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Population, Pensions, and Endogenous Economic Growth

Burkhard Heer and
Andreas Irmen

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POPULATION, PENSIONS, AND ENDOGENOUS ECONOMIC GROWTH

Burkhard Heer[1]

Free University of Bolzano-Bozen and CESifo, Munich

Andreas Irmen[2]

University of Heidelberg, CEPR, London, and CESifo, Munich

Abstract: We study the effect of a declining labor force on the incentives to engage in labor-saving technical change and ask how this effect is influenced by institutional characteristics of the pension scheme. When labor is scarcer it becomes more expensive and innovation investments that increase labor productivity are more profitable. We incorporate this channel in a new dynamic general equilibrium model with endogenous economic growth and heterogeneous overlapping generations. We calibrate the model for the US economy. First, we establish that the net effect of a decline in population growth on the growth rate of per-capita magnitudes is positive and quantitatively significant. Second, we find that the pension system matters both for the growth performance and for individual welfare. Third, we show that the assessment of pension reform proposals may be different in an endogenous growth framework as opposed to the standard framework with exogenous growth.

Keywords: Growth, Demographic Transition, Capital Accumulation, Pension Reform.

JEL-Classification: O41, C68, O11, D91, D31.

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[1]Free University of Bolzano-Bozen, School of Economics and Management, Via Sernesi 1, I-39100 Bolzano, Italy, Burkhard.Heer@unibz.it.

[2]University of Heidelberg, Department of Economics, Alfred-Weber-Institute, Grabengasse 14, D-69117 Heidelberg, Germany, airmen@uni-hd.de.

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1 Introduction

Population aging is one of the major economic challenges for today's industrialized societies. An increasing life expectancy in conjunction with declining birth rates tends to reduce the part of the population in working age and raises the part of the economically dependent old. For instance, the United Nations predicts for the US a decline in the working-age-to-population ratio, i. e., the fraction of the 20-64 year old to the 20-90 year old, from 83% in 2007 to 72% in 2075 (UN (2002)). To meet this challenge it is necessary to understand the economic consequences of such a demographic change.

This paper studies the economic consequences of the link between demographic changes and the incentives of firms to engage in innovation investments that affect total factor productivity (TFP). With a focus on the US economy and the time span between 1950 and 2200, we argue that this link may substantially modify the predicted *positive* and *normative* implications of actual policy reform proposals of the pension scheme. We obtain these implications in a new dynamic general equilibrium model with endogenous economic growth and heterogeneous overlapping generations in the spirit of Auerbach and Kotlikoff (1987).

We calibrate our model for the US economy and establish the following major results. First, we find that the net effect of a decline in population growth on the growth rate of per-capita magnitudes such as income per capita and consumption per capita is positive and quantitatively significant. For instance, under the current pension system the growth rate of per-capita income is predicted to rise from 1.8% over the period 1990-2000 to approximately 2.28% in 2100. Second, we show that the pension system indeed matters both for the growth performance of the economy and for the welfare of each generation. Finally, we show that the assessment of pension reform proposals differs in an endogenous growth framework as opposed to the standard framework where TFP growth is exogenous and costless.

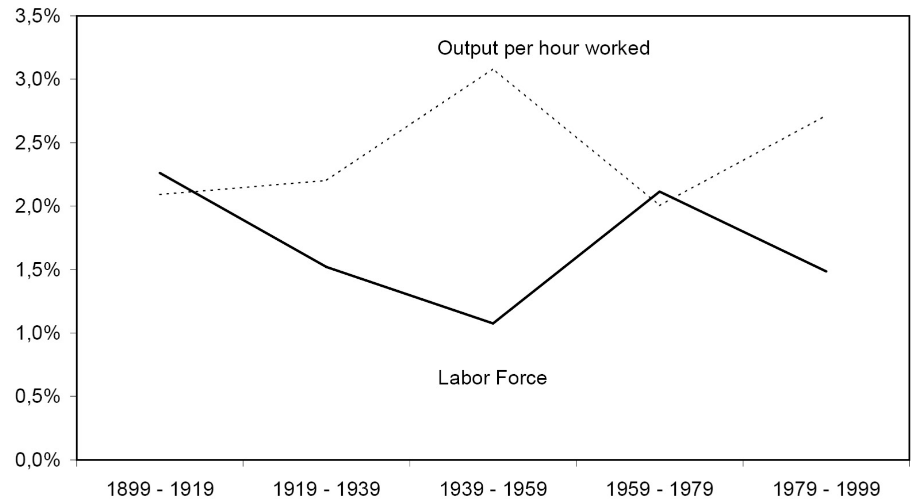
Our analytical framework has a supply side that satisfies two requirements which prove useful for our research agenda. First, endogenous growth is obtained in a competitive neoclassical economy where TFP growth is the result of labor-saving innovation investments of profit-maximizing firms and capital accumulation is subject to diminishing returns. Analytically, this economy can readily be turned into the standard neoclassical growth model of Solow (1956) with exogenous and costless technical change. This property is particularly useful for our *comparative dynamic analysis* that identifies the role of endogenous growth by comparing the predicted evolution of the US economy under endogenous and exogenous growth. Second, there is a single channel linking the demographic trend to innovation activities. This parsimony allows for clear-cut intuitions when we compare policy reform proposals under endogenous and exogenous growth.[1]

A mechanism often attributed to Hicks (1932) provides the link between the demographic trend and TFP growth. According to Hicks' argument, a demographic transition that reduces the labor force may render labor scarcer relative to capital. Accordingly, the relative price of labor increases and so does the profitability of innovation investments that raise labor productivity and TFP.

[1]We do not want to dispute the possible presence of other channels that may strengthen or weaken the mechanism expounded here. However, the analytical and the quantitative analysis of such interactions is quite involved. We leave this question for future research and shall get back to this point in the concluding Section 7.

Figure 1 provides evidence for the US economy in support of Hicks' intuition. It shows the long-run relationship between the average annual growth rates of the labor force and of output per hour worked in the US for five successive twenty-year intervals covering the 20th century.[2] The striking feature is the strong inverse relationship between both magnitudes: when the growth rate of the labor force declines then the growth rate of output per hour worked increases.

Figure 1: 20th Century U.S. Labor Force and Productivity Growth



We interpret this negative correlation as the result of two reinforcing effects. First, reduced labor force growth weakens the effect of capital dilution, i. e., a given amount of capital implies a higher capital intensity and a higher labor productivity. We follow Cutler, Poterba, Sheiner, and Summers (1990) and refer to this channel as the *Solow effect*. Second, since capital and labor are complements and capital accumulation is subject to diminishing returns, firms that anticipate a higher capital intensity due to the Solow effect will also expect a higher real wage and a lower real interest rate. Both price movements strengthen the incentives of such firms to engage in labor-saving innovation investments that raise labor productivity and TFP. We refer to this second channel as the *Hicks effect*.[3]

The relevance of Hicks' argument hinges on the economy's ability to accumulate capital, i. e., on the propensity of a declining population in working age to save. Hence, to account for the Hicks effect we have to allow firms to adjust their innovation investments and households to adjust their savings decision to a changing demographic environment. To provide for such an analytical framework we

[2] Our Figure 1 replicates and extends Figure 1 in Romer (1987), p. 182, by showing that the inverse relationship between both growth rates persists for the period 1979-1999. See Appendix 8.1 for detailed information concerning the data used.

[3] Additional evidence supporting Hicks' intuition is provided by Cutler, Poterba, Sheiner, and Summers (1990). These authors' cross-country regressions establish a negative and significant partial correlation between productivity growth and labor force growth. Moreover, several cross-country growth regressions find a negative and significant partial correlation between population growth and growth of per-capita GDP (see, e. g., Barro and Sala-í-Martin (2004), Kelley and Schmidt (1995), and Kormendi and Meguire (1985)). In a famous study, Habakkuk (1962) uses ideas similar to the Hicksian argument to explain why technological progress in the 19th century was faster in the US than in Britain.

combine the production sector of Irmen (2005) with a household sector comprising heterogeneous overlapping generations of Auerbach and Kotlikoff (1987).[4] Since the pension system is a major determinant of an economy's propensity to save, this framework is well suited for our *comparative institutional analysis* that asks how institutional characteristics of the pension scheme affect the incentive to engage in labor-saving technical change during a demographic transition.

Our research builds on and contributes to a recent and growing literature that extends the seminal framework of Auerbach and Kotlikoff (1987) to analyze the economic consequences of population aging in different settings. To the best of our knowledge, the existing literature does neither incorporate the endogenous response of firms to the demographic transition through the Hicks effect, nor does it provide an analysis of the consequences of policy reforms in endogenous growth economies.[5] The present study is a first step to fill this gap.

Important existing contributions study the consequences of policy reform proposals of the social security system in closed economies. For instance, Huang, İmrohoroğlu, and Sargent (1997), Conesa and Krüger (1999), and Fuster, İmrohoroğlu, and İmrohoroğlu (2007) deal with the transition from an unfunded to a more funded pension scheme, where the latter two emphasize political economy aspects of such a switch. Closer to our analysis are the studies of İmrohoroğlu, İmrohoroğlu, and Joines (1995) and de Nardi, İmrohoroğlu, and Sargent (1999) who focus on reform scenarios within the pay-as-you-go system and assess their welfare implications. With respect to their results, our findings suggests that i) the welfare ranking of reform proposals may depend on whether economic growth is exogenous as in their papers or endogenous as in our study, and ii) the welfare effects are much larger if economic growth is endogenous.

This paper is organized as follows. Section 2 presents the model and establishes the stationary equilibrium. Section 3 develops important channels that drive our calibration results in a simplified setting with two-period lived individuals. Section 4 provides the detailed description of our calibration strategy. Our main findings concerning the interplay between the demographic transition and economic growth appear in Section 5. We begin in Section 5.1 with the analysis of a benchmark economy where the replacement rate of the pension system is constant and the growth rate of TFP is endogenous. We establish that the feedback from population growth onto innovation incentives generates faster productivity growth. Section 5.2 contains our comparative dynamical analysis. It studies the evolution of the benchmark economy under the assumption of costless exogenous growth at the trend growth rate of the US. Here, we show that accounting for the Hicks effect substantially increases the growth rate of per-capita GDP. In Section 6, we turn to the comparative institutional analysis of pay-as-you-go pensions schemes with different characteristics. Section 6.1 considers three pension reform proposals and studies their implications for economic growth. Section 6.2 deals with the welfare implications of these reform proposals, both under endogenous and exogenous growth. Section 7 concludes. Proofs are relegated to the Appendix.

[4]The production side of Irmen (2005) satisfies the aforementioned attributes. It extends the production sector employed by Hellwig and Irmen (2001) which in turn builds on Bester and Petrakis (2003).

[5]A previous version of Ludwig, Schelke, and Vogel (2008) included an analysis of endogenous growth through endogenous human capital investments of the households and a growth mechanism along the lines of Lucas (1988). These authors do not compare different reform proposals of the pension scheme.

2 The Model

This section describes our large-scale OLG model with heterogeneous agents in the spirit of Auerbach and Kotlikoff (1987) and a production sector extending the setup studied in Irmen (2005).

There is final-good sector and an intermediate-good sector. Intermediate-good firms undertake innovation investments that increase the productivity of their workers. These productivity gains drive the evolution of the economy's level of technological knowledge and its TFP. Intermediates and capital produce final output that can be consumed or invested. If invested, the current final good may either serve as future capital or as an input in innovation investments. In each period, prices are expressed in units of the current final good.

The government collects taxes, accidental bequests, and social security contributions. In all periods, the budget of the government and the one of the pension scheme are balanced.

2.1 Demographics and Timing

A period, t, corresponds to one year. At each t, a new generation of households is born. Newborns have a real life age of 20 denoted by $s = 1$. All generations retire at age 65 ($s = R = 46$) and live up to a maximum age of 94 ($s = J = 75$). At t, all agents of age s survive until age $s + 1$ with probability $\phi_{t,s}$ where $\phi_{t,0} = 1$ and $\phi_{t,J} = 0$.

Let $N_t(s)$ denote the number of agents of age s at t. The population $N_t(s)$ of the periods $t = 0, 1, ..., 450$ is calibrated as the actual and predicted US population of the years 1950 to 2400 using the data of Krüger and Ludwig (2007).

2.2 Households

Each household comprises one worker. Households maximize intertemporal utility at the beginning of age 1 in period t

$$\max \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1}\right) \; u(c_{t+s-1}(s), l_{t+s-1}(s)), \tag{1}$$

where $\beta > 0$ denotes the discount factor, and per-period utility $u(c, l)$ is a function of consumption c and labor supply l

$$u(c, l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\theta} - 1}{1-\theta}, \quad \theta > 0, \quad \gamma \in (0, 1). \tag{2}$$

Households are heterogeneous with regard to their age, s, their individual labor efficiency, $e(s, j)$, and their wealth, ω. We stipulate that an agent's efficiency $e(s, j) = \bar{y}_s \epsilon_j$ depends on its age, $s \in \mathcal{S} \equiv \{1, 2, ..., 75\}$ and its efficiency type, $\epsilon_j \in \mathcal{E} \equiv \{\varepsilon_1, \varepsilon_2\}$. The purpose is to capture composition effects that, e. g., Gómez and Hernández de Cos (2008) find relevant in explaining cross-country growth differences. We choose the age-efficiency profile, $\{\bar{y}_s\}$, in accordance with the US wage profile. The permanent efficiency types ϵ_1 and ϵ_2 are meant to capture differences in education and ability. We use Γ to denote the unique invariant distribution of $\epsilon_j \in \mathcal{E}$.

The net wage income in period t of an s-year old household with efficiency type j is given by $(1-\tau_w-\tau_{b,t})w_t e(s,j)l_t(s)$, where w_t denotes the wage rate per efficiency unit in period t. The wage income is taxed at the constant rate τ_w. Furthermore, the worker has to pay contributions to the pension system at rate $\tau_{b,t}$ which may vary over time depending on the pension scheme. A retired worker receives pensions $b(s,j)$ that depend on his efficiency type j. Clearly, $b(s,j)=0$ for $s<R$.

Households are born without assets at the beginning of age $s=1$, hence $\omega_t(1)=0$. Parents do not leave bequests to their children, and all accidental bequests are confiscated by the government. The household earns interest r_t on his wealth $\omega_t \in \mathbb{R}$. Capital income is taxed at the constant rate τ_r. In addition, households receive lump-sum transfers tr_t from the government. As a result, the budget constraint at t of an s-year old household with productivity type j and wealth ω_t is:

$$
\begin{aligned}
b_t(s,j) + (1-\tau_w-\tau_{b,t})w_t e(s,j)l_t(s) + \left[1+(1-\tau_r)r_t\right]\omega_t(s) + tr_t \\
= c_t(s) + \omega_{t+1}(s+1).
\end{aligned}
\tag{3}
$$

2.3 Firms

Firms belong either to the final-good or to the intermediate-good sector. Both sectors are competitive.

2.3.1 Final Goods

The measure of all final-good firms is equal to one. At each t, firms produce output, Y_t, according to the following Cobb-Douglas production function

$$Y_t = K_t^{\alpha} X_t^{1-\alpha}, \quad 0<\alpha<1, \tag{4}$$

where K_t is the capital input at t and X_t denotes the amount of the intermediate good used in period-t production. Let p_t denote the price of the intermediate good at t, and $\delta \in [0,1]$ the rate at which capital depreciates within periods. Firms rent capital from households and return $1+r_t$ units of their output per unit of rented capital. Moreover, they pay $p_t X_t$ to the intermediate-goods producers. Hence, per-period profits are

$$Y_t - r_t K_t - p_t X_t - \delta K_t. \tag{5}$$

Firms take the sequence of prices $\{r_t, p_t\}$ as given and maximize the sum of the present discounted values of profits in all periods. This is equivalent to a series of one-period maximization problems and gives rise to the first-order conditions

$$\frac{\partial Y_t}{\partial K_t} = r_t + \delta = \alpha \tilde{K}_t^{\alpha-1}, \tag{6}$$

$$\frac{\partial Y_t}{\partial X_t} = p_t = (1-\alpha)\tilde{K}_t^{\alpha}, \tag{7}$$

where $\tilde{K}_t \equiv K_t/X_t$ denotes the capital intensity in the final-good sector in period t.

2.3.2 Intermediate Goods

The set of all intermediate-good firms is represented by the set $\mathbb{R}_+$ of nonnegative real numbers with Lebesgue measure.

Technology All firms have access to the same technology and face a capacity limit of 1.[6] The output of the intermediate good, x_t, is given by

$$x_t = \min\left\{1, a_t\, l_t^d\right\}, \tag{8}$$

where a_t and l_t^d denote the firm's labor productivity and its labor input, respectively. Firms hire effective labor, l_t^d, defined as the product of the efficiency factor $e(s,j)$ and the 'number' of working hours $l_t(s)$. Labor is assumed to be divisible between firms.

The individual firm's labor productivity at t depends on both the indicator of the economy-wide level of labor productivity A_{t-1} of period $t-1$ and its individual productivity growth rate q_t according to

$$a_t = A_{t-1}(1+q_t). \tag{9}$$

In order to achieve productivity growth at rate q_t, the firm must invest $i(q_t)$ units of the final good in period $t-1$. The input requirement function is increasing and strictly convex in q_t. More specifically, we stipulate

$$i(q) = v_0 q^v, \quad \text{with } v_0 > 0 \text{ and } v > 1. \tag{10}$$

Following Hellwig and Irmen (2001), the innovation reflected in a_t is proprietary knowledge of the firm only for production in period t; afterwards it becomes common knowledge.[7] The evolution of A_t over time is described below.

Profit Maximization and Zero-Profits A firm's innovation investment fully depreciates after one year. Therefore, the cost of an innovation investment undertaken at $t-1$ in units of the final good at t is $(1+r_t)i(q_t)$. With $w_t l_t^d$ denoting a firm's wage bill, a production plan $\{x_t, l_t^d, q_t\}$ for period t yields the profit

$$\pi_t = p_t\, x_t - w_t\, l_t^d - (1+r_t)\, i(q_t). \tag{11}$$

[6] The capacity limit of 1 is a shortcut that has no bearing on the evolution of any magnitude of interest in the present model. It may be endogenized along the lines set out in Hellwig and Irmen (2001).

[7] A consequence of this assumption is that an intermediate-good firm can produce at t with a technology involving $a_t = A_{t-1}$ if it decides not to undertake an innovation investment in $t-1$. However, this possibility does not arise in equilibrium.

Firms take the sequence of real prices $\{p_t, w_t, r_t\}$ as well as the sequence of aggregate productivity indicators $\{A_{t-1}\}$ as given and maximize the sum of the present discounted values of profits in all periods. Since production choices for different periods are independent of each other, for each t they choose the plan $\{x_t, l_t^d, q_t\}$ to maximize the profits π_t from this plan.

If a firm innovates then it incurs an investment cost $(1 + r_t)\, i(q_t)$ that is independent of the level of output. Hence, an innovation investment can only be profit-maximizing if the margin per unit of output is strictly positive, i. e., $p_t - w_t/a_t > 0$. Since this margin is independent of the amount of produced output, an innovating firm produces the capacity output $x_t = 1$. Accordingly, (8) delivers its demand for effective labor as

$$l_t^d = \frac{1}{A_{t-1}(1 + q_t)}. \tag{12}$$

Upon substitution of the latter in (11), the profit-maximizing productivity growth rate is

$$\hat{q}_t = \arg\max_{q \geq 0} \left[p_t - \frac{w_t}{A_{t-1}(1 + q)} - (1 + r_t)\, i(q) \right]. \tag{13}$$

The corresponding first-order condition balances the advantage of a lower wage bill and the disadvantage of a higher investment cost. It is sufficient for an interior solution and given by

$$\frac{w_t}{A_{t-1}(1 + \hat{q}_t)^2} = (1 + r_t)\, i_q(\hat{q}_t) = (1 + r_t)\, v_0\, v\, \left(\hat{q}_t\right)^{v-1}. \tag{14}$$

This condition shows that, indeed, the incentive to engage in labor saving innovation investments depends on (relative) factor prices. If firms face a higher wage, then, ceteris paribus, an increase in q_t implies a larger reduction in the wage bill. Similarly, a higher real interest rate reduces innovation incentives since, ceteris paribus, it raises marginal investment costs. This is the sense in which profit-maximizing firms adjust their innovation investments to changing factor prices as conjectured by Hicks: if $w_t/(1 + r_t)$, i. e., the relative price of labor, increases they invest more.

2.3.3 Consolidating the Production Sector

Denote n_t the measure of the set of all active firms in the intermediate-good sector at t. Since all firms have access to the same technology and face identical prices, all active firms choose the same production plan $\{1, 1/\left(A_{t-1}\left(1 + \hat{q}_t\right)\right), \hat{q}_t\}$.

In equilibrium, active intermediate-good firms must earn zero profits, i. e.,[8]

$$\pi_t\left(\hat{q}_t; p_t, w_t, r_t, A_{t-1}\right) = 0. \tag{15}$$

The following lemma shows that the conditions for profit-maximization and zero-profits in the final-good and the intermediate-good sector relate the growth rate of labor productivity $\hat{q}_t$ to the capital intensity in the final-good sector, $\tilde{K}_t$.

[8] This follows since the labor supply in each period is bounded. Therefore, the set of producing intermediate-good firms that employ more than some $\epsilon > 0$ units of effective labor must have bounded measure, i. e., it must be smaller than the set of all intermediate-good firms. Since inactive intermediate-good firms must be maximizing profits just like active ones, we need that maximum profits of intermediate-good firms at equilibrium prices are equal to zero. See Hellwig and Irmen (2001) for more details.

Lemma 1 *If all firms in the economy maximize profits and earn zero-profits, i. e., equations (6), (7), (14) and (15) hold, then there is a map* $g : \mathbb{R}_+ \to \mathbb{R}_+$ *such that* $\hat{q}_t = g\left(\tilde{K}_t\right)$ *with* $g(0) = 0$, $\lim_{\tilde{K}_t \to \infty} g\left(\tilde{K}_t\right) = \infty$, *and* $g'\left(\tilde{K}_t\right) > 0$.

Lemma 1 provides the link between the equilibrium incentive to innovate and the capital intensity. Intuitively, profit-maximization in the final-good sector as stated in (6) and (7) means that a higher $\tilde{K}_t$ lowers the real interest rate and raises the price of the intermediate good. Both price movements raise the wage that is consistent with zero-profits in the intermediate-good sector. The function $g\left(\tilde{K}_t\right)$ reflects the effect of these price movements onto the profit-maximizing level of innovation activity implied by (14). We shall later see that in equilibrium the capital intensity, $\tilde{K}_t$, is a measure of the relative abundance of capital with respect to efficient labor. Thus, Lemma 1 links the relative abundance of factors to innovation incentives as envisaged by Hicks: if capital is abundant relative to efficient labor then the incentive to engage in labor-saving innovation investments is high.

Turning to the evolution of the economy-wide indicator of technological knowledge, we assume that $A_t = \max\{a_t(n) = A_{t-1}\,(1 + \hat{q}_t(n))\,|\, n \in [0, n_t]\}$. Since in equilibrium the productivity of all producing intermediate-good firms is $a_t = A_{t-1}(1 + \hat{q}_t)$, we have $A_t = a_t$ and

$$A_t = A_{t-1}\,(1 + \hat{q}_t). \tag{16}$$

Moreover, in equilibrium the aggregate demand for innovation investment at $t-1$ is $n_t i(\hat{q}_t)$, the aggregate demand for effective labor at t is $n_t l_t^d = n_t / \left(A_{t-1}\,(1 + \hat{q}_t)\right)$, and the aggregate supply of the intermediate good at t is n_t.

2.4 Government

The government collects income taxes T_t in order to finance its expenditure on government consumption G_t and transfers Tr_t. In addition, it confiscates all accidental bequests Beq_t. The government budget is balanced in every period t, i. e.,

$$G_t + Tr_t = T_t + Beq_t. \tag{17}$$

In view of the tax rates τ_w and τ_r, the government's tax revenue is

$$T_t = \tau_w w_t L_t + \tau_r r_t \Omega_t, \tag{18}$$

where Ω_t is aggregate wealth at t.

Government spending is a constant fraction of final-good output

$$G_t = \bar{g}\, Y_t. \tag{19}$$

2.5 Social Security

The social security system is a pay-as-you-go system. The social security authority collects contributions from the workers to finance its pension payments to the retired agents. Pensions are a constant fraction of the net labor income of the productivity type j

$$b_t(s,j) = \begin{cases} 0 & s < R \\ \zeta(1-\tau_w-\tau_{b,t})w_t\epsilon_j\bar{l}_t & s \geq R, \end{cases} \tag{20}$$

where $\bar{l}_t$ denotes the average hourly labor supply of all workers in period t.

In equilibrium, the social security budget is balanced and will be defined below. As to the institutional characteristics of the pension scheme we will distinguish three different scenarios during the transition:

1. constant replacement ratio: $\zeta_t \equiv \frac{b_t}{(1-\tau_w-\tau_b)w_t\bar{l}_t} = \zeta$,
2. constant contribution rate starting in period t_0: $\tau_{b,t} = \tau_{b,t_0}$ for $t \geq t_0$,
3. constant replacement ratio and later retirement at age 70, i. e. $R = 51$.

2.6 Stationary Equilibrium

In the stationary equilibrium, individual behavior is consistent with the aggregate behavior of the economy, firms maximize profits, households maximize intertemporal utility, and factor and goods' markets clear. To express the equilibrium in terms of stationary variables only, we have to divide aggregate quantities by X_t and individual variables and prices by A_t. Therefore, we define the following stationary aggregate variables

$$\tilde{Beq}_t \equiv \frac{Beq_t}{X_t}, \quad \tilde{T}_t = \frac{T_t}{X_t}, \quad \tilde{G}_t = \frac{G_t}{X_t}, \quad \tilde{\Omega}_t = \frac{\Omega_t}{X_t}, \quad \tilde{C}_t = \frac{C_t}{X_t}, \quad \tilde{Y}_t = \frac{Y_t}{X_t},$$

and stationary individual variables

$$\tilde{c}_t \equiv \frac{c_t}{A_t}, \quad \tilde{w}_t \equiv \frac{w_t}{A_t}, \quad \tilde{b}_t \equiv \frac{b_t}{A_t}, \quad \tilde{\omega}_t \equiv \frac{\omega_t}{A_t}, \quad \tilde{tr}_t \equiv \frac{tr_t}{A_t}.$$

Let f_t denote the cross-section measure of households in period t.

A *stationary equilibrium* for a government policy $\{\tau_r, \tau_w, \tau_{b,t}, \bar{g}, \zeta_t, tr_t\}$, an initial value $A_0 > 0$, and initial measures f_0 in period 0 corresponds to a price system, an allocation, and a sequence of aggregate productivity indicators $\{A_t\}$ that satisfy the following conditions:

1. Population grows at the rate $\lambda_t = \frac{N_{t+1}}{N_t} - 1$.
2. The 'number' of intermediate-good firms, n_t, is equal to the total output of intermediates as each firms produces one unit of output, $x_t = 1$

$$n_t = X_t. \tag{21}$$

3. The aggregate productivity indicator, A_t evolves according to (16).

4. Labor market equilibrium: Aggregate efficient labor supply, L_t, is equal to the aggregate labor demand of all intermediate-good firms

$$L_t = n_t l_t^d. \tag{22}$$

5. Capital market equilibrium: aggregate wealth is equal to aggregate capital plus aggregate innovation investment

$$\Omega_t = K_t + n_t i_t(q_t).$$

6. Households maximize the intertemporal utility (1) subject to the budget constraint

$$\begin{aligned}&\tilde{b}_t(s,j) + (1-\tau_w-\tau_{b,t})\,\tilde{w}_t\, e(s,j)\, l_t(\tilde{\omega},s,j) + [1+(1-\tau_r)r_t]\,\tilde{\omega}_t + \tilde{t}r_t \\ &\qquad = \tilde{c}_t(s) + \tilde{\omega}_{t+1}(1+q_{t+1}).\end{aligned}$$

This gives rise to the first-order conditions

$$\frac{1-\gamma}{\gamma}\frac{\tilde{c}_t(s)}{1-l_t(s)} = (1-\tau_w-\tau_{b,t})\tilde{w}_t e(s,j), \tag{23}$$

$$\begin{aligned}\gamma\tilde{c}_t(s)^{\gamma(1-\theta)-1}(1-l_t(s))^{(1-\gamma)(1-\theta)} &= \beta(1+q_{t+1})^{\gamma(1-\theta)-1}\phi_{t,s}\,[1+(1-\tau_r)r_{t+1}] \\ &\quad\cdot\ \gamma\tilde{c}_{t+1}(s+1)^{\gamma(1-\theta)-1}(1-l_{t+1}(s+1))^{(1-\gamma)(1-\theta)}.\end{aligned} \tag{24}$$

Individual labor supply $l_t(\tilde{\omega},s,j)$, consumption $\tilde{c}_t(\tilde{\omega},s,j)$ and optimal next period assets $\tilde{\omega}_t'(\tilde{\omega},s,j)$ of period t are functions of the individual state variables $\tilde{\omega}$, j, and s, and also depend on the period t.

7. Firms maximize profits satisfying (6), (7) and (14). In equilibrium, firm profits are zero. Using (16), the zero-profit condition (15) can be rewritten as follows

$$\tilde{w}_t = (1-\alpha)\tilde{K}_t^{\alpha} - (1+r_t)i(q_t). \tag{25}$$

8. Aggregate variables are equal to the sum of the individual variables

$$\begin{aligned}L_t &= \int e(s,j)l_t(\tilde{\omega},s,j)\ f_t(d\tilde{\omega}\times ds\times dj), \\ \tilde{\Omega}_t &= \tilde{K}_t + i(q_t) = \frac{1}{L_t}\int \tilde{\omega} f_t(d\tilde{\omega}\times ds\times dj), \\ \tilde{B}eq_{t+1} &= \frac{A_t}{X_{t+1}}\int (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{\omega}_t'(\tilde{\omega},s,j)\ f_t(d\tilde{\omega}\times ds\times dj) \\ &= \frac{1}{L_{t+1}}\frac{1}{1+q_{t+1}}\int (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{\omega}_t'(\tilde{\omega},s,j)\ f_t(d\tilde{\omega}\times ds\times dj), \\ \tilde{C}_t &= \frac{1}{L_t}\int \tilde{c}_t(\tilde{\omega},s,j)\ f_t(d\tilde{\omega}\times ds\times dj), \\ \tilde{T}_t &= \tau_w\tilde{w}_t + \tau_r r_t(\tilde{K}_t + i(q_t)).\end{aligned}$$

9. The government budget is balanced

$$\bar{g}\tilde{K}_t^{\alpha} + \tilde{tr}_t \frac{N_t}{L_t} = \tilde{T}_t + \tilde{B}eq_t. \tag{26}$$

10. The budget of the social security system is balanced

$$\frac{1}{L_t}\int \tilde{b}_t(s,j)\, f_t(d\tilde{\omega} \times ds \times dj) = \tau_{b,t}\tilde{w}_t, \tag{27}$$

11. The market for the final good clears.

12. The cross-sectional measure f_t evolves as

$$f_{t+1}(\tilde{\mathcal{W}} \times \mathcal{S} \times \mathcal{E}) = \int P_t\left((\tilde{\omega}, s, j), \tilde{\mathcal{W}} \times \mathcal{S} \times \mathcal{E}\right)\, f_t(d\tilde{\omega} \times ds \times dj)$$

for all sets $\tilde{\mathcal{W}}$, $\mathcal{S}$, $\mathcal{E}$, where the Markov transition function P_t is given by

$$P_t\left((\tilde{\omega}, s, j), \tilde{\mathcal{W}} \times \mathcal{S} \times \mathcal{E}\right) = \begin{cases} \phi_t(s) & \text{if } \tilde{\omega}_t'(\tilde{\omega}, s, j) \in \tilde{\mathcal{W}}, \\ & j \in \mathcal{E}, s+1 \in \mathcal{S} \\ 0 & \textit{else}, \end{cases}$$

and for the newborns

$$f_{t+1}(\tilde{\mathcal{W}} \times 1 \times \mathcal{E}) = N_{t+1}(1) \cdot \begin{cases} \Gamma(\mathcal{E}) & \text{if } 0 \in \tilde{\mathcal{W}} \\ 0 & \textit{else}. \end{cases}$$

3 Population, Pensions, and Endogenous Economic Growth in a Simplified Setting

In this section, we show that the relationship between population growth and endogenous economic growth depends on the characteristics of the pay-as-you-go pension scheme. We make this point is a simplified setting that combines the production sector of Section 2.3 with a household sector à la Diamond (1965). We establish the existence of a steady state and study the steady-state response of the economy's growth rate to a decline in its population growth rate. We find that the rise of the steady-state growth rate caused by a decline in population growth is larger under a pension scheme that keeps the contribution rate constant than under a scheme with a constant replacement rate. We shall see in Section 6 that the calibration exercises bring out these qualitative findings, too.

For ease of exposition, we set the depreciation rate equal to unity and assume $\upsilon_0 = 1$ and $\upsilon = 2$. On the household side, we consider two-period lived households. Generations comprise homogeneous individuals. The labor supply when young is exogenous and normalized to one, the one when old is zero. The population growth rate, λ, is constant over time and coincides with the growth rate of aggregate labor supply, i. e., $\lambda = L_{t+1}/L_t - 1$.

3.1 Households and the Pay-As-You-Go Pension Scheme

Lifetime utility of a member of cohort t is

$$\ln c_t^y + \beta \ln c_{t+1}^o, \quad \beta \in (0,1), \tag{28}$$

where c_t^y and c_{t+1}^o denote per-capita consumption when young and old, respectively. The maximization of (28) is subject to the per-period budget constraints

$$c_t^y + s_t = (1-\tau_b)w_t, \quad \text{and} \quad c_{t+1}^o = s_t(1+r_{t+1}) + b_{t+1}, \tag{29}$$

where b_{t+1} denotes the expected benefit when old. Optimal savings of a young at t results as

$$s_t = \frac{\beta}{1+\beta}(1-\tau_b)w_t - \frac{1}{1+\beta}\left(\frac{b_{t+1}}{1+r_{t+1}}\right). \tag{30}$$

First, we consider a pay-as-you-go system with a constant contribution rate $\tau_b \in (0,1)$. Leaving intragenerational heterogeneity and taxes on wage income aside, the benefits at $t+1$ implied by the balanced budget satisfy $b_{t+1}\, L_t = L_{t+1}\tau_b\, w_{t+1}$ such that

$$b_{t+1} = (1+\lambda)\,\tau_b\, w_{t+1}. \tag{31}$$

3.2 The Dynamical System

We use $\tilde{K}_t$ as the state variable of the dynamical system. To express the savings of the young as a function of this state variable we employ Lemma 1 to find the equilibrium real wage as

$$w_t \;=\; (1-\alpha)\tilde{K}_t^{\alpha}\, A_{t-1}\left(1+g\left(\tilde{K}_t\right)\right)\left[1-\left(\frac{\alpha}{1-\alpha}\right)\frac{i\left(g\left(\tilde{K}_t\right)\right)}{\tilde{K}_t}\right]. \tag{32}$$

Accordingly, a rise in the capital intensity $\tilde{K}_t$ affects the wage in three ways. First, a higher $\tilde{K}_t$ raises final-good output. Second, a rise in $\tilde{K}_t$ renders labor scarcer such that the productivity growth rate $g(\tilde{K}_t)$ rises. Third, faster productivity growth increases investment outlays. The first two channels increase the wage, the last decreases it.[9] Observe that (32) gives the same real wage as for a neoclassical production sector with costless exogenous productivity growth if we set $g\left(\tilde{K}_t\right) = g > 0$ and $i(.) = 0$.

Expressed in units of efficient labor at t we have

$$\frac{w_t}{A_{t-1}\left(1+g\left(\tilde{K}_t\right)\right)} \;=\; \tilde{K}_t^{\alpha} - \alpha\,\tilde{K}_t^{\alpha-1}\left(\tilde{K}_t + i\left(g\left(\tilde{K}_t\right)\right)\right) \equiv \tilde{w}\left(\tilde{K}_t\right). \tag{33}$$

[9] See Appendix 8.2.2 for the proof of the last part of this statement.

In these units, the real wage at t is equal to the difference between final-good output and overall debt service payments.

In equilibrium, the savings of the young must be equal to aggregate investment, i. e.,

$$s_t\,L_t = K_{t+1} + n_{t+1}\,i\,(q_{t+1})\,. \tag{34}$$

Using Lemma 1, optimal savings (30), the budget of the pension scheme (31), equilibrium wages (32) for periods t and $t+1$, and the full employment condition (22) in (34) gives the equation of motion for $\tilde{K}_t$ as

$$\xi_1\,\tilde{K}_t^{\alpha}\left(1-\frac{\alpha}{1-\alpha}\frac{i\left(g\left(\tilde{K}_t\right)\right)}{\tilde{K}_t}\right) = \left(1+g\left(\tilde{K}_{t+1}\right)\right)\left[\xi_2\,\tilde{K}_{t+1}+\xi_3\,i\left(g\left(\tilde{K}_{t+1}\right)\right)\right], \tag{35}$$

where

$$\xi_1 \equiv \frac{\beta(1-\alpha)(1-\tau_b)}{(1+\beta)(1+\lambda)} \in (0,1), \tag{36}$$

$$\xi_2 \equiv 1+\frac{\tau_b}{1+\beta}\frac{1-\alpha}{\alpha} > 1, \tag{37}$$

$$\xi_3 \equiv 1-\frac{\tau_b}{1+\beta} \in (0,1). \tag{38}$$

The dynamical system of the economy is given by (35) in conjunction with an initial value $\tilde{K}_0 > 0$. The three statements of the following theorem establish the key results that we need for the analysis of the link between population, pensions, and endogenous economic growth.

Theorem 1 *Let* $\alpha > 1/3$.

1. *Equation (35) gives rise to a function* $\tilde{K}_{t+1} = \phi\left(\tilde{K}_t\right)$ *with* $\phi'\left(\tilde{K}_t\right) > 0$ *for all* $\tilde{K}_t > 0$.
2. *There is at least one locally stable steady state* $\tilde{K}^* = \phi\left(\tilde{K}^*\right) > 0$.
3. *There is a function* $\tilde{K}^* = \tilde{K}^*\left(\lambda, \tau_b\right)$ *with* $\partial\tilde{K}^*/\partial\lambda < 0$ *and* $\partial\tilde{K}/\partial\tau_b < 0$.

Statement 1 assures a unique equilibrium value $\tilde{K}_{t+1}$ given $\tilde{K}_t$.[10] It guarantees that $\tilde{w}\left(\tilde{K}_t\right)$ is increasing on $\mathbb{R}_{++}$, i. e., as $d\tilde{K}_t > 0$ the increase in output is more pronounced than the increase in investment costs, both measured in units of efficient labor.

[10]The condition on α is sufficient but by no means necessary for this results.

Statement 2 assures the existence of at least one locally stable steady state. Hence, without a closed form solution at hand, the focus of our analysis on steady-state effects is admissible.

The comparative statics of Statement 3 reveal that faster population growth and a higher contribution rate reduce the steady-state capital intensity. Intuitively, in a steady state savings per unit of efficient labor are such that all generations are equipped with the same amount of capital and innovation investments per unit of efficient labor. Moreover, since the steady state is locally stable, an increase in $\tilde{K}^*$ augments savings per unit of efficient labor by less than the investment needs per unit of efficient labor. Then, a higher population growth rate has two reinforcing effects on $\tilde{K}^*$. First, it increases the amount of aggregate investments necessary to maintain a given steady-state capital intensity. Given savings per unit of efficient labor this reduces $\tilde{K}^*$ (*Solow effect*). Second, savings per unit of efficient labor falls since, ceteris paribus, faster population growth means a higher pension benefit in accordance with (31). The second finding of Statement 3 follows from the observation that the contribution rate affects the individual incentive to save through the well-known income effects. An increase in τ_b means a lower net wage when young and a higher benefit when old. Both effects reduce savings per unit of efficient labor. If such a hike occurs in a locally stable steady state the value of $\tilde{K}^*$ must fall.

Finally, observe that the steady-state growth rate of all per-capita magnitudes in this economy is equal to $q^* = g\left(\tilde{K}^*\right)$. This follows from Lemma 1, the updating condition (16), and the equilibrium conditions for the intermediate good market and for the labor market, (21) and (22), respectively. Equilibrium output at t becomes $Y_t = K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}$ such that output per worker at t may be written as $Y_t/L_t = \tilde{K}_t^{\alpha} A_t$. Since $\tilde{K}_t$ is constant in the steady state, Y_t/L_t grows at rate $g\left(\tilde{K}^*\right)$.

3.3 Population, Pensions, and Endogenous Economic Growth

What is the effect of population growth on the steady-state growth rate of the economy? It turns out that the answer depends on particular properties of the pension scheme. The following proposition has the result for the pension scheme with a constant contribution rate discussed above.

Proposition 1 *Suppose the economy is in a locally stable steady state. If the pension system has a constant contribution rate, then* $q^* = g\left(\tilde{K}^*\left(\lambda, \tau_b\right)\right)$ *and it holds that*

$$\left.\frac{dq^*}{d\lambda}\right|_{\tau_b=const.} < 0. \tag{39}$$

Hence, a decline in the population growth rate increases the steady-state growth rate of the economy, q^*. The intuition is straightforward. According to Statement 3 of Theorem 1 a decline in the population growth rate implies a higher steady-state capital intensity (*Solow effect*). Moreover, in accordance with Lemma 1, the increased relative scarcity of labor means faster productivity growth (*Hicks effect*).

Next, we consider the same economy except that the pay-as-you-go pension system stipulates a constant replacement rate and not a constant contribution rate. How does the steady-state growth rate of such an economy respond to a demographic change?

Consider, in accordance with (20), a constant replacement ratio $\zeta \in (0,1)$ that ties the benefit when old to the net wage of the current young, i. e.,

$$b_{t+1} = \zeta\, w_{t+1}(1-\hat{\tau}_{b,t+1}), \tag{40}$$

where $\hat{\tau}_{b,t+1}$ adjusts such that the budget of the pension scheme is balanced. The $\hat{}$ -notation distinguishes the contribution rate under a constant replacement rate from τ_b of the constant-contribution-rate case. At all t the contribution rate consistent with a balanced budget of (31) and (40) satisfies $(1+\lambda)\hat{\tau}_{b,t} = \zeta\,(1-\hat{\tau}_{b,t})$, or, suppressing the time argument, we have

$$\hat{\tau}_b = \frac{\zeta}{\zeta+1+\lambda} \equiv \hat{\tau}_b\,(\lambda,\zeta)\,, \quad \text{with} \quad \frac{\partial\hat{\tau}_b}{\partial\lambda} < 0 \quad \text{and} \quad \frac{\partial\hat{\tau}_b}{\partial\zeta} > 0. \tag{41}$$

Since the population growth rate is constant, so is the contribution rate. Faster population growth reduces the dependency ratio such that a given replacement ratio can be supported with a lower contribution rate. A higher replacement rate requires a higher contribution rate. On the one hand, such an increase reduces benefits when old, on the other hand, it increases the contributions paid by the young.

Another important implication of (41) is that Statement 1 and 2 of Theorem 1 also apply here. Indeed, given $\tilde{K}_0 > 0$ the evolution described by the dynamical system of (35) is the same under a constant contribution rate and under a constant replacement rate if the right-hand sides of (31) and (40) are equal, i. e., if

$$(1+\lambda)\tau_b = \zeta(1-\hat{\tau}_b). \tag{42}$$

Statement 3 of Theorem 1 needs to be modified. In view of (41) the relationship between the steady-state capital intensity and population growth is now described by $\tilde{K}^* = \tilde{K}^*\,(\lambda, \hat{\tau}_b\,(\lambda,\zeta))$. It follows that

$$\frac{d\tilde{K}^*}{d\lambda} = \frac{\partial\tilde{K}^*\,(\lambda,\hat{\tau}_b\,(\lambda,\zeta))}{\partial\lambda} + \frac{\partial\tilde{K}^*\,(\lambda,\hat{\tau}\,(\lambda,\zeta))}{\partial\hat{\tau}_b}\frac{\partial\hat{\tau}_b\,(\lambda,\zeta)}{\partial\lambda}. \tag{43}$$

According to Theorem 1 and (41), all three partial derivatives are negative. Hence, the overall effect of a decline in population growth on the steady-state capital intensity is ambiguous. The reason is to be found in the relationship between changes in the population growth rate and the incentive to save. Under a fixed contribution rate, a decline in population growth reduces the expected pension benefit (see, (31)). Accordingly, savings increase. Under a constant replacement rate this effect is weakened since a lower population growth rate also raises the contribution rate. Indeed, from (31) and (41), we have $b_{t+1} = (1+\lambda)\,\hat{\tau}_b\,(\lambda,\zeta)\; w_{t+1} \equiv b_{t+1}\,(\lambda,\hat{\tau}_b\,(\lambda,\zeta)\,, w_{t+1})$. Therefore, $db_{t+1}/d\lambda = \partial b_{t+1}/\partial\lambda + (\partial b_{t+1}/\partial\hat{\tau}_b)\,(\partial\hat{\tau}_b\,(\lambda,\zeta)\,/\partial\lambda)$, where the first term is positive and the second is negative. Of course, (40) implies that the overall effect remains positive such that the incentive to save is weaker. Moreover, since a higher contribution reduces the net wage of the current young, they also save less.

As to the effect of population growth on the steady-state growth rate of the economy under a constant replacement rate we obtain the following result.

Proposition 2 *Consider an economy in a locally stable steady state. If the pension system has a constant replacement rate, then* $q^* = g\left(\tilde{K}^*\left(\lambda, \hat{\tau}_b\left(\lambda, \zeta\right)\right)\right)$ *and it holds that*

$$\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.} < 0. \tag{44}$$

Proposition 2 holds since the direct effect of λ on $\tilde{K}$ in (43) dominates the indirect effect through $\hat{\tau}_b$ such that the overall effect is negative. Hence, under a constant replacement rate we expect the economy's steady-state growth rate to increase following a decline in the population growth rate just as under a constant contribution rate. However, according to the following proposition, the strength of the steady-state response of the economy's growth rate to a demographic change depends on the institutional design of its pension scheme.

Proposition 3 *Consider an economy in a locally stable steady state. Suppose (42) is satisfied such that* $q^* = g\left(\tilde{K}^*\left(\lambda, \tau_b\right)\right) = g\left(\tilde{K}^*\left(\lambda, \hat{\tau}_b\left(\lambda, \zeta\right)\right)\right)$. *Then, the impact of a permanent decline in the population growth rate on the steady-state growth rate is greater under a constant contribution rate than under a constant replacement rate, i. e.,*

$$\left|\left.\frac{dq^*}{d\lambda}\right|_{\tau_b=const.}\right| > \left|\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.}\right|. \tag{45}$$

Hence, the prediction is that a decline in the population growth rate increases the steady-state growth rate of the economy by more if the pension scheme keeps the contribution rate constant. The necessary increase of the contribution rate under a constant replacement rate discourages savings. Hence, the induced change in $\tilde{K}^*$ will be less pronounced. We shall confirm this result in our evaluation of the quantitative model in Section 6 (see, in particular Figure 7).

4 Calibration Strategy

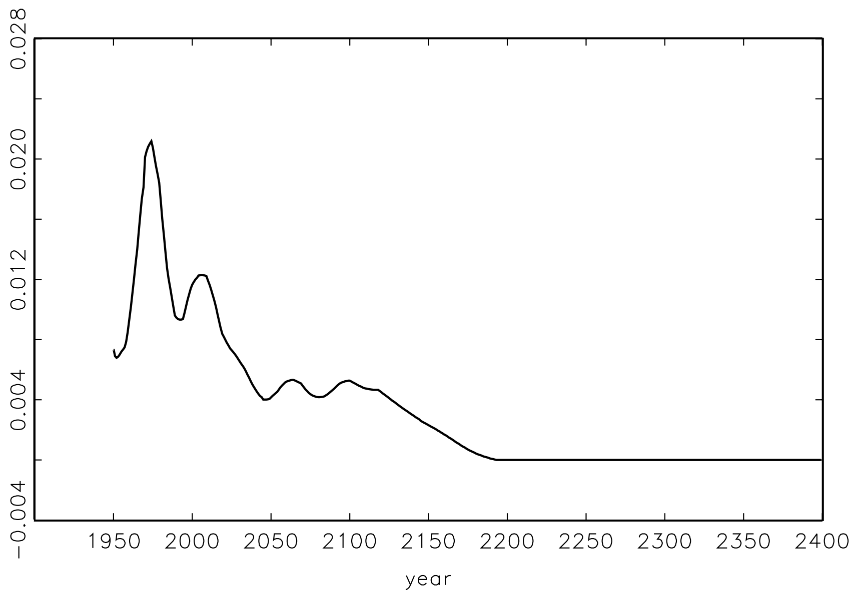
In this section, we describe our calibration of the model parameters.

4.1 Demographics

Our projection of the US population demographics for 1950-2050 is taken from UN (2002). The forecast for the US population development until 2400 is taken from Krüger and Ludwig (2007). These projections are based on the assumptions that life time expectancy increases at constant rates until the year 2100 and that the number of newborns is constant after 2200. Figure 2 illustrates the time profile of the population growth rate 1950-2400.[11] We take this evolution as exogenous and study the consequences for economic growth, pensions, and welfare.

[11] The depicted growth rates are computed for the population aged 20-94 in compliance with our model of Section 2.

Figure 2: Actual and Predicted US Population Growth Rates



4.2 Endowments and Preferences

We choose the discount factor $\beta = 1.011$ in accordance with the empirical estimates of Hurd (1989) who explicitly accounts for mortality risk.[12] For our choice of β, the real interest rate before taxes amounts to 7.33% (6.25%) during 1990-2000 (at the end of the demographic transition). We set the coefficient of risk aversion $\theta = 2.0$. The parameter $\gamma = 0.32$ of the utility function is calibrated so that the average labor supply of the workers is approximately 0.30.

The s-year old household of type j has the productivity $e(s,j) = \bar{y}_s \epsilon_j$. The age-efficiency profile $\{\bar{y}_s\}_{s=1}^{45}$ is taken from Hansen (1993), interpolated to in-between years and normalized to one. The set of the equally distributed productivity types $\{\epsilon_1, \epsilon_2\} = \{0.57, 1.43\}$ is taken from Storesletten, Telmer, and Yaron (2004). With this calibration, we are able to replicate the empirical distribution of US wages. In our model, the Gini coefficient of the wage distribution is equal to 0.301 which compares favorably with empirical values reported by, e. g., Díaz-Giménez, Quadrini, and Ríos-Rull (1997).

4.3 Production

The labor income share and the depreciation rate are set in accordance with Prescott (1986) implying the values $\alpha = 0.35$ and $\delta = 0.08$, respectively. The labor income (capital income) share in our

[12]Related research that uses such a value for β includes İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Huggett (1996). In the literature, the discount factor β has often been set smaller than one. In the Appendix, we study how a lower value, $\beta = 0.99$, affects our result. Besides an unrealistically high interest rate both our qualitative and our quantitative results remain virtually unaffected.

model amounts to 64.1% (35.0%), while research expenditure relative to GDP are equal to 0.93%.[13] A central parameter of our model is the elasticity of the growth rate of labor productivity q with respect to innovation investment i, $1/v$. With an increasing value of $1/v$, the endogenous growth rate q becomes more sensitive to the exogenous variables of our model, in particular to the population growth rate and to policy parameters in the hand of the government. The intuition for this comes from Lemma 1 which states the link between the capital intensity in the final-good sector, $\tilde{K}$, and the equilibrium growth rate of technological knowledge, $\hat{q}$. Indeed, the elasticity of the function $g\left(\tilde{K}\right)$ with respect to $\tilde{K}$ can be derived from equation (50) of the proof of Lemma 1 as

$$\frac{dg\left(\tilde{K}\right)}{d\tilde{K}}\frac{\tilde{K}}{g\left(\tilde{K}\right)}=\frac{1}{\alpha+(1-\delta)\tilde{K}}\left(\frac{1}{v}\right)\frac{1+\left(1+\frac{1}{v}\right)g\left(\tilde{K}\right)}{1+g\left(\tilde{K}\right)-\frac{1}{v}\left(1-g\left(\tilde{K}\right)\right)}. \tag{46}$$

It measures the response of the equilibrium growth rate of technological knowledge to a change in the capital intensity at $\tilde{K}$. This elasticity is increasing in $1/v$ for the relevant values of $g\left(\tilde{K}\right)$.[14] Hence, as v decreases the effect of a higher $\tilde{K}$ due to population aging on labor productivity is more pronounced.

We derive an estimate for the value of $v = 1.14$ when we regress the log of the GDP per-capita growth rate on the log of the R&D investment expenditure divided by GDP. We use annual time series data for the US economy from 1973-2000. Data on business enterprize R&D expenditure in current national prices are from OECD (1997) and OECD (2002a). Moreover, the price deflator and GDP data are from OECD (2002b).[15] The parameter v_0 is set equal to 1.407 implying an average productivity growth equal to 1.80% over the years 1990-2000.

4.4 Government

The government share $\bar{g} = G/Y$ is set equal to the average ratio of government consumption in GDP, $\bar{g} = 0.195$ in the US during 1959-93 according to the Economic Report of the President (1994). The tax rates $\tau_w = 24.8\%$ and $\tau_r = 42.9\%$ are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza, Razin, and Tesar (1994). The pension net replacement rate $\zeta = 0.50$ is taken from İmrohoroğlu, İmrohoroğlu, and Joines (1995). Government transfers, tr, and the social security contribution rate, τ_b, are computed using the equilibrium condition of the government budget (17) and the social security budget (20).

[13] One reason why the fraction of research expenditure in GDP is relatively small is that the model does not explicitly account for above average salaries of researchers.

[14] To be precise, this statement is true for the relevant range $g\left(\tilde{K}\right) \in [0,1]$. Moreover, in deriving this finding we have to take into account the dependency of g on v as implied by (50).

[15] In our model, total investment expenditure, $n_t i_t$, divided by total output, Y_t, is equal to $i_t/\tilde{K}_t^{\alpha}$. Therefore, in our regression, we make the identification assumptions that 1) the capital intensity $\tilde{K}$ is approximately constant in the US during 1973-90, and 2) the productivity growth rate q is approximately equal to the GDP per-capita growth rate. Moreover, $R^2 = 0.42$ and the standard deviation of the parameter $1/v$ (v) amounts to 0.218 (0.348).

4.5 Computation of the Transition Dynamics

To compute the transition dynamics presented in the next section, we assume that the US economy attains a new steady state with zero population growth in the year 2400.[16] In addition, we assume an arbitrary steady state with 1.1% population growth in the year 1950. Given that we simulate the economy starting in 1950, the initialization is found to have a small effect on the transition after 2000.

In a first step, we make an initial guess of the time path of $\{\tilde{K}_t, \bar{l}_t, \tau_{b,t}, \tilde{tr}_t, q_t\}_{t=1950}^{t=2400}$. We iterate backwards in time and compute the household decisions of the newborn generation in each period $t = 2400, 2399, \ldots, 1950$.[17] In each period, we aggregate savings of all generations and compute new values for $\{\underline{\tilde{K}}_t^i, \underline{\bar{l}}_t^i, \underline{\tau}_{b,t}^i, \underline{\tilde{tr}}_t^i, \underline{q}_t^i\}$. We update the time path using the Gauss-Seidel-Quasi-Newton algorithm presented by Ludwig (2007). In particular, we use the derivatives of the final steady-state equilibrium conditions with respect to $\tilde{K}$, $\bar{l}$, τ_b, $\tilde{tr}$, and q to initialize the Jacobian matrix in the Broyden algorithm. Moreover, we solve the model blockwise by first solving the transition for $\{\tilde{K}_t^1\}_{t=1950}^{t=2400}$ given the initial guess $\{\bar{l}_t^0, \tau_{b,t}^0, \tilde{tr}_t^0, q_t^0\}$. In the next step, we solve the transition for $\{\bar{l}_t^1\}$ given the remaining variable values $\{\tilde{K}_t^1, \tau_{b,t}^0, \tilde{tr}_t^0, q_t^0\}$ and so forth. When we tried to solve the model for all endogenous aggregate variables simultaneously, the program did not converge. Therefore, computational time is considerable and amounts to approximately 3 hours on a Pentium III, 860 MHz, for an accuracy chosen to be equal to 10^{-5} for both the individual decision rules and the time path of the endogenous aggregate variables.

5 Demographic Transition and Economic Growth

In this section we study the evolution of the US economy implied by the demographic transition depicted in Figure 2. As to the pension scheme we assume a constant pension replacement ratio equal to $\zeta = 50\%$ such that the social security contribution rate $\tau_{b,t}$ adjusts to balance the budget of the social security system. We refer to this setup as the benchmark economy.

The analysis proceeds in two steps. First, we establish and interpret the evolution of the labor force, the capital intensity, the factor prices, and the growth rate of technological knowledge, q, for the period 1950-2200. Second, to identify the role of endogenous growth, we compare the benchmark economy to an economy that differs in that growth is exogenous at a constant rate and costless, i. e., $q = 1.8\%$ and $i(1.8\%) = 0$.

5.1 Demographic Transition and Endogenous Growth

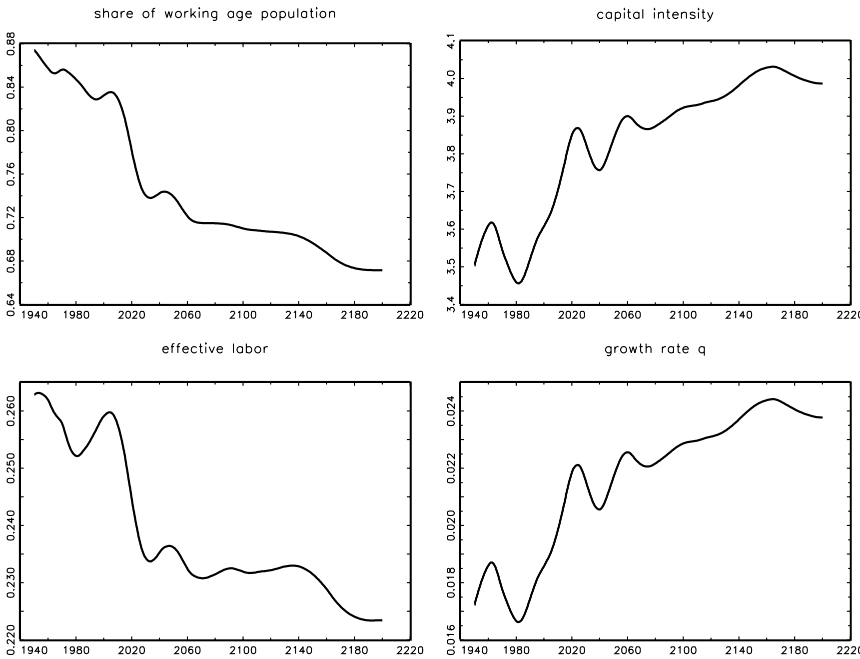
The transition from 1950 until 2200 is graphed in Figures 3 and 4 for the benchmark economy. As the population ages, the share of the working age population falls from 88% in 1950 to 67% in 2200.

[16] In fact, the deviation of the steady-state capital intensity in 2400 from the computed capital intensity at the end of the transition in period 2399 is less than 0.001%. Details on the transition until 2400 are given in the Appendix.

[17] To compute the individual household problem, we have to solve a non-linear equation system in two variables which are the capital stocks of the 21-year old households with productivity ϵ_1 and ϵ_2, respectively. The methods are described in more detail in Heer and Maussner (2008).

Therefore, the contribution rate to the social security system $\tau_{b,t}$ increases from 5.2% to 14.0%.

Figure 3: Transition in the Benchmark Economy, Results I



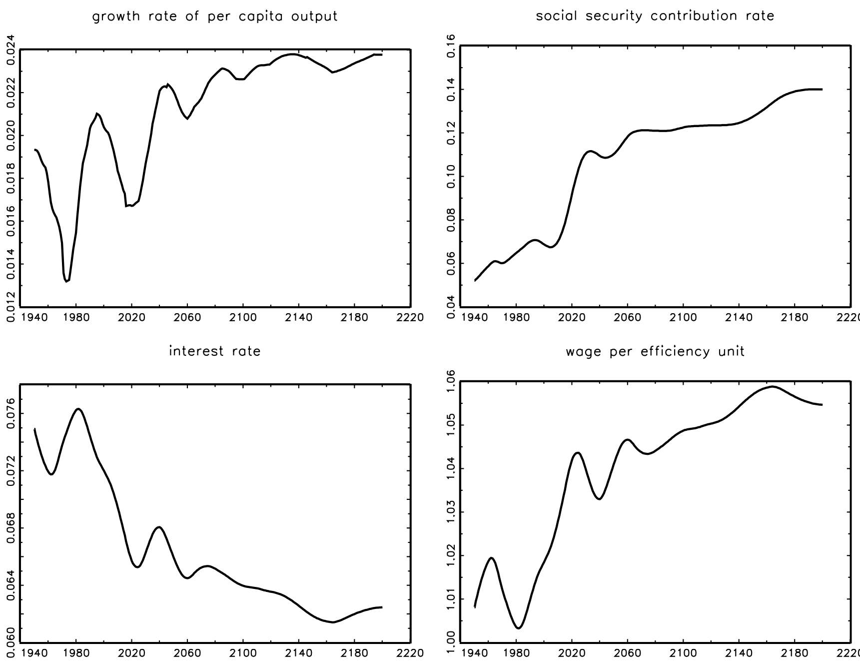
The dynamics of the capital intensity, $\tilde{K} = K/AL$, seems to reflect the evolution of the labor force share; as the aggregate efficient labor supply, L, declines, $\tilde{K}$ increases. However, the capital intensity is also influenced by general equilibrium effects that impinge on, e. g., the ability and willingness of a declining labor force to save and the evolution of the level of technological knowledge, A. Overall, the capital intensity is predicted to increase over time, though the evolution is not monotonous. The dynamics of factor prices reflects the one of the capital intensity; while the real wage rate w increases in $\tilde{K}$, the real interest rate r falls.

Since the growth rate of technological knowledge is tied to the capital intensity by the monotonous function $\hat{q}_t = g\left(\tilde{K}_t\right)$, the evolution of q mimics the one of $\tilde{K}$. The lower right diagram in Figure 3 shows that $\hat{q}_t$ increases from an average rate of 1.8% during 1991-2000 to 2.28% in 2100. As we approach the steady state after 2200, $\hat{q}_t$ rises even further.[18] In the steady state $\hat{q}_t = q^* = 2.38\%$ is constant and coincides with the growth rate of all per-capita magnitudes such as income and consumption. Hence, our first main result, we conclude that the demographic transition is associated with an economically significant increase of the long-run growth rate by 0.58 percentage points.

We may use these observations to shed some light on our interpretation of the data shown in Figure 1. From Lemma 1, the updating condition (16), and the equilibrium conditions for the intermediate good market and for the labor market, (21) and (22), respectively, we find the equilibrium output

[18]All endogenous variables of the model like the capital stock per capita or the aggregate efficient labor supply per capita stabilize around the year 2200. Indeed, in 2200 the transition is almost complete. The complete transition of $\tilde{K}_t$ and $\hat{q}_t$ until the year 2400 is shown in Figure 9 in Appendix 8.3.

Figure 4: Transition in the Benchmark Economy, Results II



at t to equal $Y_t = K_t^{\alpha}\,(A_t L_t)^{1-\alpha}$. Then, output per efficient hour worked at t may be written as $Y_t/L_t = \tilde{K}_t^{\alpha}\,A_t$. Suppressing time subscripts and denoting the growth rate of some variable x by g_x, the growth rate of Y/L at all t is

$$g_{Y/L} = \alpha\, g_{\tilde{K}} + g\left(\tilde{K}\right). \tag{47}$$

Hence, growth of labor productivity has two sources. First, there is the direct contribution of growth in $\tilde{K}$, second there is the indirect contribution of $\tilde{K}$ on the growth of technological knowledge. As population and effective labor decline, both channels contribute to the growth of labor productivity. However, the impact of a rise in $g\left(\tilde{K}\right)$, i. e., the Hicks effect, is quantitatively much larger. For instance, the model predicts average annual growth rates of labor productivity and technological knowledge between 2000 and 2100 of 2.18% and 2.15%, respectively. Accordingly, growth in Y/L is almost entirely due to the Hicks effect.

In the new steady state with a constant population, both the capital stock per capita K/AN and the output per capita Y/AN (both measured in efficiency units and not illustrated) are much smaller than the corresponding values in 1950 where we started the computation assuming a steady state with population growth equal to 1.1%. The per-capita capital stock and output decline by 13.7% and 4.6%, respectively. Even though a decline in the fertility rate results in substantial capital deepening and higher output per worker,[19] the effect of the decline in the work force share in the

[19] Furthermore, we have a positive composition effect in the labor force as the share of high-productive old-age workers in the total labor force increases.

total population dominates. As the share of the working-age population falls, tax revenues also decline. In order to balance the budget, government transfers are cut, especially during the first decades of 21st century. In 2200, government transfers are reduced by approximately 0.5% of total consumption compared to 1950.

5.2 Endogenous versus Exogenous Growth

To highlight the role of endogenous innovation investments, we simplify the benchmark economy and assume a production sector where q is constant at 1.8% and disregard the investment requirement function $i(q)$. Then, the growth rate of technological knowledge is exogenous, constant, and technical change is costless. Indeed, these assumptions turn the production side of the economy into the one of Solow (1956) with exogenous technical change and $A_t = 1.018A_{t-1}$.

At first sight, the transition of the variables under exogenous growth looks similar to the one in the benchmark economy with endogenous growth. For instance, Figure 5 shows the dynamics of the capital intensity for the benchmark economy both with endogenous and exogenous growth. When growth is exogenous, the capital intensity increases from the present over the next 100-200 years showing similar swings as under endogenous growth. However, form 2000 onwards the level of $\tilde{K}$ is smaller under endogenous growth. The first reason for this difference can be deduced from the lower left graph of Figure 5: from 1993 onwards the growth rate of A is higher reflecting the Hicks effect which is absent when growth is exogenous. Then, similar to the neoclassical growth model with exogenous technological change of Solow (1956), the capital intensity is smaller for higher growth rates. Second, in the benchmark model not all invested resources are channeled towards capital accumulation since innovation investments are costly.[20]

When growth is exogenous, a declining population affects labor productivity only through the Solow effect in equation (47). Quantitatively, the increase in Y/L is much smaller even though $\tilde{K}$ rises faster when growth is exogenous. In fact, the average annual growth rate of labor productivity between 2000 and 2100 is only 1.87%, i. e., 14% lower than under endogenous growth.

This intuition helps to understand the differential evolution of per-capita income under endogenous and exogenous growth. The lower right graph of Figure 5 shows that, from 2007 onwards, Y/N grows faster under endogenous growth. Using the same steps that let to the decomposition of equation (47) we obtain per-capita output at t as $Y_t/N_t = (L_t/N_t)\,\tilde{K}_t^{\alpha}\,A_t$. Accordingly, at t the growth rate of Y_t/N_t is

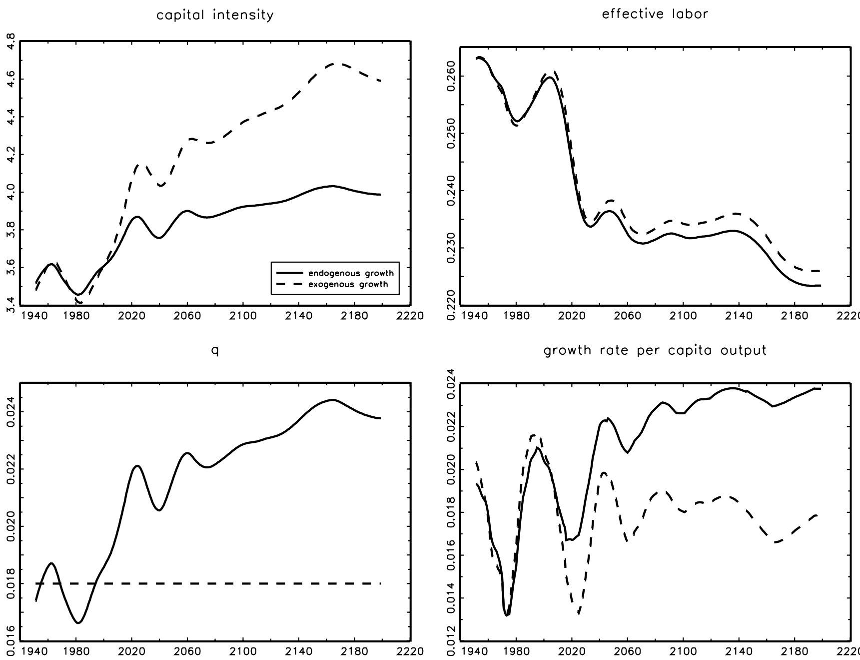
$$g_{Y/N} = g_{L/N} + g_{Y/L}, \tag{48}$$

i. e. the sum of the growth rate of efficient labor per capita and the growth rate of output per unit of efficient labor.

Both under endogenous and exogenous growth we have in the steady state that $g_{L/N} = g_{\tilde{K}} = 0$. Hence, in the long run $g_{Y/N} = 2.38\% > 1.8\%$. In the short and medium run, (48) suggests that the

[20] The upper right graph of Figure 5 reveals that the evolution of L is very similar under endogenous and exogenous growth. Hence, the effective labor supply cannot account for the differential evolution of $\tilde{K}$.

Figure 5: Transition of $\tilde{K}$, L, q, and the growth rate of Y/N in the Model with Endogenous and Exogenous Growth



Hicks effect also dominates the evolution output per capita since the upper right graph of Figure 5 shows that during the transition $g_{L/N}$ is approximately the same under endogenous and exogenous growth. Hence, starting in 2007, $g_{Y/N}$ is higher under endogenous growth because of the Hicks effect.

6 Population, Pensions, and Growth

In the previous section, we found that a decline in the population growth rate increases the growth rate of technological knowledge. In this section, we ask whether and how the design of the pension system affects this result. In Section 6.1 the focus is on possible growth effects. Since faster growth is no end in itself, Section 6.2 turns to the comparison of individual welfare under different designs of the pension scheme.

6.1 Comparative Analysis of Pension Reform Proposals

We consider the three scenarios of the pay-as-you-go pension scheme already introduced in Section 2:

1. constant replacement ratio, i. e., $\zeta_t = \frac{b_t}{(1-\tau_w-\tau_b)w_t \bar{l}_t} = \zeta = 50\%$. This is the benchmark economy (solid line in the following figures),

2. constant contribution rate after 2010 (broken line). More precisely, the contribution rate adjusts until 2010 before it remains constant at the level of $\tau_b = 7.1\%$. We assume that this policy change is fully anticipated by all economic agents.[21]

3. constant replacement ratio and later retirement at age 70, i. e., $R = 51$, for those agents born in 2010 and afterwards (dotted line).

Figure 6 illustrates the dynamics of the US economy for the three pension reform scenarios under endogenous growth. In the scenarios 1 and 3 (2), the contribution rate τ_b (the replacement ratio of the pensions ζ) adjusts in order to balance the social security budget. Consequently, pensions fall in scenario 2 as ζ is reduced from 50.0% to 23.9%. In scenario 3, agents retire at age 71 if they are born after 2010. Therefore, the burden on the pension system declines significantly during 2055-59 when the cohorts of the 66 through 70-year old workers remain in the labor market. During this period, the effective labor supply increases (see upper right picture of Figure 6) and the dependency ratio falls. The social security contribution rates in the new steady states amount to 14.0%, 7.1%, and 10.9% in the scenarios 1, 2, and 3, respectively.

In the scenarios 2 and 3, the lower social security contribution rates also increase the incentives to supply labor as the net wage rate increases and the substitution effect dominates the income effect. Therefore, the aggregate supply of efficient labor increases. In scenario 2, households also accumulate higher savings for old age, while in scenario 3 households accumulate savings over a longer working life. As a consequence, savings are also much higher in the scenarios 2 and 3. Since both savings and the labor force increase, the net effect on the capital intensity is ambiguous. As can be seen from the upper left picture in Figure 6, the capital intensity is higher in scenario 2 than in the benchmark case, while it is approximately equal in the benchmark case and in the scenario 3 with the exception of the period 2055-70.

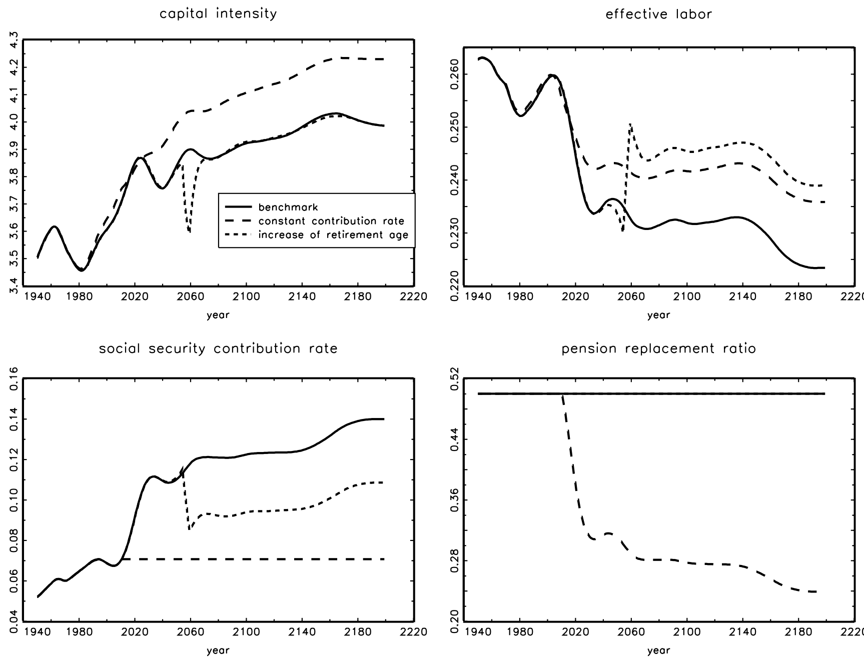
In all three scenarios, the evolution of the growth rate of technological knowledge q mimics the evolution of $\tilde{K}$. Therefore, the long-run change in q is largest in scenario 2 that keeps the contribution rate τ_b constant after 2010. As can be seen from Figure 7, q_t rises from 1.80 during 1990-2000 to 2.73% in 2200.[22] In both scenarios 1 and 3, q_t is approximately at 2.38% in 2200. Since in 2200 the transition is almost complete we may interpret these numbers as the steady-state growth rates of the respective scenarios. Then, the calibration exercise confirms the qualitative predictions of Proposition 3: the effect of the demographic transition on the steady-state growth rate of the economy is larger under a constant contribution rate (scenario 2) than under a constant replacement rate (scenario 1).

Summing up, we state as our second main result that the design of the pension system may have large quantitative effects on the economy's growth rate.

[21] Appendix 8.4.2 has the case where this reform is not anticipated.

[22] However, even though the growth rate increases significantly in scenario 2, it takes another 123 years until the absolute level of the pensions overtakes the one in the benchmark case. The effect of the lower replacement ratio dominates and lasts for many decades. For example in 2100, pensions are still 26% lower in scenario 2 than compared to the benchmark.

Figure 6: Transition Dynamics for Different Public Pension Reform Scenarios under Endogenous Growth



6.2 Welfare under Endogenous and Exogenous Growth

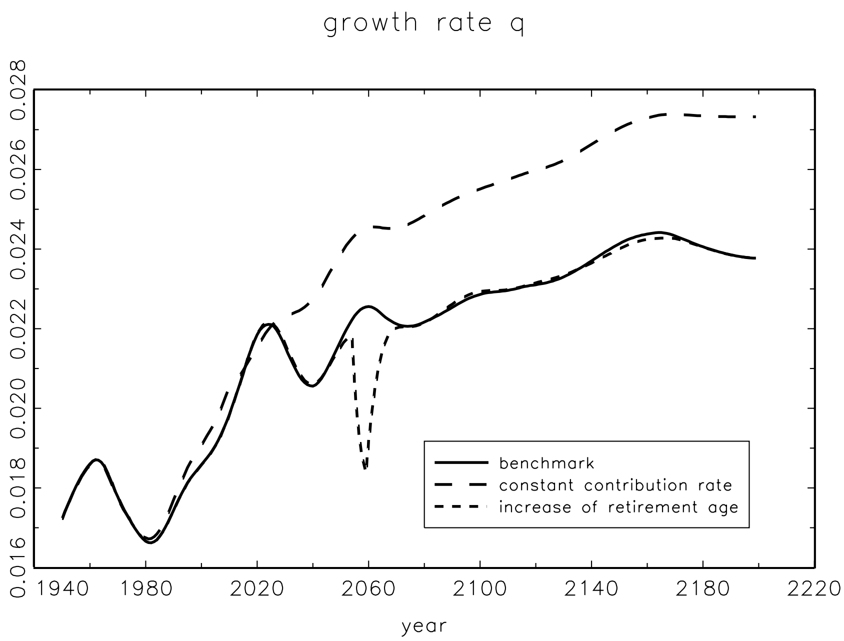
This section addresses two questions. First, we ask how the three pension reform designs compare in terms of welfare. Second, we want to know how the ranking of these proposals depends on whether the growth rate is endogenous or exogenous. Our results suggest that such a ranking can be established and that the negligence of growth rate effects of the social security system matters for the rank order of the proposals.

To address these questions, we hypothesize that the average newborn of each generation knows her lifetime utility associated with a life under each scenario. Equipped with this knowledge, she is able to rank the scenarios according to the lifetime utility they deliver. Figure 8 displays this welfare ranking for each generation born between 1960 and 2050. To provide an interpretable measure of welfare, the vertical axis states the consumption equivalent change relative to the benchmark economy with endogenous growth and a constant replacement rate. In other words, for all 6 scenarios it states the percentage change of the life-time consumption profile for the average newborn that is necessary to make her indifferent to a life in the benchmark economy.[23]

Obviously, the welfare effects are large and public pension reform matters. For example, consider the generation born in the year 2050. Under endogenous growth, the compensation in consumption necessary to make the average newborn indifferent between the benchmark economy and the economy with a constant contribution rate amounts to 13.21% of total lifetime consumption, the

[23]Similarly, de Nardi, İmrohoroğlu, and Sargent (1999) use wealth equivalent changes.

Figure 7: Growth Rate Effects of Different Public Pension Reform Scenarios



difference between the upper broken line and the solid horizontal line at 2050. Under exogenous growth this compensation declines to 1.11%, the difference between the lower solid and the lower broken curve at 2050. Of course, these huge welfare effects represent differences in the growth effects of the individual pension reforms.

Our results with regard to the welfare effects of the public pension reform can be summarized as follows:

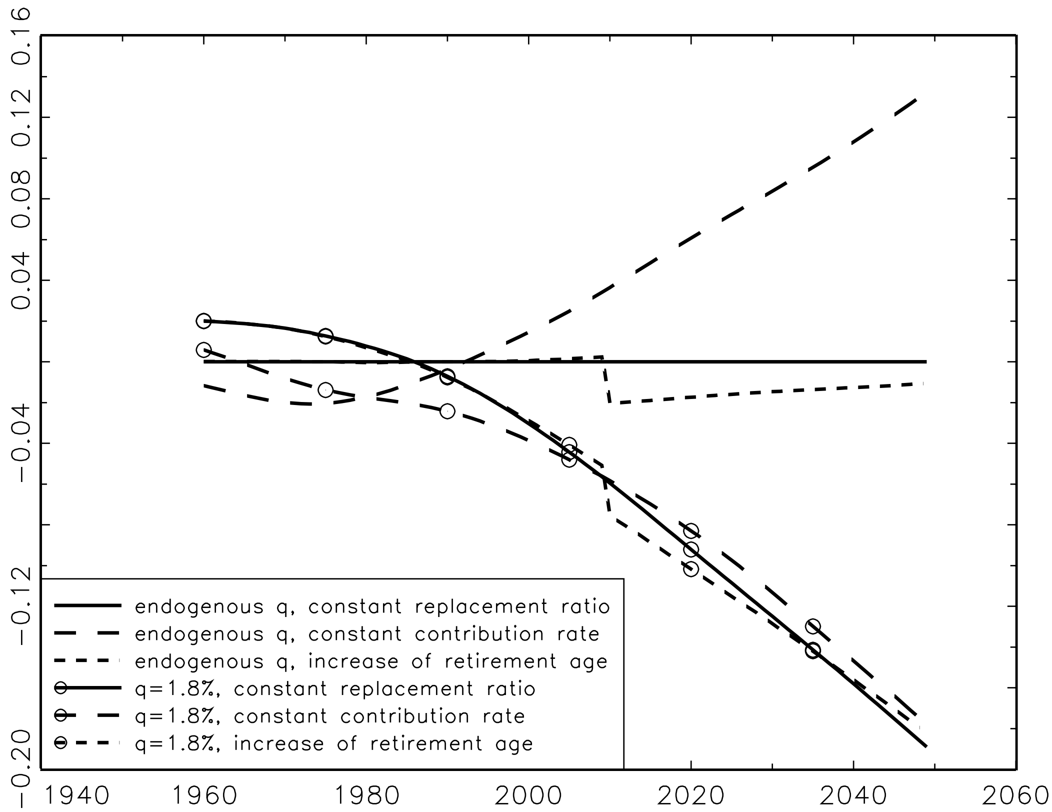
1. The quantitative welfare effects of public pension reform proposals might be much larger if growth is endogenous.

2. Endogenous productivity growth matters for the ranking of the pension reform proposals. For instance, generations that enter the labor force between 1990-2010 support a pension reform proposal that keeps the contribution rate τ_b constant after 2010 under endogenous growth, and they reject it under exogenous growth.

3. In the case of an increase in the retirement age, the generations born in the immediate decades following the policy change suffer. In the case of exogenous growth, welfare decreases for those born during the next 25 years. In the case of endogenous growth, all future generations suffer until the year 2354.

4. To keep the burden of the pension system low, a freezing of the social security contribution rate is preferable to an increase in the retirement age. Indeed, when growth is endogenous all generations born after 1993 prefer scenario 2 where the contribution rate is constant.

Figure 8: Welfare of Newborn Generations – Consumption Equivalent Change



As a caveat, let us mention that we do not interpret these results as being based on a comprehensive welfare analysis of public pension reforms. Clearly, our analysis neglects features like public debt and a pension system where benefits are tied to previous contributions. Our point, however, is more subtle. In fact, our results suggest that quantitative welfare results may depend on the assumption of endogenous or exogenous growth. Therefore, welfare results concerning public pension reforms that appear, e. g., in Huang, İmrohoroğlu, and Sargent (1997) or de Nardi, İmrohoroğlu, and Sargent (1999) under exogenous growth may not be robust.

7 Concluding Remarks

In an economy where the production technology is endogenous, the investment decisions of firms are guided by profit incentives. The evolution of the labor force, taken as exogenous by firms, is then an important determinant of aggregate investment and growth. Moreover, a declining labor force also affects an economy's ability to save and invest. The purpose of the present paper is to study the interdependency of these aspects in a dynamic general equilibrium model and to assess quantitatively the resulting effects on economic growth, pensions, and welfare for the US economy.

We find that, as a consequence of the demographic transition, the US growth rates increases from

1.8% during 1990-2000 to 2.28% (2.55%) in 2100 if the pension replacement ratio (contribution rate) is kept at its current level. In addition, welfare increases for all generations born after 1993 if the government keeps the pension contribution rate constant at the current level while pensions are allowed to decrease. At the conceptual level, our results suggest that allowing for TFP growth to be endogenous affects both the positive and the normative implications of large-scale OLG models employed to study policy reform proposals of the social security system.

Our findings suggests several promising routes for future research:

1. In our analytical framework technical change must be labor-saving at the level of intermediate-good firms. In a next step, one would want to fully endogenize the direction of technical change, e. g., in the spirit of Acemoglu (2003). While the literature on capital versus labor-saving technical change may be interpreted as providing a reason for the technology to be more labor than capital augmenting, both forms of technical progress occur along the transition towards a steady state and affect the economy's growth rate.

2. On the household side, we assume a constant labor force participation rate. However, over the last three decades, this rate has increased in the US from 58.9% in 1966 to 67.1% in 1998. Since 1998, it has declined to 66.0% in 2005 basically due to cyclical effects (see, McEwen, Orrenius, and Wynne (2005)). Intuitively, the labor force participation rate is likely to depend on the demographic transition and the design of the public pension system. While a rising participation rate reduces the relative scarcity of labor it is likely to increase aggregate savings and capital accumulation through a rise in the economy's wage incomes. The net effect on investment incentives is hitherto undetermined.

3. The level of technological knowledge used in an economy is the result of innovation investments made throughout the world. Recently, Jones (2002) documented the relevance of this channel for the evolution of the US economy during the period from 1950 to 1993. However, to incorporate this link one needs to know the appropriate diffusion process. In light of the general formulation of such a process given in Benhabib and Spiegel (2005), our specification of the evolution of technological knowledge (see equation (16)) is the one of the world's single technological leader. Clearly, more work needs to be done to quantify the relevant parameters of a richer diffusion process that applies to the US economy.

4. Globalization and market liberalization in many emerging countries, notably China and India, account for a rising global labor force. As a matter of fact, population ageing is already a predominant phenomenon in many of these countries including China, Japan, and Europe. In an open economy, these developments are likely to affect wages and interest rates in a way that may mitigate the effects emphasized in our paper. However, Krüger and Ludwig (2007) find that the decline in the labor force in Europe and Japan implies a small increase of US relative factor prices. With the Hicks effect in their framework such a small increase is likely to raise the growth rate of the US economy. We leave the quantitative assessment of this effect for future research.

8 Appendix

8.1 Details on the Data for Figure 1

Figure 1 replicates and extends Figure 1 in Romer (1987). Accordingly, for the period 1899-1979 we use the same data as described in footnote 23, p. 182, of Romer's paper. For the period 1979-1999, we use the same data sources that Romer uses for the time span 1949-1979. Hence, data on *man-hours* for 1979-1999 is taken from *Department of Labor, Bureau of Labor Statistics, Table B-47: Hours and Earnings in Private Nonagricultural Industries.* The output measure is *Gross Private Domestic Product* 1979-1999 as provided by the Department of Commerce. *Labor Force* corresponds to the *Total Civilian Labor Force* as reported by the *Department of Labor, Bureau of Statistics, Table B-12: Labor Force, Employment and Unemployment* and *Table B-35: Civilian Population and Labor Force.* Growth of output per man-hour is equal to the growth of output minus the growth in the labor force plus the growth in the rate of employment and minus growth in average hours worked.

8.2 Proofs

8.2.1 Proof of Lemma 1

Consider equations (6), (7), (14) and (15) hold. Upon combining the latter two, we obtain

$$p_t = \left[(1+q_t)\, i_q(q_t) + i(q_t)\right](1+r_t) \tag{49}$$

Acknowledging the dependency of p_t and r_t on $\tilde{K}_t$ through (6) and (7) in (49) gives

$$\frac{(1-\alpha)\tilde{K}_t}{(1-\delta)\tilde{K}_t^{1-\alpha}+\alpha} = (1+\hat{q}_t)\, i_q(\hat{q}_t) + i(\hat{q}_t). \tag{50}$$

Let $LHS(\tilde{K}_t)$ and $RHS(\hat{q}_t)$ denote the left and the right-hand side of (50). One readily verifies that $LHS(\tilde{K}_t)$ satisfies

$$LHS(0) = 0, \quad \lim_{\tilde{K}_t \to \infty} LHS(\tilde{K}_t) = \frac{\tilde{K}_t^{\alpha}}{1-\delta} = \infty, \text{ and } LHS'(\tilde{K}_t) > 0 \text{ for all } \tilde{K}_t > 0. \tag{51}$$

Similarly, in view of the specification of the input requirement function i of (10) we find for $RHS(\hat{q}_t)$ the properties

$$RHS(0) = 0, \quad \lim_{\hat{q}_t \to \infty} RHS(\hat{q}_t) = \infty, \quad \text{and } RHS'(\hat{q}_t) > 0 \text{ for all } \hat{q}_t > 0. \tag{52}$$

Hence, for each value $\tilde{K}_t \geq 0$ there is exactly one corresponding value $\hat{q}_t$ that satisfies (50). Moreover, the function g must have the indicated properties. ■

8.2.2 Proof of Theorem 1

We prove each statement of the theorem separately starting with Statement 1. Recall that $\delta = 1$, $v_0 = 1$, and $v = 2$ such that $i(q) = q^2$.

1. Implicit differentiation of (35) gives

$$\left[\xi_1 \, \alpha \tilde{K}_t^{\alpha-1} \left(1 + \frac{i(g(\tilde{K}_t))}{\tilde{K}_t} - \frac{i_q(g(\tilde{K}_t))g_{\tilde{K}}(\tilde{K}_t)}{1-\alpha}\right)\right] d\tilde{K}_t = Z(\tilde{K}_{t+1})d\tilde{K}_{t+1}. \tag{53}$$

Here,

$$Z(\tilde{K}_{t+1}) \equiv g_{\tilde{K}} \, \left[\xi_2 \, \tilde{K}_{t+1} + \xi_3 \, i\right] + (1+g) \left[\xi_2 + \xi_3 \, i_q \, g_{\tilde{K}}\right], \tag{54}$$

and the arguments of i is g and the one of g is $\tilde{K}_{t+1}$. The properties of the function i and the fact that (50) with $\delta = 1$ implies

$$\frac{d\hat{q}_t}{d\tilde{K}_t} = \frac{\frac{1-\alpha}{\alpha}}{2i_q(g(\tilde{K}_t)) + (1+g(\tilde{K}_t))i_{qq}(g(\tilde{K}_t))} \equiv g_{\tilde{K}}(\tilde{K}_t) > 0 \tag{55}$$

for all $\tilde{K}_t > 0$ leads to the conclusion that $Z(\tilde{K}_{t+1}) > 0$. Hence,

$$\frac{d\tilde{K}_{t+1}}{d\tilde{K}_t} = \frac{\xi_1 \, \alpha \tilde{K}_t^{\alpha-1} \left(1 + \frac{i(g(\tilde{K}_t))}{\tilde{K}_t} - \frac{i_q(g(\tilde{K}_t))g_{\tilde{K}}(\tilde{K}_t)}{1-\alpha}\right)}{Z(\tilde{K}_{t+1})} \equiv \phi'(\tilde{K}_t) \tag{56}$$

exists. To prove that $\phi'(\tilde{K}_t) > 0$, we have to make sure that the numerator of (56) is greater than zero for all $\tilde{K}_t > 0$. Before doing this it is useful to prove the following claim.

Claim 1 *Consider $g_{\tilde{K}}(\tilde{K})$ of (55) and the investment requirement function (10) with $v_0 = 1$ and $v = 2$. Then*

$$\lim_{\tilde{K}\to 0} g_{\tilde{K}}(\tilde{K}) = \left(\frac{1-\alpha}{2\alpha}\right). \tag{57}$$

Proof *If $\tilde{K} > 0$, equation (55) implies that*

$$g_{\tilde{K}}(\tilde{K}) = \left(\frac{1-\alpha}{2\,\alpha}\right)\left(\frac{1}{2g(\tilde{K}) + (1+g(\tilde{K}))}\right). \tag{58}$$

Since $i(0) = i_q(0) = 0$, (50) implies that $\lim_{\tilde{K}\to 0} g(\tilde{K}) = 0$. Hence,

$$\lim_{\tilde{K}\to 0} g_{\tilde{K}}(\tilde{K}) = \left(\frac{1-\alpha}{2\,\alpha}\right) \lim_{\tilde{K}\to 0} \frac{1}{2g(\tilde{K}) + (1+g(\tilde{K}))} = \frac{1-\alpha}{2\,\alpha}. \tag{59}$$

■

The numerator of (56) is greater than zero for all $\tilde{K}_t > 0$ if

$$1 + \frac{i}{\tilde{K}_t} - \left(\frac{1}{2\,\alpha}\right) \frac{i_q}{2 i_q + (1+g) i_{qq}} > 0, \tag{60}$$

where the argument of i is g and the argument of g is $\tilde{K}_t$. The fraction $i/\tilde{K}_t$ is increasing in $\tilde{K}$ since $i_q\, g_{\tilde{K}}\, \tilde{K} - i > 0$. To see this, consider (55) and (50) to conclude that $(1+g)\left[(i_q)^2 - i\, i_{qq}\right] > i\, i_q$ holds for the investment requirement function specified in (10). Finally, using l'Hôpital's rule and Claim 1,

$$\lim_{\tilde{K}\to 0} \frac{i(g(\tilde{K}_t))}{\tilde{K}_t} \quad = \quad \lim_{\tilde{K}\to 0} i_q(g(\tilde{K}_t)\, g_{\tilde{K}}(\tilde{K}_t) = 0. \tag{61}$$

Hence, a lower bound on the left-hand side (60) is

$$1 - \left(\frac{1}{\alpha}\right)\left(\frac{g}{1+3g}\right) \tag{62}$$

which can be shown to be strictly positive for all $\alpha > 1/3$ as stated in the theorem.

■

2. The steady state is determined by (35) for $\tilde{K}^* = \tilde{K}_t = \tilde{K}_{t+1}$. We show that the right-hand side, $RHS(\tilde{K})$, and the left-hand side, $LHS(\tilde{K})$, of (35) intersect on $\tilde{K} > 0$ at leaste once. First, consider

$$RHS(\tilde{K}) \equiv \left(1 + g(\tilde{K})\right)\left[\xi_2\, \tilde{K} + \xi_3\, i(g(\tilde{K}))\right]. \tag{63}$$

This function is bounded from below, i. e., $RHS(\tilde{K}) > \xi_2\, \tilde{K}$ for all $\tilde{K} > 0$. From the proof of Claim 1, we also have $RHS(0) = 0$. Moreover,

$$RHS'(\tilde{K}) = g_{\tilde{K}}(\tilde{K})\left[\xi_2\, \tilde{K} + \xi_3\, i(g(\tilde{K}))\right] + \left(1 + g(\tilde{K})\right)\left[\xi_2 + \xi_3\, i_q(g(\tilde{K}))g_{\tilde{K}}(\tilde{K})\right] > 0, \tag{64}$$

and

$$\begin{aligned} \lim_{\tilde{K}\to 0} RHS'(\tilde{K}) \quad &= \quad \lim_{\tilde{K}\to 0} \left\{ g_{\tilde{K}}(\tilde{K})\left[\xi_2\, \tilde{K} + \xi_3\, i(g(\tilde{K}))\right] + \left(1 + g(\tilde{K})\right)\left[\xi_2 + \xi_3\, i_q(g(\tilde{K}))g_{\tilde{K}}(\tilde{K})\right]\right\} \\ &= \quad \xi_2 \end{aligned} \tag{65}$$

since $\lim_{\tilde{K}\to 0} g_{\tilde{K}}(\tilde{K})$ is positive and finite.

Next, consider the function

$$LHS(\tilde{K}) \equiv \xi_1\, \tilde{K}^{\alpha} \left(1 - \frac{\alpha}{1-\alpha}\frac{i(g(\tilde{K}))}{\tilde{K}}\right). \tag{66}$$

This function is bounded from above, i.e., $\xi_1\,\tilde{K}^\alpha > LHS(\tilde{K})$ for all $\tilde{K} > 0$ and satisfies $\lim_{\tilde{K}\to 0} LHS(0) = 0$. To verify the latter we note that

$$\begin{aligned}\lim_{\tilde{K}\to 0} LHS(\tilde{K}) &= \xi_1 \lim_{\tilde{K}\to 0} \tilde{K}^\alpha - \frac{\alpha\,\xi_1}{1-\alpha} \lim_{\tilde{K}\to 0} \frac{i\left(g(\tilde{K})\right)}{\tilde{K}^{1-\alpha}} \\ &= 0 - \frac{\alpha\,\xi_1}{(1-\alpha)^2} \lim_{\tilde{K}\to 0} \frac{i_q\, g_{\tilde{K}}}{\tilde{K}^{-\alpha}} \\ &= \frac{\alpha\,\xi_1}{(1-\alpha)^2} \lim_{\tilde{K}\to 0} i_q\, g_{\tilde{K}}\, \tilde{K}^\alpha = 0, \end{aligned} \tag{67}$$

where the second step uses l'Hôpital's rule and the last step applies arguments of the proof of Claim 1.

Since $\alpha > 1/3$, the term in brackets on the left-hand side of (53) is strictly positive for $\tilde{K} > 0$. Therefore, $LHS'(\tilde{K}) > 0$. Moreover, we have

$$\lim_{\tilde{K}\to 0} LHS'(\tilde{K}) = \lim_{\tilde{K}\to 0} \left[\xi_1\, \alpha \tilde{K}^{\alpha-1}\left(1 + \frac{i(g(\tilde{K}))}{\tilde{K}} - \frac{i_q(g(\tilde{K}))g_{\tilde{K}}(\tilde{K})}{1-\alpha}\right)\right] \tag{68}$$

$$= \xi_1\,\alpha\left[\lim_{\tilde{K}\to 0} \frac{1}{\tilde{K}^{1-\alpha}} + \lim_{\tilde{K}\to 0}\left(\frac{(1-\alpha)i - i_q\, g_{\tilde{K}}\, \tilde{K}}{(1-\alpha)\tilde{K}^{2-\alpha}}\right)\right]. \tag{69}$$

The first limit is plus infinity whereas l'Hôpital's rule applied to the second limit gives

$$\frac{-1}{(1-\alpha)(2-\alpha)}\left[\lim_{\tilde{K}\to 0}\left(\frac{\alpha\, i_q\, g_{\tilde{K}}}{\tilde{K}^{1-\alpha}}\right) + \lim_{\tilde{K}\to 0}\left(\frac{i_{qq}\,(g_{\tilde{K}})^2\,\tilde{K}}{\tilde{K}^{1-\alpha}}\right) + \lim_{\tilde{K}\to 0}\left(\frac{i_q\, g_{\tilde{K}\tilde{K}}\,\tilde{K}}{\tilde{K}^{1-\alpha}}\right)\right]. \tag{70}$$

To study the last two terms of (70) we use (50) to derive

$$g_{\tilde{K}\tilde{K}} = -g_{\tilde{K}}^2\left(\frac{3i_{qq} + (1+g)i_{qqq}}{2i_q + (1+g)i_{qq}}\right). \tag{71}$$

Using the latter and $v = 2$, these two terms become

$$\begin{aligned}&\lim_{\tilde{K}\to 0} (g_{\tilde{K}})^2\,\tilde{K}^\alpha\left[\frac{(1+g)(i_{qq})^2 - i_{qq}\, i_q - (1+g)i_q\, i_{qqq}}{2i_q + (1+g)i_{qq}}\right] \\ =\; &\lim_{\tilde{K}\to 0} (g_{\tilde{K}})^2\,\tilde{K}^\alpha\left[\frac{4(1+g) - 4g}{4\,g + 2(1+g)}\right] \\ =\; &\lim_{\tilde{K}\to 0} (g_{\tilde{K}})^2\,\tilde{K}^\alpha\left[\frac{2}{1+3\,g}\right]. \end{aligned} \tag{72}$$

From Claim 1 we know that $\lim_{\tilde{K}\to 0} g_{\tilde{K}} = (1-\alpha)/(2\alpha)$. Hence, the limit in (72) is zero.

Turning to the first term in (70), we have

$$\lim_{\tilde{K}\to 0}\left(\frac{\alpha\, i_q\, g_{\tilde{K}}}{\tilde{K}^{1-\alpha}}\right) = (1-\alpha)\lim_{\tilde{K}\to 0}\left(\frac{1}{\tilde{K}^{1-\alpha}}\right)\left(\frac{v\, g^{v-1}}{2v\, g^{v-1} + (1+g)v(v-1)\, g^{v-2}}\right). \tag{73}$$

Evaluated at $v = 2$ and using l'Hôpital's rule gives

$$\begin{aligned}
& (1-\alpha) \lim_{\tilde{K}\to 0} \left(\frac{g}{\tilde{K}^{1-\alpha}(1+3g)} \right) \\
= \quad & (1-\alpha) \lim_{\tilde{K}\to 0} \left(\frac{g_{\tilde{K}}}{3g_{\tilde{K}}\, \tilde{K}^{1-\alpha} + (1+3g)(1-\alpha)\tilde{K}^{-\alpha}} \right) \\
= \quad & (1-\alpha) \lim_{\tilde{K}\to 0} \left(\frac{\tilde{K}^{\alpha}\, g_{\tilde{K}}}{3g_{\tilde{K}}\, \tilde{K} + (1+3g)(1-\alpha)} \right) \\
= \quad & 0. \qquad (74)
\end{aligned}$$

Hence, the expression (70) vanishes in the limit such that

$$\lim_{\tilde{K}\to 0} LHS'(\tilde{K}) = \xi_1\, \alpha \left[\lim_{\tilde{K}\to 0} \frac{1}{\tilde{K}^{1-\alpha}} \right] = \infty. \qquad (75)$$

Since ξ_2 is finite, the comparison of (65) to (75) leads to the conclusion that $LHS(\tilde{K}) > RHS(\tilde{K})$ for $\tilde{K}$ arbitrary close to $\tilde{K} = 0$. In addition, the fact that $RHS(\tilde{K}) > \xi_2\, \tilde{K}$ and $\xi_1\, \tilde{K}^{\alpha} > LHS(\tilde{K})$ for all $\tilde{K} > 0$ implies that there is at least one steady state $\tilde{K}^* \in \left(0, (\xi_1/\xi_2)^{(1/(1-\alpha))}\right)$ that satisfies $LHS'(\tilde{K}^*) < RHS(\tilde{K}^*)$. Then, at $\tilde{K}^*$ we have $\phi'(\tilde{K}^*) = LHS'(\tilde{K}^*)/RHS(\tilde{K}^*) < 1$ and the steady state is locally stable. ■

3. Consider equation (35) at $\tilde{K}_t = \tilde{K}_{t+1} = \tilde{K}^*$ and define the left-hand side of this equation as $LHS\left(\tilde{K}^*, \xi_1\right)$ and its right-hand side as $RHS\left(\tilde{K}^*, \xi_2, \xi_3\right)$. Taking the differential with respect to $d\tilde{K}^*$ and $d\xi_1$ gives

$$\frac{d\tilde{K}^*}{d\xi_1} = \frac{-LHS_{\xi_1}\left(\tilde{K}^*, \xi_1\right)}{LHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_1\right) - RHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_2, \xi_3\right)} > 0. \qquad (76)$$

The sign follows since $LHS_{\xi_1} > 0$ and local stability implies $LHS_{\tilde{K}^*} < RHS_{\tilde{K}^*}$.

Then, the negative relationship between $\tilde{K}^*$ and λ obtains from the fact that $\partial\xi_1/\partial\lambda < 0$ such that

$$\frac{d\tilde{K}^*}{d\lambda} = \frac{d\tilde{K}^*}{d\xi_1}\, \frac{\partial\xi_1}{\partial\lambda} < 0. \qquad (77)$$

Turning to the relationship between $\tilde{K}^*$ and τ_b we need to take into account that ξ_1, ξ_2, and ξ_3 depend on τ_b. Total differentiation of (35) with respect to $\tilde{K}^*$ and τ_b delivers

$$\frac{d\tilde{K}^*}{d\tau_b} = \frac{-LHS_{\xi_1}\left(\tilde{K}^*, \xi_1\right) \frac{\partial\xi_1}{\partial\tau_b} + RHS_{\xi_2}\left(\tilde{K}^*, \xi_2, \xi_3\right) \frac{\partial\xi_2}{\partial\tau_b} + RHS_{\xi_3}\left(\tilde{K}^*, \xi_2, \xi_3\right) \frac{\partial\xi_3}{\partial\tau_b}}{LHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_1\right) - RHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_2, \xi_3\right)}. \qquad (78)$$

Again, due to local stability the denominator is negative. The first term of the numerator is positive since $\partial\xi_1/\partial\tau_b < 0$. The sum of the second and third term is also positive. To see this, observe that

$$\begin{aligned} RHS_{\xi_2}\frac{\partial\xi_2}{\partial\tau_b} + RHS_{\xi_3}\frac{\partial\xi_3}{\partial\tau_b} &= \frac{1+g(\tilde{K}^*)}{1+\beta}\left[\frac{1-\alpha}{\alpha}\tilde{K}^* - i\left(g(\tilde{K}^*)\right)\right] \\ &= \frac{1+g(\tilde{K}^*)}{1+\beta}\frac{1-\alpha}{\alpha}\tilde{K}^*\left[1-\frac{\alpha}{1-\alpha}\frac{i\left(g(\tilde{K}^*)\right)}{\tilde{K}^*}\right] \end{aligned} \tag{79}$$

From the proof of Statement 2, the term in brackets is positive. Hence, it follows that $d\tilde{K}^*/d\tau_b < 0$. ■

8.2.3 Proof of Proposition 1

From Lemma 1 and Theorem 1, we have

$$\left.\frac{dq^*}{d\lambda}\right|_{\tau_b=const.} = g_{\tilde{K}}(\tilde{K}^*(\lambda,\tau_b))\,\frac{\partial K^*(\tilde{\lambda},\tau_b)}{\partial\lambda} < 0. \tag{80}$$

■

8.2.4 Proof of Proposition 2

Under a constant replacement rate we have $q^* = g(\tilde{K}^*(\lambda,\hat{\tau}_b(\lambda,\zeta)))$. Hence,

$$\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.} = g_{\tilde{K}}\left(\tilde{K}^*\right)\left[\frac{\partial\tilde{K}^*}{\partial\lambda} + \frac{\partial\tilde{K}^*}{\partial\hat{\tau}_b}\frac{\partial\hat{\tau}_b(\lambda,\zeta)}{\partial\lambda}\right], \tag{81}$$

where the argument of $\tilde{K}^*$ is $(\lambda,\hat{\tau}_b(\lambda,\zeta))$. To proof the proposition, we show that the term in brackets of (81) is negative for all admissible parameter values.

Define $\hat{\xi}_i \equiv \xi_i|_{\tau_b=\hat{\tau}_b(\lambda,\zeta)}$, $i=1,2,3$. Using (77) and (78) we find that $d\tilde{K}^*/d\lambda < 0$ holds if and only if

$$-LHS_{\hat{\xi}_1}\frac{\partial\hat{\xi}_1}{\partial\lambda} + \left(-LHS_{\hat{\xi}_1}\frac{\partial\hat{\xi}_1}{\partial\hat{\tau}_b} + RHS_{\hat{\xi}_2}\frac{\partial\hat{\xi}_2}{\partial\hat{\tau}_b} + RHS_{\hat{\xi}_3}\frac{\partial\hat{\xi}_3}{\partial\hat{\tau}_b}\right)\frac{\partial\hat{\tau}_b}{\partial\lambda} > 0, \tag{82}$$

where the argument of LHS is $\left(\tilde{K}^*,\hat{\xi}_1\right)$, the one of RHS is $\left(\tilde{K}^*,\hat{\xi}_2,\hat{\xi}_3\right)$, and the one of $\tilde{K}^*$ is $(\lambda,\hat{\tau}_b(\lambda,\zeta))$. From the definitions of $\hat{\xi}_i$, $i=1,2,3$ and the fact that the derivatives are evaluated at the steady state, we derive that (82) can be written as

$$\frac{\hat{\xi}_2\tilde{K}^*+\hat{\xi}_3 i}{1+\lambda} + \left(\frac{\hat{\xi}_2\tilde{K}^*+\hat{\xi}_3 i}{1-\hat{\tau}_b} + \tilde{K}^*\frac{1-\alpha}{\alpha(1+\beta)} - \frac{i}{1+\beta}\right)\frac{\partial\hat{\tau}_b}{\partial\lambda} > 0. \tag{83}$$

Using (41) to substitute for $\hat{\tau}_b$, condition (83) becomes

$$\frac{\hat{\xi}_2 \tilde{K}^* + \hat{\xi}_3 i}{1+\lambda} \left(1 - \frac{\zeta}{\zeta+1+\lambda}\right) + \left(\tilde{K}^* \frac{1-\alpha}{\alpha(1+\beta)} - \frac{i}{1+\beta}\right) \frac{-\zeta}{(\zeta+1+\lambda)^2} > 0. \tag{84}$$

With $\hat{\xi}_i$, $i = 1, 2, 3$, one readily verifies that inequality (84) holds for all admissible parameter values. Hence, the term in brackets of (81) is negative. With Lemma 1 the proposition follows. ■

8.2.5 Proof of Proposition 3

Under a constant replacement rate where $q^* = g(\tilde{K}^*) = g(\tilde{K}^* (\lambda, \hat{\tau}_b (\lambda, \zeta)))$, we have

$$\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.} = g_{\tilde{K}}(\tilde{K}^*) \left[\frac{\partial \tilde{K}^* (\lambda, \hat{\tau}_b (\lambda, \zeta))}{\partial \lambda} + \frac{\partial \tilde{K}^* (\lambda, \hat{\tau}_b (\lambda, \zeta))}{\partial \hat{\tau}_b} \frac{\partial \hat{\tau}_b (\lambda, \zeta)}{\partial \lambda}\right] \tag{85}$$

$$= g_{\tilde{K}}(\tilde{K}^*) \frac{\partial \tilde{K}^* (\lambda, \hat{\tau}_b (\lambda, \zeta))}{\partial \lambda} + g_{\tilde{K}}(\tilde{K}^*) \frac{\partial \tilde{K}^* (\lambda, \hat{\tau}_b (\lambda, \zeta))}{\partial \hat{\tau}_b} \frac{\partial \hat{\tau}_b (\lambda, \zeta)}{\partial \lambda} \tag{86}$$

$$= \left.\frac{dq^*}{d\lambda}\right|_{\tau_b=const.} + g_{\tilde{K}}(\tilde{K}^*) \frac{\partial \tilde{K}^* (\lambda, \hat{\tau}_b (\lambda, \zeta))}{\partial \hat{\tau}_b} \frac{\partial \hat{\tau}_b (\lambda, \zeta)}{\partial \lambda}, \tag{87}$$

where the last step makes use of Proposition 1 and the fact that $\tau_b = \hat{\tau}_b (\lambda, \zeta)$. Since the last term in (87) is positive the proposition is proved. ■

8.3 Computation Details

As mentioned in the main text, we assume that the transition is complete in the year 2400. Figure 9 presents the complete transition from 1950 to 2400 for the capital intensity, $\tilde{K}_t$, and the endogenous growth rate, q_t. These time paths suggest that the economy is in its new steady state from 2200 onwards.

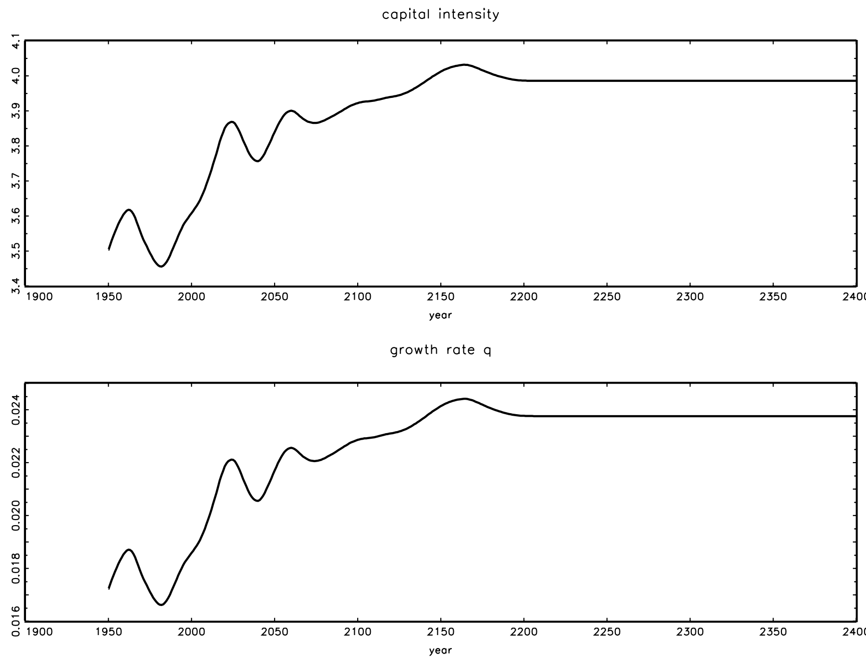
8.4 Sensitivity Analysis

8.4.1 Investment Requirement Function

The critical parameter in our analysis is the elasticity of the growth rate of productivity, q, with respect to innovation investment, i. Since $i = v_0\, q^v$, this elasticity is equal to $1/v$. From our regression of the log of the GDP per capita growth rate on the log of the R&D investment expenditure divided by GDP we estimated a value equal to $v = 1.14$. The closer the value v is to one, the higher will be the quantitative effect of population ageing on the growth rate.

In this section, we analyze the sensitivity of the growth rate effect of population aging on the choice of v. In particular, we determine an estimate for the lower bound of the quantitative growth rate.

Figure 9: Complete Transition in the Benchmark Economy



We choose $v = 1.14 + \sigma_v = 1.49$, where σ_v is the standard deviation of v derived from the above empirical regression analysis for the US economy during 1973-2000.[24] The parameter $v_0 = 4.385$ is chosen so that the average growth rate during 1990-2000 is equal to 1.80% as in the benchmark economy. In this case, the share of expenditure on research and development relative to GDP falls to 0.71% and is in less accordance with empirical numbers than for the case with $v = 1.14$.

Figure 10 presents our results. For $v = 1.49$ ($v = 1.14$), the growth rate rises from 1.8% during 1990-2000 to 2.11% (2.38%) in 2200. The relative increase of the growth rate remains substantial, however, it is certainly smaller than the one computed for the benchmark case.

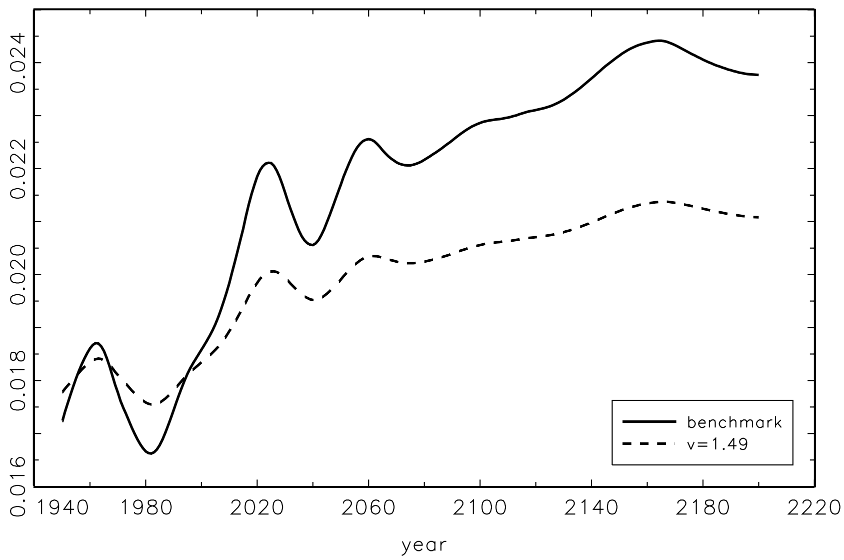
8.4.2 Perfect Foresight versus Unexpected Policy Changes

In Section 6, we analyze the pension reform that switches from a constant replacement ratio of pensions relative to net average wages ($\zeta_t = 0.5$ for $t \leq 2010$), to a constant contribution rate ($\tau_{b,t} = \tau_{b,2010}$ for $t > 2010$) in the year 2010. In our computations, we assume that the change in the pension policy is fully anticipated. In this Appendix, we study the alternative scenario where the households do not learn about this policy change until the end of the year 2010.

In Figures 11 and 12, the transition dynamics of the aggregate supply of efficient labor, L, and

[24]In our regression, we estimate $\widehat{\frac{1}{v}} = 0.953$ with a standard deviation equal to 0.218. In order to derive the mean and the standard deviation of our estimate for v, we generate 1 million normally distributed random variables x_i with a standard deviation equal to 0.218, compute the values $v_i = 1/(0.953 + x_i)$ and the associated sample mean and standard deviation. Basically the same value results from a computation that employs bootstrapping methods.

Figure 10: The Growth Rate q in the Benchmark Economy for $v = 1.14$ and $v = 1.49$



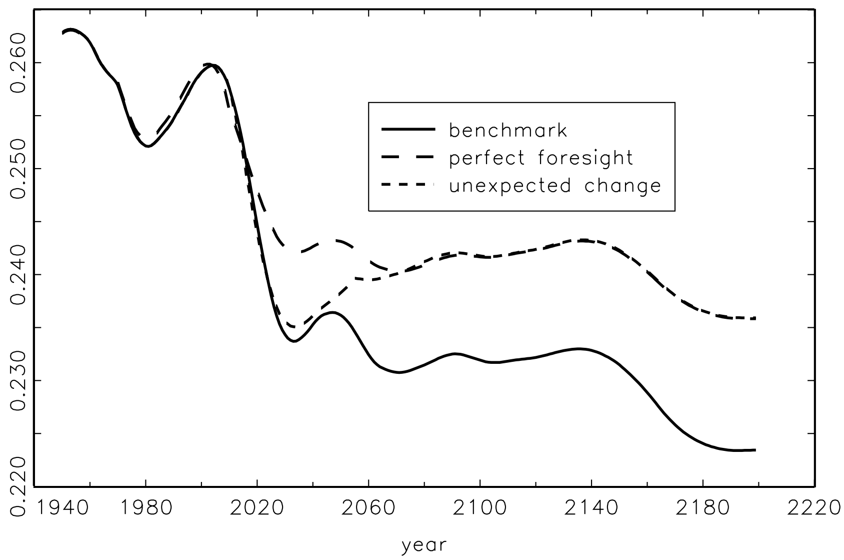
the capital intensity, $\tilde{K}$, are illustrated for the benchmark case (with constant replacement ratio $\zeta_t = 0.5$ for all t), and for the two cases of an unexpected and an expected policy change involving a constant contribution rate for $t > 2010$. If the policy change is unexpected, the time paths of the two variables coincide with the one of the benchmark case until the year 2010. Afterwards, L decreases temporarily (the dotted line falls below the solid line until 2020), even though only to a very small extent. Households increase their relative labor supply in old age at the expense of their labor supply in young age because, in comparison to the benchmark case, the contribution rates $\tau_{b,t}$ in old age are much lower so that the net wages are higher. As the substitution effect is stronger than the income effect, labor supply in the later years of their working life increases.

In addition, households learn in 2010 that their pensions will be lower than expected. As a consequence, they increase savings, especially in old age. Compared to the case with perfect foresight (broken line), the capital intensity $\tilde{K}$ is always lower. As the growth rate, q_t, is a monotone function of the capital intensity, growth is also higher in the case of an expected policy change. In 2037, the two transition paths display the maximum difference between the two growth rates amounting to 0.14 percentage points (not illustrated). Clearly, the unexpected change in policy results in a short and medium-run decline of the compounded growth rate.

8.4.3 Sensitivity with Respect to the Discount Factor β

Figures 13 and 14 display the transition dynamics for a lower discount factor $\beta = 0.99$. For this choice of the discount factor, the real interest rates are much higher than for our benchmark calibration and increase to values between 9.4%-11.1% during 1950-2100. Apart from this, all our qualitative results hold. In particular, the growth rate of productivity q increases from 1.8% during 1990-2000 to 2.20% in 2100 and even 2.27% in the long run. All remaining qualitative and

Figure 11: Effects of Policy Announcement on Aggregate Effective Labor



quantitative results concerning the pension reform under endogenous and exogenous growth are available upon request from the authors.

Figure 12: Effects of Policy Announcement on the Capital Intensity

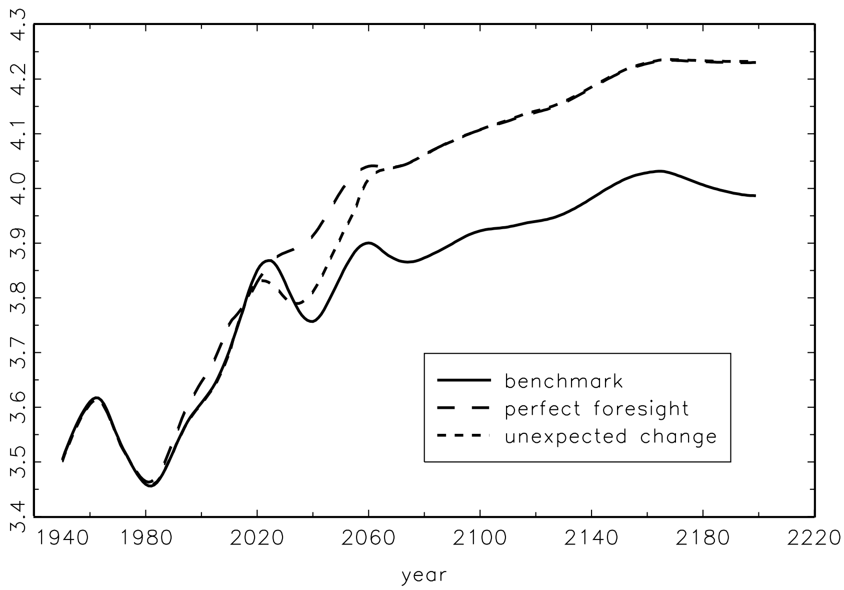


Figure 13: Transition in the Economy with $\beta = 0.99$, Results I

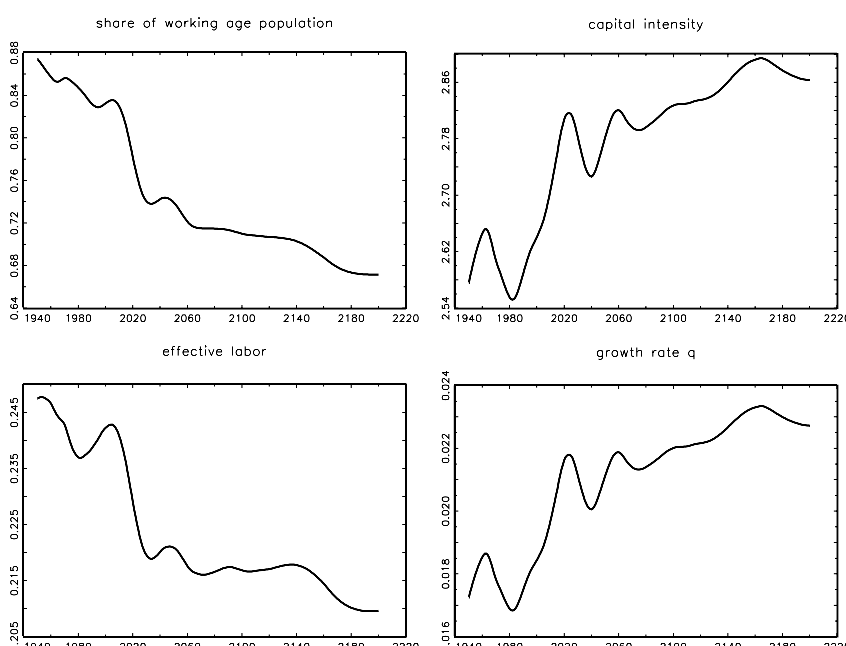
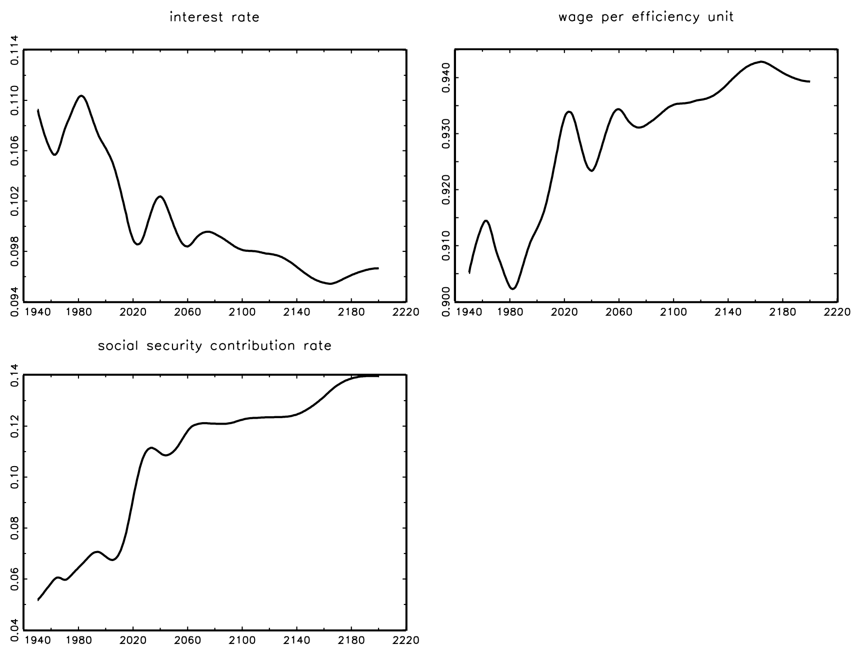


Figure 14: Transition in the Economy with $\beta = 0.99$, Results II



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