Ambiguity, Efficiency and Bank Bailouts

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Abstract
The paper examines the effects of ambiguity in regulation on the equilibrium allocation. Under ambiguous bailout policy, agents’ suffer from a lack of information with regards to the insolvency resolution method, which would be chosen by the regulator if a financial institution fails. In this case, beliefs of bankers regarding whether an insolvent bank is liquidated, may differ from those of depositors. The beliefs may be asymmetric even if bankers and depositors possess absolutely symmetric information about the policy of the regulator. It is shown that such asymmetry in beliefs can generate an allocative inefficiency of the bank based economy.

JEL Classification: G28

Keywords: bank bailouts; constructive ambiguity; decision-making, uncertainty

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1 Introduction

Central Bankers often follow a policy of "constructive ambiguity", which means that the bailout policy is not announced ex-ante (Goodhart and Schoenmaker [1995], Santomero and Hoffman [1998], Bennett [2001] provide empirical evidence). This is mostly justified by the objective of avoiding excessive risk-taking by banks (see a review by Enoch et al. [1997]) or by social benefits (see Freixas [2000] for a costs-benefits analysis). Researchers address the issue of constructive ambiguity in assuming that banks may be bailed out with some probability, which is known to the public. In general, however, there is no reason to assume that this piece of information is available to the agents. Cukierman and Meltzer [1986] present one of the first models to encompass political ambiguity, in which they assume that the public forms rational expectations about the policy indicators on the basis of the historical path of signals. What happens if such path of signals is absent or if agents do not trust the information from the past? The issue becomes more relevant in an environment, where the public is heterogenous, and the outcome crucially depends on what beliefs each group of agents has. The current paper addresses the issue of ambiguous policy from the point of view of allocative efficiency of the resulting market equilibrium. The purpose of the current paper is twofold. First, it suggests a novel explanation for disintermediation, which can arise as an equilibrium outcome if the public is not informed about the regulatory policy. Second, it introduces decision-making under ambiguity in a market-equilibrium framework. Constructive ambiguity with regards to bank failures resolutions is a nice example of a political ambiguity in a heterogenous framework; similar results may be
obtained in other equilibrium situations with political ambiguity.

The issue of expectations (beliefs) formation under ambiguity has been paid a great attention in the literature. Uncertainty is often modeled in the von Neumann-Morgenstern [1944] Expected Utility (EU) framework. Ambiguity may be viewed as a special case of uncertainty, when the probabilities of events are unknown to the decision-makers. One of the approaches to deal with ambiguity, which received an increasing attention of economists, is Choquet Expected Utility (CEU) approach suggested by Schmeidler [1989]. Recently, Chateuaneuf et al. [2007] introduced a special CEU representation, which counts for both pessimism and optimism. An optimistic agent counts for the best outcome, whereas a pessimistic agent counts for the worst one. This aspect of the decision-making under ambiguity will play an important role in the current paper.

Generally, it is not a trivial task to formally introduce ambiguity in an equilibrium framework. Eichberger and Kelsey [2000] and Marinacci [2000] introduce ambiguity in the concept of a game equilibrium. Eichberger et al. [2007] suggest various social interaction games, in which ambiguity either distorts equilibria (e.g. it affects prices in Cournot competition) or selects equilibria in coordination games with multiple equilibria. However, it is unclear, what is the source of ambiguity in such games, especially if they only have equilibria in pure strategies. In Eichberger and Kelsey [2000], it is rather a behavioral phenomenon, with players building beliefs with regards to their rivals’ strategic behavior. In Marinacci [2000], games are assumed to be context-dependent and players face ambiguity with regards to which game is actually played. In contrast to these and similar studies, the ambiguity in the current paper naturally arises from

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1 Sometimes ambiguity is called Knightian (1921) uncertainty. See Ellsberg (1961) for discussion of the literature on risk and uncertainty.
the fact that one of the agents (Regulator) selects the subgame, which other market participants (bankers and depositors) play. Even if bankers and depositors are rational and informed about the objectives of the regulator, they cannot reveal, which action will the regulator choose, if the latter does not commit to any of them, because of the multiplicity of the regulator’s optima. For the time of writing, the author is unaware of other attempts to study decision-making by heterogenous agents under ambiguity in the context of a standard market equilibrium, as opposed to games.\textsuperscript{2}

The focus of the current paper is an economy with agents (depositors) wishing to invest their fixed endowment into a risky asset, which dominates the risk-free one. However, they have no access to the market of the risky asset, which justifies the existence of banks in the economy. Banks are assumed to be completely financed through [uninsured] deposits. Banks act as the second group of decision-makers in the economy, whose investment decision is explicitly modelled. Finally, there exists a regulator, which can either liquidate or bail out banks in case of their insolvency. The objective of the regulation is to provide depositors with the same investment opportunities, which would be available for them if the market access would not be restrained. Explicit modelling of the decision-making by banks and depositors allows one to study the market equilibrium.

An important difference of the model here from other studies of constructive ambiguity in banking regulation is the treatment of the ambiguity. When the bailout policy is not announced ex-ante, agents work in an environment, where the probability distribution over possible outcomes is not known. For example, in the models of Freixas [2000] and Shim [2006], the only "public" are bankers who are supposed to know the

\textsuperscript{2} Of course, market equilibrium can also be modeled in game-theoretic terms. Then the model might be seen as the one providing an alternative introduction of ambiguity in a standard Nash-equilibrium framework.
probability of bailouts. Even if they are not informed about the actual policy of the regulator, they have an identical subjective probability distribution about it and thus form homogenous expectations (beliefs) about the policy outcomes as does the public in Cukierman and Meltzer [1986]. Introducing depositors into the model makes the public heterogenous, which implies that beliefs may differ among agents. Akerlof [1970] was the first to indicate that an asymmetry in information may lead to suboptimal market outcomes. In contrast, both bankers and depositors in the current paper have identical information and are equally rational in their decisions. The model takes into consideration the asymmetry in beliefs of bankers and depositors regarding the bailout policy of the regulator. If such an asymmetry is present, the equilibrium outcome may result in disintermediation. To improve the situation, the Regulator may still freely choose the probability of bailouts, but should signal it to the public in order to align the public’s beliefs.

The paper begins with a simple game-theoretical example in Section 2, which illustrates the idea of the equilibrium distortion through ambiguity. Section 3 introduces the economic environment. In a risky environment, it is possible that banks are insolvent, therefore insolvency resolution is discussed in Section 4. Market equilibrium and optimal regulatory policy are identified in Section 5. Section 6 studies beliefs of agents and market equilibrium under ambiguity. The paper concludes with a summary of results.

2 An example

Before proceeding with a formal description of the model, it might be useful to consider a simple game, which gives intuition for the principal idea of the paper.
Experiments show that players not always land at Nash-Equilibria. Goeree and Holt [2001] give the following example: in a two-players coordination game, players obtain the payoff of 90 if they both choose action $L$, or they obtain the payoff of 180 if both choose action $H$, and the payoff of zero if one of them plays $H$ whereas the other one plays $L$. Besides, there exists an outside option for one of the players, who obtains 40 if he does not enter the game, in which case the payoff of the other player is $x$. Obviously, the outside option is dominated by a mix of $H$ and $L$, hence only strategy combinations $LL$ and $HH$ are Nash equilibria. In experiments, 80 percent of outcomes coordinated on the high-payoff equilibrium. However, if $x$ is large enough ($x = 400$ in the experimental treatment), only 32 percent of outcomes coordinated on this equilibrium, 16 percent coordinated on the low-outcome equilibrium, and more than half of all the cases were uncoordinated, non-Nash outcomes. Goeree and Holt [2001] explain this by a reasoning that when $x$ is high, the player with the outside option expects his rival to choose $L$ and therefore avoids choosing $H$ himself. This expectation is of a behavioral nature and only limitedly rational: a rational player should obviously expect that the rival understands that the outside option is dominated and will never be played.

Now, consider a game with three players: $R$, $B$, and $D$. First, player $R$ decides, which subgame players $B$ and $D$ should play. Then, players $B$ (rows) and $D$ (columns) play one of the following subgames:

<table>
<thead>
<tr>
<th></th>
<th>Subgame 1</th>
<th>Subgame 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D$</td>
<td>$D$</td>
</tr>
<tr>
<td>$B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>0.2 ($\theta$)</td>
<td>0.2 ($\theta$)</td>
</tr>
<tr>
<td></td>
<td>0.4 ($I$)</td>
<td>0.4 ($I$)</td>
</tr>
<tr>
<td>$b$</td>
<td>-2.2 ($I$)</td>
<td>-2.4 ($I$)</td>
</tr>
<tr>
<td></td>
<td>1.2 ($I$)</td>
<td>1.3 ($I$)</td>
</tr>
</tbody>
</table>

The number in brackets is the payoff of player $R$, it does not depend on his
strategic choice. The game possesses two Nash equilibria in pure strategies: \((1, t, d)\) and \((2, b, d)\). In a sequential game, when player \(R\) makes his action known to players \(B\) and \(D\), both equilibria are subgame perfect, and one would expect players landing at one of the two equilibria, depending on the choice of \(R\). Should we also expect players landing at one of the two Nash equilibria in a simultaneous game, or when players \(B\) and \(D\) are not informed about the action of player \(R\)?

Like in the example of Goeree and Holt [2001] above, players might build expectations about their counterpart’s choice. Assume that players \(B\) and \(D\) try to guess, which subgame they will be actually playing. There is an important difference between them: pairwise comparison of the payoffs of player \(B\) in each strategy combination in both subgames shows that player \(B\) is better off, if subgame 2 is played; the same exercise for player \(D\) shows that he is better off, if subgame 1 is played. If both players are pessimistic and count for the worst payoff structure, then player \(B\) would act as though he were playing subgame 1. Player \(D\), in his turn, would act as though he were playing subgame 2. As a result, this distortion in beliefs leads to the following perceived payoff structure:

<table>
<thead>
<tr>
<th></th>
<th>(g)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
<td>0, 2</td>
<td>0, 1</td>
</tr>
<tr>
<td>(b)</td>
<td>−2, 2</td>
<td>−2, 3</td>
</tr>
</tbody>
</table>

The unique Nash equilibrium in this perceived subgame is \((t, g)\), which is distinct from the equilibria in the original game. Formally, the payoffs in the last matrix are the perceived payoffs, but as soon as we obtain \((t, g)\) being the equilibrium combination of strategies, we can find actual payoffs in the original game for the one or the other
action of player $R$. Incidentally, perceived payoffs in equilibrium are the same as in the original game. Player $R$ would play either 1 or 2, or a mix of them, since his payoff is equal in both cases. However in this case his payoff is lower than before, when players $B$ and $D$ were informed about the action of player $R$. It turns out that it does matter for the outcome of the game, whether players $B$ and $D$ are informed about the strategic choice of player $R$.

Let player $R$ be a regulating body, and players $B$ and $D$ be market agents, say bankers and depositors, like in the model below. The above game may be treated in the following way. Subgames 1 and 2 are types of the regulatory policy conducted by the regulator, e.g. commitment to bailing out banks in financial distress, or to liquidating them. If insolvent banks are liquidated (Subgame 2), bankers enjoy limited liability, and reap positive expected profit if invest risky (action $b$), or zero expected profit if invest safe (action $t$). Depositors then either get a high payoff if deposited (action $d$) with the bank investing risky, or enjoy a lower payoff otherwise. If insolvent banks are bailed out (Subgame 1), bankers are penalized for insolvency and their payoff is negative, if they invest risky. Depositors, however, enjoy higher payoffs due to guarantees on deposits. If depositors and bankers know the policy of the regulator, the resulting equilibria deliver regulator the payoff of 1. If they are not informed about the policy of the regulator, the game can end with a $(t,g)$-equilibrium, which is strictly worse for all the players. Note that the regulator cannot improve the situation through his policy choice.

The model below views the interactions between bankers and depositors from a market perspective. Though it focuses at the deposit market, similar results may be obtained for other cases of regulatory ambiguity or political intransparency. Such
examples may be market-entry decisions in a monopolized industry, which depend on the antimonopolist policy of the regulator; R&D or launch of new products conditional on the governmental support/subsidization; macroeconomic performance conditional on the monetary policy, etc.

3 The Model

Consider an economy with a continuum of risk-neutral agents distributed at $[0; 1]$, and two types of financial assets, one risky and one risk-free. The model describes two periods: in the first period, decisions and investments are made, and in the second period, a state of nature $s \in \{H, L\}$ is realized and investment gains reaped. Each household is endowed with one unit of wealth in the beginning of the first period.

3.1 Markets

The markets of both risky and risk-free assets are characterized by an absolutely elastic supply of assets. The risky asset yields a gross rate of return of $r^s$ in state of nature $s$, the risk-free asset yields $r^F$ in each state of nature. The probability of state $s = H$ is $p$, and the probability of state $s = L$ is $1 - p$. It is assumed that

$$r^H > r^F > r^L$$  \hspace{1cm} (A-1)

and

$$pr^H + (1 - p)r^L > r^F$$  \hspace{1cm} (A-2)

Short sales are not allowed, hence the amount invested in financial assets is non-negative. Assumptions A-1 and A-2 guarantee that a financial portfolio of a risk-neutral agent would only contain the risky asset. Since the supply of the asset is perfectly elastic, market equilibrium would result in the allocation of funds entirely in the risky asset.

\footnote{It may be convenient to think of the risky asset as of an investment project like a production technology, which yields different outcomes in two different states of nature.}
asset. This is a reference point for the analysis.

3.2 Banks

Assume, transaction costs prevent agents from entering the market for the risky asset. They still have an access to the market of the risk-free asset. Transaction costs justify the existence of banks, which offer a deposit contract with a duration of one period and without a premature withdrawal option. The banking sector is assumed to be of a unit size, perfectly competitive and homogenous. Banks belong to a small part of agents, who manage banks and are called bankers.4

The sequence of interactions between banks and depositors is shown in Fig. 1. In the first period, three actions take place: first, banks are created, then deposits are collected and, finally, banks invest. In the beginning of the second period, the state of nature is realized. Other three actions take place in the second period: first, banks reap portfolio gains, then deposits are repaid, and the banks are closed. The economy terminates at the end of the second period.

There exists also a regulatory authority (Regulator), which chooses in the first

4 Throughout this paper, banks and bankers are synonyms. Bankers are infinitesimal in the population.
period to either liquidate or bail out insolvent banks. The term "liquidation" is used to describe the insolvency resolution, as opposed to the duly closure of each bank at the end of the second period, when the economy terminates.

4 Insolvency Resolution

In the first period, banks collect deposits in the amount of $D$ and invest them in a portfolio with share $x$ of the risky asset and share $(1 - x)$ of the risk-free one. In the second period, if the state of nature $s \in \{H, L\}$ is realized, the value $V_s$ of a bank is

$$V_s = [xr^s + (1 - x)r^F]D$$

(1)

The bank is insolvent if $V_s < r^D D$. If the insolvent bank is liquidated, each depositor receives $\frac{V_s}{D}$ per unit of the initial deposit and thus faces the state-contingent rate of return $xr^s + (1 - x)r^F$. Bankers enjoy limited liability and obtain the payoff of zero: $\max{(xr^s + (1 - x)r^F - r^D; 0)} D = 0$ if the bank is insolvent.

If the insolvent bank is bailed out, the Regulator injects liquidity in the amount of

$$\max{(r^D D - V_s; 0)} = \max{(r^D - (xr^s + (1 - x)r^F); 0)} D > 0$$

(2)

and depositors are repaid in full.\footnote{For the sake of brevity, the source of such a subsidy is not discussed here. It may be thought as taxes collected by the Regulator from future generations, which are not considered in the two-period setting here.} Bankers are assumed to internalize the costs of bailout proportionally to the size of the liquidity injection with coefficient $\theta \in [0; 1]$.

Payoff of bankers is thus

$$\left[\max{(xr^s + (1 - x)r^F - r^D; 0)} - \theta\max{(r^D - (xr^s + (1 - x)r^F); 0)}\right] D$$

(3)

If $\theta = 1$, we obtain complete internalization of costs by bankers, which corresponds to "unlimited liability".\footnote{To avoid possible negative consumption in the second period, we might assume that agents obtain in the second period a lump-sum payment additionally to the investment payoff. In this case the penalty of} If $\theta = 0$, the payoff of bankers is the same as in the
liquidation case, and as a result, bankers’s choice does not depend on the regulatory policy. The introduction of the costs internalization makes bankers’ choice dependent on the choice of the regulator. It is a metaphor for other distortions in the decision-making by bankers, which may be caused by the regulatory policy.

Hoggarth et al. [2003] stress that government liquidity injections are mostly conditional on changes in senior management, who lose their jobs; at the same time, shareholders bear some losses as well. Although management’s losing jobs is not relevant for the one-period setting in this paper, the losses of shareholders still play an important role. Specifically, in order to prevent moral hazard, government can mandate an infusion of private sector capital, when performing an open bank assistance. This idea is captured by the internalization of costs above. Although there is no moral hazard in the model, costs internalization prevents negative effects of the limited liability (which is then a special case with $\theta = 0$). Brown and Dinç [2005] provide an empirical evidence on costs internalization for bank failures in emerging markets. They report, in particular, that if a failed bank is taken over by the state, prefailure owners and top management lose the most; depositors tend to lose much less, if anything. Bailouts in the current paper do not capture takeovers, but reflect the same idea of penalizing bankers for the bad outcome.

5 Optimal Bailouts

In this section, we derive the optimal bailout policy of the regulator. The objective of the regulator is to replicate the allocation of funds, which would be achieved, if depositors had access to the market of the risky asset. Recall from Section 3 that bankers (internalized bailout costs) is deducted from this amount. This additional payment does not change the decision-making in the first period, this is why it is superfluous for the analysis and not considered in the text.
risky asset dominates the safe one in terms of expected gains, and hence households would invest their entire endowment in the risky asset. Therefore, the Regulator should choose the probability of bailout to ensure in the deposit market equilibrium that (1) agents deposit their entire endowment with banks, and (2) banks portfolios consist entirely of the risky asset.

In period 1, the Regulator announces bailout probability $z$. Depositors are aware of $p$, $z$ and $x$, therefore they can build expectations about future deposit repayments, given the deposit rate $r^D$.

5.1 Households

Households decide upon the composition of their portfolio with share $a$ of deposits and $(1-a)$ of the safe asset and search for $\max_a G^e$, s.t. $0 \leq a \leq 1$, with $G^e$- expected gains of households:

$$G^e = z ar^D + p (1 - z) a \min \left\{ r^D; x r^H + (1 - x) r^F \right\}$$

$$+ (1 - p) (1 - z) a \min \left\{ r^D; x r^L + (1 - x) r^F \right\} + (1 - a) r^F$$

In (4), the first term corresponds to the deposit payoff in the bailout case: $a$ units of deposit are repaid in full with interest rate $r^D$ no matter whether the bank is solvent or not. The second and the third terms correspond to state-contingent deposit payoffs in the liquidation case: if the bank is insolvent, depositors only obtain $x r^s + (1 - x) r^F$ pro unit of deposit in each state of nature $s$. The fourth term describes the payoff through investment in the safe asset, which depends neither on the state of the nature nor on the liquidation/bailout decision of the Regulator.

Since there is a unit mass continuum of households possessing a unit endowment, a solution of the individual optimization problem above determines the aggregate supply
of deposits:

\[ D^* = a^* \in \arg \max_a G^e \]  \hspace{1cm} (5)

Solving for \( a^* \) is straightforward due to the linearity of \( G^e \) in \( a \): depositors place their entire endowment as deposits with banks, as soon as the expected return from depositing is higher than the risk-free rate of interest. If the expected deposit payoff equals to the risk-free return, households are assumed to invest in the deposit contract. This assumption simplifies the exposition. A possible interpretation of it could be infinitesimal transaction costs, induced by a purchase of the risk-free asset. This leads to the following proposition:

**Proposition 1** For a given probability of bailouts \( z \), aggregated deposit supply is

\[ D^* (r^D, x) = \begin{cases} 
1 & \text{if } r^D \geq r^F + \frac{(1-p)(1-z)}{p+z(1-p)}x (r^F - r^L) \\
0 & \text{if } r^D < r^F + \frac{(1-p)(1-z)}{p+z(1-p)}x (r^F - r^L)
\end{cases} \]  \hspace{1cm} (6)

Note that the demand for deposits depends on the deposit interest rate \( r^D \) and on the financial quality of the bank \( x \), and is parametrized on the bailout policy \( z \). Term \( \frac{(1-p)(1-z)}{p+z(1-p)}x (r^F - r^L) \) represents the interest margin, which depositors require in order to switch from risk-free assets to deposits. It is distinct from the risk premium. It is easy to check that the risk premium in this case is zero since agents are risk-neutral.

**5.2 Banks**

In period 1, each bank decides upon its portfolio composition \( x \) and the amount of deposits \( D \) to be collected. The banks are aware of two possible actions of the Regulator: bailout with probability \( z \), and liquidation with probability \( 1 - z \). The state contingent payoff of banks is conditioned on the state of nature \( s \) and on the action of the Regulator and discussed in Section 4. Banks’ payoff in state of nature \( s \) is \( \max [xr^s + (1 - x)r^F - r^D; 0] D \). In addition, if the Regulator plays "bailout", banks
partly internalize bailout costs. This results in the following expected payoff function of bankers:

\[
\Pi^e = p \max\left[ x r^H + (1-x) r^F - r^D ; 0 \right] D + \\
(1 - p) \max\left[ x r^L + (1-x) r^F - r^D ; 0 \right] D - \\
pz\max\left[ r^D - (x r^H + (1-x) r^F) ; 0 \right] D - \\
(1 - p) z\max\left[ r^D - (x r^L + (1-x) r^F) ; 0 \right] D
\] (7)

The first two terms correspond to the expected profit of banks under limited liability. The third and fourth terms stand for the costs internalization. Note that with no internalization, the probability of bailout would vanish from the expected payoff of banks.

Each bank seeks for \( \max_{x,D} \Pi^e \) subject to \( D \geq 0 \) and \( 0 \leq x \leq 1 \). The solution of this optimization problem is again straightforward due to its linearity in both \( D \) and \( x \). It is important to distinguish between the cases \( z = 0 \) and \( z > 0 \). If \( z = 0 \), bank is liquidated with certainty and enjoys limited liability. Its objective function is then zero if \( r^D > r^H \), which implies indifference with regards to \( D \) and \( x \). The same applies to the case \( \theta = 0 \).

**Proposition 2**  If the Regulator bails out insolvent banks with probability \( z \in (0,1) \), and if banks internalize the cost of bailouts \( (\theta > 0) \), the optimal choice \( (x^*,D^d) \) of each competitive bank is:

\[
x^* \in \begin{cases} 
[0;1] & \text{if } r^D > \frac{1}{p + z\theta(1-p)} \left( pr^H + z\theta (1-p) r^L \right) \\
\{1\} & \text{if } r^D \leq \frac{1}{p + z\theta(1-p)} \left( pr^H + z\theta (1-p) r^L \right) 
\end{cases} \\
D^d \in \begin{cases} 
\{0\} & \text{if } r^D > \frac{1}{p + z\theta(1-p)} \left( pr^H + z\theta (1-p) r^L \right) \\
[0,\infty) & \text{if } r^D = \frac{1}{p + z\theta(1-p)} \left( pr^H + z\theta (1-p) r^L \right) \\
\{\infty\} & \text{if } r^D < \frac{1}{p + z\theta(1-p)} \left( pr^H + z\theta (1-p) r^L \right) 
\end{cases}
\]
Commitment to liquidation \((z = 0)\) or no costs internalization \((\theta = 0)\) leads to:

\[
x^* = 1
\]
\[
D^d \in \begin{cases} [0, \infty) & \text{if } r^D \geq r^H \\ \{\infty\} & \text{if } r^D < r^H \end{cases}
\]

Note that in a banking sector of a unit size, \(D^d\) from Proposition 2 describes the aggregate demand for deposits.

5.3 Equilibrium and Optimal Bailout Rule

We need now to define the deposit market equilibrium and compare the resulting allocation with the reference point for different bailout policies. If we denote with \(X^*\) equilibrium aggregate investment in the risky asset, and with \(D^*\) - equilibrium aggregate amount of deposits, then the optimal policy of the regulator is the one, for which \(X^* = D^* = 1\).

**Definition 1** For a given bailout policy \(z\), competitive equilibrium is the allocation of funds \((X^*, D^*)\) and the interest rate \(r^D_c\), which provides

1. \(X^* = x^* D^d\) with \((x^* (r^D_c), D^d (r^D_c)) \in \arg \max \Pi^e\)
2. \(D^s (r^D_c, x^*) = a^* \in \arg \max G^e\)
3. \(D^* = D^s (r^D_c, x^*) = D^d (r^D_c)\)

The definition of equilibrium requires that deposit supply equals deposit demand. Note that equilibrium is parametrized on the bailout policy of the regulator. The portfolio choice \(x^*\) by banks is uniquely determined by the equilibrium interest rate \(r^D_c\) and the regulator’s choice of \(z\). Given \(x^*\) and \(D^*\), the equilibrium investment in the risky asset is determined by \(X^* = x^* D^*\).

**Proposition 3** If the Regulator bails out insolvent banks with probability \(z \in (0, 1]\),
and if banks internalize the cost of bailout ($\theta > 0$), the competitive equilibrium is:

$$X^* = D^* = 1$$
$$r_c^D = \frac{1}{p + z\theta (1 - p)} (pr^H + z\theta (1 - p) r^L)$$

Commitment to liquidation ($z = 0$) or no costs internalization ($\theta = 0$) yields

$$X^* = D^* = 1$$
$$r_c^D \in [r^H; +\infty)$$

The proposition straightforwardly follows from propositions 1 and 2. Fig. 2 illustrates the proposition. In the figure, $r_D^D = r^F + \frac{(1-p)(1-z)}{p+z(1-p)} (r^F - r^L)$ and $r_B^D = \frac{1}{p+z\theta(1-p)} (pr^H + z\theta (1 - p) r^L)$ are the critical rates of interest for both depositors’ supply of and bankers’ demand for funds. It is easy to check that $r_D^D < r_B^D$.\(^7\)

Multiple equilibria in the commitment to liquidation case are characterized by excessive interest rates (higher than the best possible risky outcome), but all of them provide investment of the entire households’ endowment into the risky asset. This allocation is efficient since it replicates the one, which would result from the households’ optimization problem if they had direct access to the market of the risky asset.

This result ensures that the objective of the regulator is achieved with any policy $z \in [0; 1]$. However, it was assumed throughout the discussion that both depositors and bankers are informed about the policy of the Regulator, even if it is a stochastic bailout-liquidation rule $0 < z < 1$. The next section explores, what happens in the economy if the policy choice of the regulator is unknown to the agents and they have to make decisions under ambiguity.

\(^7\) First, note that $\frac{\partial r_B^D}{\partial \theta} < 0$, which implies that it suffices to check for $r_D^D < r_B^D (\theta = 1)$. Multiplying both sides with $p + z (1 - p)$ yields

$$(p + z (1 - p)) r^F + (1 - p) (1 - z) (r^F - r^L) < pr^H + z (1 - p) r^L$$

which is equivalent to

$$r^F - (1 - p) (1 - z) r^L < pr^H + z (1 - p) r^L$$

which holds due to Assumption A-2. Since $r^F + \frac{(1-p)(1-z)}{p+z(1-p)} x (r^F - r^L)$ strictly increases in $x$ (except for certainty $p = 1$ or regulatory commitment to bailouts $z = 1$), it never exceeds $r_D^D$. Since the latter is always smaller than $r_B^D$, bankers’ choice in the corresponding area is always $x = 1$.\(17\)
6 Ambiguous Bailouts

Assume the Regulator does not commit to any bailout rule. Uncertainty about regulatory policy induces uncertainty about the payoff structure in the model. Note that the analysis above does include uncertainty in form of a possible mixed strategy of the regulator, i.e. a stochastic bailout-liquidation rule. Now it is assumed that depositors and bankers possess less information than before, but still are symmetrically informed about the economy. To be precise, depositors and bankers are informed about the following: (1) the set of players in the economy, (2) set of strategies of each player, and (3) payoff functions of all players. Payoff functions are stochastic and determined by the realization of the random variable $s$, which determines the state of nature, and consequently, the realization of the return of the risky asset. As we have seen above, uncertainty in terms of stochasticity does not destroy efficiency of the equilibrium allocation. Ambiguity is an uncertainty, which is distinct from stochasticity.

6.1 Nature of Ambiguity and Decision-making

Under assumption of rationality, agents should be able to predict, which policy the regulator chooses, if they know the payoff function of the latter. Since it was assumed that

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8 One might wish to see nature as a fourth player in the game. This would require additional discussion, which is not in the focus of the current paper.
the objective of the regulator is to provide for efficiency of the equilibrium allocation, both depositors and bankers can identify that this objective may be achieved through any policy in the range $z \in [0; 1]$. However, which probability should they count for, when solving their respective optimization problems? There is no reason, why both depositors and bankers should count for the same probability of bailouts. Clearly, depositors and bankers operate under uncertainty, which is represented by a continuum of probability distributions over the regulatory policy. This kind of uncertainty is a special case of ambiguity.

One of the relevant concepts for decisions under ambiguity is the notion of pessimism and optimism. Wakker [2001] defines optimism and pessimism on the basis of choices, which agents would make, if their actions lead to different outcomes in different states of the world, probabilities of which are unknown. For example, if households in the current paper have access to the market of the risky asset, but are not aware of the probability distribution $p$, they would also face ambiguity. Knowing that two states of nature are possible, they might prefer to invest in the risky asset (which corresponds to optimism) or to invest in the risk-free asset (which corresponds to pessimism). The reason for that is that for an optimist, the best possible outcome overweights the worst one, and for a pessimist the opposite is true.9 There are several ways to capture optimism and pessimism in the decision-making.10 To show that an equilibrium outcome under ambiguity may differ from the one under a stochastic bailout rule, we assume that agents weight best and worst. If we denote the degree of pessimism with $\alpha$, then

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9 Gneezy et al. (2006) provide a paradoxical experimental evidence that a lottery over the best and the worst may be valued significantly lower than the worst outcome itself. Decisions under ambiguity, as described in the text, are not related to such behavioral effects.

10 Chateauneuf et al. (2007) introduce non-extreme outcome additive capacities (neo-additive capacities) to represent the CEU as a weighted sum of the EU-term, a pessimistic term, and an optimistic term. Simple capacities (see, e.g. Eichberger and Kelsey, 2000) also capture the same possibility.
depositors maximize the following functional (Chateauneuf et al., 2007):

\[ \alpha \min_{z \in [0;1]} G^e (z) + (1 - \alpha) \max_{z \in [0;1]} G^e (z) \quad (8) \]

with \( G^e (z) \) denoting the expected gains of depositors (4) for a given bailout policy \( z \).

The first term in (8) corresponds to pessimism and counts for the worst outcome, and the second term corresponds to optimism and counts for the best outcome. Extreme pessimism corresponds to \( \alpha = 1 \). The ambiguity itself is captured by the fact that bailouts may follow any probability distribution \( z \in [0;1] \). More generally, \( z \in \Delta_z \subseteq [0;1] \) with \( \Delta_z \) capturing the degree to which agents are informed about the regulatory policy. If \( \Delta_z = \{ \tilde{z} \} \) then (8) turns into \( G^e (\tilde{z}) \), and we obtain the above discussed case without ambiguity.

Differentiating (4) with respect to \( z \) yields \( \frac{\partial G^e}{\partial z} = 0 \) if \( r^D > xr^H + (1 - x) r^F \), or \( \frac{\partial G^e}{\partial z} > 0 \) in all other cases. Therefore, the worst expected outcome for depositors is associated with liquidation of banks: \( \min_{z \in [0;1]} G^e (z) = G^e (0) \). The best expected outcome takes place if the regulator bail out insolvent banks: \( \max_{z \in [0;1]} G^e (z) = G^e (1) \). This implies that under ambiguity depositors maximize

\[ \alpha \min_{z \in [0;1]} G^e (z) + (1 - \alpha) \max_{z \in [0;1]} G^e (z) = \]

\[ (1 - \alpha) a (1 + r^D) + \alpha pa \min \{ r^D; xr^H + (1 - x) r^F \} + \]

\[ \alpha (1 - p) a \min \{ r^D; xr^L + (1 - x) r^F \} + (1 - a) (1 + r^F) \]

Note that technically, (9) repeats (4) if we replace \( z := 1 - \alpha \).

We can do the same exercise for banks, by replacing \( G^e (z) \) in functional (8) with expected payoff of bankers \( \Pi^e (z) \) from (7). To keep generality, we assume that the degree of pessimism of bankers is given by \( \beta \), which is not necessarily equal to \( \alpha \). Again, we need to identify, what is the worst outcome for bankers, who internalize
bailout costs ($\theta > 0$):

$$\frac{\partial \Pi^e}{\partial z} = -p\theta \max \left[ r^D - (xr^H + (1-x)r^F) ; 0 \right] D -$$

$$- (1-p) \theta \max \left[ r^D - (xr^L + (1-x)r^F) ; 0 \right] D$$

For positive values of $D$ we obtain $\frac{\partial \Pi^e}{\partial z} < 0$ for all $r^D$, except for the case $xr^L + (1-x)r^F > r^D$, in which $\frac{\partial \Pi^e}{\partial z} = 0$. Therefore, the worst expected outcome for bankers is associated with bailouts: $\min_{z \in [0;1]} \Pi^e (z) = \Pi^e (1)$. The best expected outcome is associated with liquidation: $\max_{z \in [0;1]} \Pi^e (z) = \Pi^e (0)$. This is the result of the internalization of bailout costs by bankers. Similarly to depositors, bankers maximize now the functional, which technically repeats (7) with $z := \beta \theta$

$$\beta \min_{z \in [0;1]} \Pi^e (z) + (1-\beta) \max_{z \in [0;1]} \Pi^e (z) =$$

$$(1-p) \max \left[ (xr^L + (1-x)r^F - r^D) ; 0 \right] D -$$

$$\beta \theta p \max \left[ xr^H + (1-x)r^F - r^D ; 0 \right] D -$$

$$\beta \theta (1-p) \max \left[ (xr^L + (1-x)r^F - r^D) ; 0 \right] D$$

Note that similarity between (9) and (4) as well as between (10) and (7) is only technical and does not arise through substitution of some perceived probability of bailouts instead of $z$. This would be the case if we consider asymmetric information leading to different degenerated priors $\Delta_z = \{ z \}$ for depositors and bankers. Instead, the information is symmetric, and both face the same prior $\Delta_z = [0; 1]$ for the bailout policy. Even more, depositors and bankers treat the missing information in the same way, and as a special case we can obtain equal degrees of pessimism $\alpha = \beta$. It is the degree of pessimism that technically replaces $z$ in objective functions.
6.2 Equilibrium under Ambiguity

As noticed above, technically the objective function of depositors (9) under ambiguous bailout policy repeats their objective function (4) with \( z = 1 - \alpha \). Their optimization problem is the same as before. To determine the supply of deposits, it suffices to substitute for \( z = 1 - \alpha \) in Proposition 1:

\[
D^* (r^D, x) = \begin{cases} 
1 & \text{if } r^D \geq r^F + \frac{(1-p)\alpha}{p+(1-\alpha)(1-p)} x \left( r^F - r^L \right) \\
0 & \text{if } r^D < r^F + \frac{(1-p)\alpha}{p+(1-\alpha)(1-p)} x \left( r^F - r^L \right)
\end{cases}
\]

(11)

The same applies to banks. Their objective function corresponds to (7) with \( z = \alpha \). To determine the optimal choice of banks, it suffices to substitute for \( z = \beta \theta \) in Proposition 2:

\[
x^* \in \begin{cases} 
[0; 1] & \text{if } r^D > \frac{1}{p+\beta \theta(1-p)} \left( pr^H + \beta \theta (1 - p) r^L \right) \\
\{1\} & \text{if } r^D \leq \frac{1}{p+\beta \theta(1-p)} \left( pr^H + \beta \theta (1 - p) r^L \right)
\end{cases}
\]

(12)

\[
D^d \in \begin{cases} 
\{0\} & \text{if } r^D > \frac{1}{p+\beta \theta(1-p)} \left( pr^H + \beta \theta (1 - p) r^L \right) \\
[0, \infty) & \text{if } r^D = \frac{1}{p+\beta \theta(1-p)} \left( pr^H + \beta \theta (1 - p) r^L \right) \\
\{\infty\} & \text{if } r^D < \frac{1}{p+\beta \theta(1-p)} \left( pr^H + \beta \theta (1 - p) r^L \right)
\end{cases}
\]

We can define an equilibrium in a similar way as before:

Definition 2  For given degrees of pessimism \( \alpha \) and \( \beta \), the equilibrium under ambiguity with regards to the bailout policy is the allocation of funds \((X^*, D^*)\) and the interest rate \( r^D \), which provide

1. \( X^* = x^* D^d \)
2. \( D^* (r^D_a, x^*) = a^* \)
3. \( D^* = D^s (r^D_a, x^*) = D^d (r^D_c) \)

where \( (x^* (r^D), D^d (r^D_a)) \) maximizes \( \beta \cdot \min_{z \in [0;1]} \Pi^e (z) + (1 - \beta) \cdot \max_{z \in [0;1]} \Pi^e (z) \)

and \( a^* \) maximizes \( \alpha \cdot \min_{z \in [0;1]} G^e (z) + (1 - \alpha) \cdot \max_{z \in [0;1]} G^e (z) \).

Note that the equilibrium is not anymore parametrized on the bailout policy, since it is not announced. Instead, the equilibrium is parametrized on the degree of pessimism of the agents. The following proposition establishes that under extreme
pessimism of agents, the economy may settle in an inefficient equilibrium.

**Proposition 4** Let $\alpha$ be the degree of pessimism of depositors and bankers. If bankers internalize the bailout costs ($\theta > 0$), then the equilibrium under ambiguity is given by:

$$
X^* = D^* = \left\{ \begin{array}{ll}
1 & \text{if } p \cdot \frac{p+(1-\alpha)(1-p)}{p+\beta(1-p)} \geq \frac{r_F - r_L}{r_H - r_L} \\
0 & \text{if } p \cdot \frac{p+(1-\alpha)(1-p)}{p+\beta(1-p)} < \frac{r_F - r_L}{r_H - r_L} \\
\{x\} & \text{if } p \cdot \frac{p+(1-\alpha)(1-p)}{p+\beta(1-p)} \geq \frac{r_F - r_L}{r_H - r_L} \\
[\pi; \overline{\pi}] & \text{if } p \cdot \frac{p+(1-\alpha)(1-p)}{p+\beta(1-p)} < \frac{r_F - r_L}{r_H - r_L}
\end{array} \right.
$$

with $r = \frac{1}{p+\beta(1-p)} (pr^H + \beta (1-p) r^L)$ and $\overline{\pi} = r^F + \frac{(1-p)\alpha}{p+(1-\alpha)(1-p)} (r^F - r^L)$.

Condition $p \cdot \frac{p+(1-\alpha)(1-p)}{p+\beta(1-p)} \geq \frac{r_F - r_L}{r_H - r_L}$ characterizes the investment climate in the economy: it relates risk, pessimism and rates of return. With extreme optimism of both depositors and bankers ($\alpha = \beta = 0$), the left side of it turns to unity and the condition is true for any rates of return $r^H > r^F > r^L$. In this case, regulatory ambiguity has no effect on the market equilibrium. With extreme pessimism ($\alpha = \beta = 1$) and complete internalization of bailout costs by bankers ($\theta = 1$), the condition turns to $p^2 \geq \frac{r_F - r_L}{r_H - r_F}$, which stresses that in economies with high investment risk (low $p$) and relatively low risk-free rate $r^F$, ambiguity in bailout policy would result in a disintermediation: $X^* = D^* = 0$. If the risky asset represents productive firms, the intermediated economy would fail to link creditors (depositors) and borrowers (firms), although the market economy would provide for such a link under the same parameters $p$, $r^H$, $r^L$ and $r^F$.

To highlight the intuition behind the proposition, we assume $\alpha = \beta = \theta = 1$. Competitive banks choose $x^* = 1$ and set the deposit rate so that their expected profit is zero. If $D > 0$, this implies deposit interest rate of $pr^H + (1-p) r^L$, which should exceed or be equal to the rate $r^F + \frac{1-p}{p} (r^F - r^L)$, required by depositors. This is only possible if $p^2 \geq \frac{r_F - r_L}{r_H - r_F}$. In fact, inefficient equilibria appear because pessimistic banks
Figure 3. Equilibrium under ambiguity: extreme pessimism, $\alpha = 1$, and high investment risk, $p^2 < \frac{r_H - r_L}{r_H - r_L}$.

exhibit cautionary behavior and avoid acquiring deposits at high interest rates. At the same time, pessimistic depositors exhibit cautionary behavior as well, and avoid depositing at interest rates, which make the expected return on deposits lower than the risk-free rate.$^{11}$

Figure 4. Degrees of pessimism, costs internalization and inefficient equilibria

Figure 4 provides an illustration of the result for arbitrary degrees of pessimism and costs internalization. Condition $p \cdot \frac{p + (1 - \alpha)(1 - p)}{p + \beta \theta (1 - p)} \geq \frac{r_H - r_L}{r_H - r_L}$ corresponds to the area below the threshold line $\beta \theta = \frac{p}{1 - p} \cdot \frac{r_H - r_F}{p^2 r_H - r_L}$, which intersects the axes in points

$^{11}$ The expected return as given by the probability of the states of nature, not by the bailout policy.
\[
\frac{p}{1-p} \frac{r_H - r_F}{r_F - r_L} > 1 \quad \text{and} \quad \frac{1}{1-p} \frac{r_H - r_F}{r_F - r_L} > 1
\] (both are above unity due to Assumption A-2). For some given degree of costs internalization \(\theta\), the dotted area in the picture represents inefficient equilibria. In order for disintermediation to arise in equilibrium, neither do depositors and bankers need to be extremely pessimistic \((\alpha = \beta = 1)\), nor do they need to have equal ambiguity attitude \((\alpha = \beta)\) at all. The area above the line \(\beta = \alpha\) represents higher pessimism of bankers, whereas the area below the line corresponds to depositors being more pessimistic than bankers. If \(p^2 \geq \frac{r_F - r_L}{r_H - r_L}\), the threshold line lies above the area \([0; 1]^2\), thus making inefficient equilibria impossible for any values of parameters \(\alpha, \beta\) and \(\theta\). This explains why the issue of disintermediation due to the ambiguity in banking regulation can be less (if anything) relevant for economies with low aggregate investment risk (high \(p\)).

If \(p^2 < \frac{r_F - r_L}{r_H - r_L}\), threshold \(\overline{\theta} = \frac{p}{1-p} \frac{pr_H(1-p)r_F}{r_F - r_L} + \frac{(1-p)r_L(1-p)r_F}{r_F - r_L}\) represents the highest degree of costs internalization, which still allows to avoid disintermediation for any ambiguity attitudes of the public: the penalty for bankers is not high enough to prevent them from operating under a deposit interest rate, which would be satisfactory for depositors, even if both exhibit extreme pessimism \((\alpha = \beta = 1)\). Note that the internalization of costs by bankers is in this case conditional on the investment climate: with high investment risk (low \(p\)) it might be optimal for the regulator to encourage banking activity through setting low \(\theta\).\(^{12}\) However, this is unnecessary if the regulator informs the public about the bailout policy.

Recall that objective functions of depositors (9) and bankers (10) under ambiguity technically coincide with their objective functions (4) and (7) under announced bailouts if \(z = 1 - \alpha\) for depositors and \(z = \beta \theta\) for bankers. If the bailout policy \(z\) is announced

\(^{12}\) \(\theta\) should still be positive to avoid negative effects of limited liability.
to the public, beliefs of depositors and bankers align along the line $\beta\theta = 1 - \alpha$ in Figure 4, which lies entirely in the area of efficient equilibria, no matter how (un)favorable the investment climate is and what degree of costs internalization is imposed by the insolvency regulation.

7 Conclusions

Regulatory ambiguity and intransparency have been for a long time being in the center of economic debates. The common approach to the issue is representing intransparent regulatory policy with a probability distribution over its possible realizations. This approach fails to capture possible heterogeneity of beliefs of uninformed agents. If the policy of the regulator is not announced, the public estimates the likelihood of the future outcomes according to their degrees of pessimism or optimism. Even if the public are homogenous in their ambiguity attitude, they can form different beliefs, if the regulation has an asymmetric impact on them.

In the current paper, regulatory ambiguity is studied in the market equilibrium framework. It is shown that even if agents are perfectly rational and symmetrically informed about each other, as well as about the macroeconomic environment, some missing piece of information can play a crucial role in determining the equilibrium outcome. The fact that the regulator is better informed about his policy than the public, does not create a problem of asymmetric information, since the regulator does not participate in the market interactions. If the perfectly rational public are informed about the objective function of the regulator, they may wish to find the optimal regulatory policy, which they would count for in their decision-making. However, if there are multiple optima, the public have to make decisions under ambiguity.
Regulatory ambiguity is studied here in application to the deposit market. An ambiguous bailout policy creates an asymmetry in beliefs of depositors and bankers with regards to the action of the regulator in case of banks’ insolvency. This may result in a suboptimal allocation of funds as compared with the market outcome. Informing agents about the probability of bailouts eliminates the asymmetry in beliefs and restores the optimal allocation of funds. This result provides a reason for limiting the "constructive ambiguity" to a stochastic bailout rule with a probability of bailouts known to both bankers and depositors.

The inefficiency result seems to be more likely for economies with high aggregate investment risk and high internalization of bailout costs by banks (penalty on bankers). If the regulator cannot credibly signal about his policy, and as a result the beliefs of the public cannot align, efficient equilibria still can be ensured, if the internalization of bailout costs by banks is low. This suggests an additional aspect of a penalty based regulation.

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