1), 65–94.

Tilton, J. E. (1971): *International Diffusion of Technology: The Case of Semiconducters.* Brookings Institution, Washington, D.C.

Weil, D. N. (2005): *Economic Growth.* Pearson, Addison - Wesley, Boston.

Welch, F. (1970): "Education in Production," *Journal of Political Economy*, 78, 35–59.