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Abstract

We study the optimal R&D trajectory in a setting where new technologies are never perfect backstops in the sense that there is no perfectly clean technology that eventually solves the pollution problem once and for all. New technologies have stings attached, i.e. each emits a specific stock pollutant. Damages are convex in individual pollution stocks but additive across stocks, creating gains from diversification. The research and pollution policies are tightly linked in such a setting. We derive the optimal pollution path and R&D program. Pollution stocks overshoot and in the long run all available technologies produce. Research is sequential and the optimal portfolio of technologies is finite.

JEL classification: Q55, Q53, O32, C61
Keywords: horizontal innovation, stock pollution, backstop technology, multi-stage optimal control, pollution thresholds, overshooting

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1 Introduction

In much of the literature studying the linkage between technological change and the environment, we assume the possibility of a 'clean' technology or 'environmental backstop' that will solve the pollution problem once and for all [2, 8]. This assumption of a 'magic bullet' constitutes a very productive modeling shortcut that has allowed the formulation of novel results on the optimal timing of R&D [for example 2, 18], on the subtle links between R&D policy and pollution control [17, 1] and other relevant issues. In practice, however, those technologies that are developed and adopted to provide a solution to an existing pollution problem themselves commonly involve the production of new pollutants. In time, therefore, these new technologies require the development of a new solution. Two examples of many are chlorofluorocarbons (CFCs) and carbon dioxide (CO_2). CFCs are a pollutant blamed for the depletion of the ozone layer. Their introduction resulted from the search for a substitute for poisonous refrigerants. After their effective ban by the Montreal Protocol new substitutes (e.g. HCFC-123) have been developed that are suspected to cause cancer and decay into toxic substances such as trifluoroacetic acid. CO_2 is a major greenhouse gas accumulating in the atmosphere due to fossil fuel based energy production. The primary substitute, nuclear power, generates radioactive waste with substantial half-life periods. Modeling R&D as the successful quest for a 'magic bullet' may therefore not be a very realistic depiction of what is achievable through R&D.

If new technologies are imperfect in the sense that new technologies will themselves frequently generate new technology-specific types of pollutants that will accumulate in the environment, then this may have important implications for environmental R&D. In such a setting, R&D will consist of a sequence of innovative steps taken against the background of an evolving pollution situation that is itself a product of previous technological choices.

In order to provide some initial insights into an optimal pollution and R&D policy without 'magic bullets', we construct a simple model for studying potentially infinite sequences of innovation targeted at solving stock pollution problems. The model consists of a production sector producing a single good up to a fixed output constraint. Production generates a technology-specific stock pollutant. Damages are convex in each pollutant and additive across pollutants. At any given time, a new technology with a zero stock of existing pollution can be provided at a fixed cost. In this set-up, we study the optimal R&D sequence and pollution stock dynamics.

Our approach is closely related to the literature on stock pollution (starting with [14]) and optimal "green" R&D. [17] study the problem of a stock pollutant and the conditions under which a switch to a perfect backstop should be made. [6] consider the optimal transition to a perfect backstop technology in terms of the stock of production capacity. [1] examines the interplay between abatement policies and innovation policies for a stock pollutant when a clean technology can be generated through a stochastic R&D process. In related papers developed in the context of non-renewable resources, [9] study the impact of backstops of different quality on the optimal timing of adoption and the path of resource depletion while [18] analyze the optimal timing of the switch to a backstop technology when the cost of that technology decreases with the amount of R&D carried out. [19] investigate the impact of such continuous technological change on the growth paths of economies.

While focusing on similar questions as these papers, our modeling context differs on account of the assumption that no single innovation ever fully resolves the stock pollution problem. This implies that we need to consider not just the next technology to be phased in, but entire technology trajectories instead. We also differ in other areas: In contrast to [1], R&D in our model is deterministic; however, the technology is not yet available as in [6] and needs to be acquired at a cost. In contrast to [18, 19], technological progress in this model is discrete rather than continuous. While we initially assume a time-invariant fixed cost of R&D, we also consider - in the spirit of [18] - the implications of the cost of R&D decreasing in the stock of R&D already carried out on the optimal timing of R&D.

Our contributions take the form of a number of propositions and a methodological novelty. As our main result, we characterize the optimal R&D path as involving a finite sequence of innovations carried out at increasing interval length. Not more than one technology is developed at any single point in time, so that there is no front-loading of the technology portfolio. In short, therefore, the finite portfolio of production technologies is built up over time at decreasing speed. Secondly, we confirm the 'overshooting' results observed by [6] in that excess stock of pollution is built up early in the planning period. Thirdly, except for finite adjustment periods, the optimal pollution policy involves the joint use of several production technologies rather than full replacement of technologies of earlier vintage as observed elsewhere. Fourthly, positive feedback between R&D activities and the cost of R&D leads to initiating all R&D activities earlier similar to [18]. Lastly, on the methodological side we introduce recent results from multi-stage dynamic optimization theory [11] into the literature on R&D policy.

The structure of the paper is as follows: We present the simple model in section 2, and derive the optimal pollution policy in section 3. In section 4 we study the optimal R&D sequence, derive the main results and demonstrate the empirical relevance of the policy. We discuss alternative specifications of the R&D process and the effects of more general types of technologies in section 5. In section 6 we conclude.

2 The Model

The model consists of two fundamental components, one describing the nature of the stock pollution problem and the other the process of innovation. Jointly, they describe the social planner's problem of developing a simultaneous pollution and innovation policy.

The environmental side of our model consists of standard pollution stock dynamics common in this literature (for example [6, 1]).[1] At time t, there are $n(t)$ different potential pollutants $i \in \{1, ..., n(t)\}$ with associated stock levels

[1]See [15] for alternative specifications.

$S_i(t)$ with stock dynamics of the type:

$$\dot{S}_i(t) = \alpha_i q_i(t) - \delta_i S_i(t) \tag{1}$$

with α_i denoting the rate of accumulation on the basis of emissions of volume q_i and δ_i denoting the rate of decay in the stock of pollutant i.

Pollutants are technology-specific and, in the interest of tractability, do not interact with each other. Hence, i denotes both the technology and the single pollutant generated by this technology. The pollution damage function is additively separable in the square of individual stocks of pollutants such that pollution damage $D\left(S_1(t), ..., S_{n(t)}(t)\right)$ caused by stocks $S_1(t)$ to $S_{n(t)}(t)$ at time t is

$$D(S_1(t), ..., S_{n(t)}(t)) = \sum_{i=1}^{n(t)} \frac{d_i}{2} S_i(t)^2 \tag{2}$$

with d_i denoting the marginal damage coefficient of pollutant i.

The general form of the instantaneous welfare from production at time t is assumed to be additively separable

$$W(t) = \sum_{i=1}^{n(t)} \left[q_i\left(t\right)^{\beta} - c_i\left(q_i, t\right) - \frac{d_i}{2} S_i(t)^2 \right] \tag{3}$$

with $c_i(q_i, t)$ denoting the production cost at time t given output q_i and $0 < \beta \leq 1$. Given the general form of (3), there are at least five reasons for conducting R&D in such a setting: (1) Cost reduction [16], thus targeting $c_i(q_i, t)$; (2) improvements in the output-emission ratio [e.g. 3, 5] through searching for products with lower α_i; (3) amelioration of environmental damages through finding less harmful technologies, implying a lower d_i, or less persistent pollutants, implying a higher δ_i; (4) technological diversification that increases the variety of consumer goods on account of new technologies [4] if $\beta < 1$ and marginal welfare is hence decreasing in the output of each individual product; and finally (5) technological diversification that increases the variety of existing pollutants because damage is convex in each individual pollution stock but additive across stocks.

All of the reasons mentioned above individually provide positive incentives for resources to be devoted to R&D. Most relevant for a policy problem involving imperfect technologies is the last setting where diversification in pollutants is the primary reason for devoting resources to R&D. Therefore, we design a model that strips out all these other well established drivers before exploring the implications of additional factors in section 6. The mechanism underpinning R&D investments then is similar in spirit to the well-known product differentiation models of the "horizontal innovation" type [7], with one important difference: Instead of increases in the variety of products, it is increases in the variety of pollutants that generates welfare gains by decreasing marginal damages associated with production. In this sense, our paper models a process of "green" horizontal innovation of pollution differentiation.

As a consequence, the model that follows contains some important simplifications: Technologies and pollutants respectively are assumed to be symmetric in terms of rate of accumulation $\alpha_i = \alpha$, rate of decay $\delta_i = \delta$, and the marginal damage coefficient $d_i = d$, thus eliminating R&D motives (2) and (3). Costs are assumed symmetric and zero such that $c_i\left(q_i, t\right) = 0$, eliminating motive (1). In order to strip out product differentiation gains (4), technologies are perfect substitutes ($\beta = 1$) and symmetric in terms of net marginal benefits which are normalized to 1 per unit of output. Aggregate output is exogenously bounded from above as in [1]. This is an indirect way to take capital stock constraints into account.

$$\sum_{i=1}^{n(t)} q_i(t) \leq 1 \tag{4}$$

$$0 \leq q_i(t) \leq 1 \quad , \forall i \in \{1, ..., n(t)\} \tag{5}$$

The symmetry of the technologies in terms of the production-pollution side of the model then simplifies the instantaneous welfare function (3) to

$$W(t) = \sum_{i=1}^{n(t)} \left[q_i(t) - \frac{d}{2} S_i(t)^2 \right] \tag{6}$$

in which technologies now differ in terms of vintage only.

Innovation is modeled in the simplest fashion as a deterministic process: At any time t, society can choose to spend resources $R(t)$ which will make available instantaneously and with certainty the $n+1$st technology. Call this point in time t_{n+1}. We also allow for the development of more than one technology at the same point in time. In this case t_{n+1} and t_{n+2} coincide. The number of technologies $n(t)$ available for production at t therefore depends on the sequence of past investments $\{t_1, ..., t_n\}$. All new technologies start with an initial stock of pollution $S_n(t_n) = 0$ and can at once be used at any level of intensity. We therefore abstract from questions about the optimal accumulation of technology specific capital which has been studied by [6]. For convenience, we assume initially that the current cost of R&D is independent of time such that $R(t) = R$

and that at the beginning of the planning period, one technology is available such that $n(0) = 1$. Furthermore, we assume that there is an arbitrarily large but finite number of technologies M that can possibly be developed. One can think of a finite pool of ideas to solve a particular problem.

The social planner's problem is to maximize net welfare from production over an infinite time horizon, subject to the effects of stock pollution and subject to the deterministic and instantaneous R&D process. Its choice variables are on the one hand the production intensities $q_i(t)$ of the currently available technologies $i \in \{1, ..., n(t)\}$ and on the other hand, the timing of R&D activities $\{t_2, t_3 ...\}$ that expand the set of available technologies $n(t)$ from $n(0) = 1$ up to a finite $N \leq M$ that is also endogenously determined. The problem is then

$$\begin{aligned}\max_{\{q_i(t)\},\{t_2,t_3,\ldots,t_N\},\{N\}} &= \int_0^{t_2} e^{-rt}\left[\left(q_1 - \frac{d}{2}S_1^2\right)\right]dt - e^{-rt_2}R \\ &+ \int_{t_2}^{t_3} e^{-rt}\left[\sum_{i=1}^{2}\left(q_i - \frac{d}{2}S_i^2\right)\right]dt - e^{-rt_3}R \\ &+ \ldots \\ &+ \int_{t_N}^{\infty} e^{-rt}\left[\sum_{i=1}^{N}\left(q_i - \frac{d}{2}S_i^2\right)\right]dt \end{aligned} \tag{7}$$

subject to conditions (1), (4), (5) and the transversality condition

$$\lim_{t\to\infty} H_N^*(t) = 0. \tag{8}$$

To sum up, the nature of the planner's problem describes a situation in which the choice of pollution policy and R&D policy are linked. This is because the past history of R&D determines the planner's degrees of freedom in allocating production shares to different technologies. The solution strategy involves separating the problems into an optimal pollution policy given a number of technologies and the optimal R&D policy that determines the number of technologies that should be available at any given point in time.

3 The Optimal Pollution Policy

The optimal pollution policy between any two innovation events is a standard deterministic Markov-process where the number of state variables equals the number of available technologies. Conditional on this number and the pollution stocks at the beginning of the considered planning period the optimal policy can be derived. This is done in this section while the optimal R&D policy is studied in Section 4. Note that while studying the optimal pollution policy the number of technologies remains fixed at $n = n(t_i)$ for all $t \in [t_i, t_{i+1}]$, $i = \{1, ..., N\}$, where $t_1 = 0$ is the arrival time of the first (free) technology.

Given the number of technologies n and their pollution stock levels $S_i(t)$, the Hamiltonian of this problem is

$$H = e^{-rt}W(t) + \sum_{i=1}^{n} \left[\mu_i(t)\left(\alpha q_i(t) - \delta S_i(t)\right)\right] + e^{-rt}\kappa_n(t)\left(1 - \sum_{i=1}^{n} q_i(t)\right)$$

where μ_i is the shadow price of pollution stock S_i and κ_n is the shadow price of the output constraint (4). The corresponding first order conditions are

$$\frac{\partial H}{\partial q_i} = e^{-rt} + \alpha\mu_i(t) - e^{-rt}\kappa_n(t) = 0 \tag{9}$$

$$-\frac{\partial H}{\partial S_i} = e^{-rt}dS_i(t) + \delta\mu_i(t) = \dot{\mu}_i. \tag{10}$$

Condition (9) requires that the present value of the marginal benefits of production equals the sum of marginal future damages and shadow price of the capacity constraint, which is the present value of the opportunity costs of not allocating more output to another technology. Note that $\mu < 0$ while $\kappa_n \geq 0$. (9) gives rise to the following switching function

$$\sigma_i(t) = \alpha\mu_i(t) + e^{-rt}[1 - \kappa_n(t)] \begin{cases} < 0 & , \quad q_i(t) = 0 \\ = 0 & , \quad q_i(t) = q_i^*(t) \\ > 0 & , \quad q_i(t) = 1 \end{cases} \tag{11}$$

There are three relevant cases to be considered: (a) the singular case where all pollution stocks will be symmetric, (b) a non-singular case where one technology has initially a zero stock while stocks of other technologies are at the same positive level and (c) a non-singular case where one technology has initially a zero pollution stock and there are different positive stock levels. This selection is exhaustive because new technologies always start with a zero pollution stock. Case (a) applies before the first innovation as well as after pollution stocks of existing technologies have converged. If innovation occurs while the economy is in an (a) phase, case (b) is relevant. However, if the economy is in phase (b) or (c) when innovation occurs, (c) is appropriate.

(a) The Singular Solution

Technologies for which the switching function (11) is zero

$$\sigma_i(t) = 0 \tag{12}$$

are on a singular path. Observe that, due to (9) and (10), the switching function is zero for more than one technology only if their stocks are symmetric.

The following shadow price dynamics apply to all technologies on a singular path

$$\mu_i(t) = \frac{e^{-rt}}{\alpha} \left(\kappa_n(t) - 1\right) \tag{13}$$

$$\dot{\mu}_i(t) = -\frac{e^{-rt}}{\alpha} \left[r \left(\kappa_n(t) - 1\right) - \dot{\kappa}_n(t)\right]. \tag{14}$$

Three relevant cases are considered:[2]

Case 1: $\kappa_n = 0$ and $\dot{\kappa}_n = 0$

Case 2: $\kappa_n > 0$ and $\dot{\kappa}_n = 0$

Case 3: $\kappa_n > 0$ and $\dot{\kappa}_n \neq 0$

Case 1

Here, supply falls short of the fixed unit production capacity and the constraint (4) is not binding ($\kappa_n = 0$). Using the first order condition (10) and the shadow price dynamics (13) and (14) one gets

$$S_i(t) = \frac{r + \delta}{\alpha d} \tag{15}$$

$$q_i(t) = \frac{\delta(r + \delta)}{\alpha^2 d}. \tag{16}$$

This is a steady state that is "incomplete" in the sense that the marginal damage of pollution outweighs the marginal benefit of production before the capacity constraint becomes binding. A higher discount rate, lower persistence of pollution, lower emission intensity and lower marginal damages increase output

[2]These are the relevant cases because κ_n can not become negative in this problem.

and stock levels of the incomplete steady state. Both, equilibrium output and pollution stock of technologies are independent of the number of technologies. However, the condition for this steady state to exist

$$n\frac{\delta(r+\delta)}{\alpha^2 d} \leq 1, \tag{17}$$

which follows from (16) and (4), is a function of n. For each set of exogenous parameters thus, there is an upper bound of n above which the incomplete steady state is not feasible.

Case 2

This is the complete steady state as the demand constraint (4) is binding ($\kappa_n > 0$) while the corresponding shadow price is constant ($\dot{\kappa}_n = 0$). Again, using (10), (13) and (14) one gets by imposing symmetry

$$S_i(t) = \frac{\alpha}{\delta n} \tag{18}$$

$$q_i(t) = \frac{1}{n}. \tag{19}$$

Equilibrium output is completely determined by the number of available technologies. The steady state pollution stocks are a function of the pollution intensity α, the depreciation rate of pollution δ and the number of technologies. The discount rate r and the slope of the damage function d do not affect the steady state. The complete steady state is feasible if and only if

$$n\frac{\delta(r+\delta)}{\alpha^2 d} > 1 \tag{20}$$

holds, since $\kappa_n > 0$. Note, that (17) and (20) are mutually exclusive and exhaustive.

Case 3

Supply is at full capacity ($\kappa_n > 0$) but the shadow price of a marginal increase of the production capacity is changing. Case 3 is therefore not a steady state. From (1) and by imposing symmetry one gets

$$S_i(t) = \frac{\alpha}{\delta n} - \frac{\alpha}{\delta n} e^{-\delta t} \tag{21}$$

$$q_i(t) = \frac{1}{n} \tag{22}$$

This is the most rapid approach path to a steady state when all technologies have equal initial pollution stocks. In $t = 0$ the economy has to be in this case because by assumption $n(0) = 1$.[3] As stocks accumulate according to (21), the economy either reaches the incomplete steady state (Case 1) or approaches the complete steady state (Case 2). Conditions (17) and (20) determine which steady state is relevant.

(b) Innovation with Symmetric Stocks

So far only situations where all technologies have the same pollution stock were analyzed. However, if innovation of k technologies occurs at some point in time $t_n > 0$ this is no longer the case. While the incumbent technologies $\{1, ..., n-k\}$ have already accumulated some stock, that of new ones $\{n-k, ..., n\}$ is still zero. Hence, pollution stocks and their respective shadow prices differ across new and established technologies. Here we will assume that this is the first innovation at some strictly positive point in time. However, it will be shown later, that the

[3] The same holds for $n(0) > 1$. Since for all $i \in \{1, ..., n(0)\}$ it holds that $S_i(0) = 0$.

analysis also applies to all subsequent sequences of innovation.

$$S_i(t_n) = \frac{\alpha}{\delta(n-k)} - \frac{\alpha}{\delta(n-k)} e^{-\delta t_n} \quad , i = 1, ..., n-k \tag{23}$$

$$S_j(t_n) = 0 \quad , j = n-k+1, ..., n \tag{24}$$

Here, the singular condition (12) cannot hold for all technologies simultaneously but only for one of the two sets of technologies. Since $S_i(t_n) > S_j(t_n)$ and therefore $\mu_i(t_n) < \mu_j(t_n)$ it has to hold that $\sigma_i(t_n) < \sigma_j(t_n)$. Due to (4), (12) can only hold for the k new technologies while for all $n-k$ old technologies $\sigma_i(t_n) < 0$ and hence

$$q_i(t) = 0 \quad , \forall t \in \left[t_n, \hat{t}_n\right] \quad , i = 1, ..., n-1 \tag{25}$$

$$q_j(t) = \frac{1}{k} \quad , \forall t \in \left[t_n, \hat{t}_n\right] \tag{26}$$

This is the most rapid approach path to a situation where pollution stocks of all technologies are equal. The corresponding stock dynamics are

$$S_i(t) = S_i(t_n) e^{-\delta(t-t_n)} \quad , \forall t \in \left[t_n, \hat{t}_n\right] \tag{27}$$

$$S_j(t) = \frac{\alpha}{\delta k} - \frac{\alpha}{\delta k} e^{-\delta(t-t_n)} \quad , \forall t \in \left[t_n, \hat{t}_n\right] \tag{28}$$

where $\hat{t}_n$ is the point in time where $S_i(\hat{t}_n) = S_j(\hat{t}_n)$. Using (27) and (28) the point of convergence is at

$$\hat{t}_n = t_n + \frac{1}{\delta} \ln \left[\frac{\delta k}{\alpha} S_i(t_n) + 1 \right]. \tag{29}$$

From $\hat{t}_n$ until the next innovation all technologies are used at equal shares and stocks grow according to the following 'Case 3'-process

$$S_l(t) = \frac{\alpha}{\delta n} - \frac{\alpha}{\delta n} e^{-\delta(t-\bar{t}_n)} \quad , t > \hat{t}_n, l = 1, ..., n. \tag{30}$$

The virtual starting point of this process $\bar{t}_n$ is determined by

$$S_l(\hat{t}_n) = S_i(\hat{t}_n) \quad , i = 1, ..., n-1, l = 1, ..., n \tag{31}$$

which yields

$$\bar{t}_n = 0. \tag{32}$$

Hence, the path of the pollution stock after innovation and convergence (30) is exactly the same as the one were all n technologies are available at $t = 0$ (21). Subsequent innovations can therefore be analyzed by exactly the same procedure substituting in the respective new values for n and k. This, however, hinges on the condition that innovation occurs after convergence has occurred. The alternative case is analyzed next.

(c) Innovation with Asymmetric Stocks

Assume innovation occurs at $t_n \in \{t_{n-1}, \hat{t}_{n-1}\}$ where pollution stocks of technologies $\{1, ..., n-k\}$ have not yet converged. Again, it is optimal to follow the most rapid approach path, i.e.

$$q_i(t) = 0 \quad , \forall t \in \left[t_n, \hat{t}_n\right], i = 1, ..., n-k \tag{33}$$

$$q_n(t) = \frac{1}{k} \quad , \forall t \in \left[t_n, \hat{t}_n\right], i = n-k+1, ..., n. \tag{34}$$

Analog to the procedure used to derive $\hat{t}_n$ it is possible to get the point in time where the stocks of technologies $n-k$ and $\{n-k+1, ..., n\}$ converge.

$$\hat{t}_n = t_n + \frac{1}{\delta} \ln \left[\frac{\delta k}{\alpha} S_i(t_n) + 1\right] \tag{35}$$

Whether or not this case actually occurs depends on the optimal timing of R&D. This is analyzed in the next section.

4 The Optimal Timing of R&D

Given the optimal contingent pollution policies derived in the previous section, the social planner faces the problem at which points in time to invest into R&D and thereby buy a new technology with a zero pollution stock.

The following analysis is based on recent results on multi-stage dynamic optimization techniques derived by [11]. Given the initial endowment of $n(0) = 1$ technologies the optimization problem is as follows

$$\begin{aligned}\max_{\{t_2,t_3,\dots,t_N\},\{N\}} &= \int_0^{t_2} e^{-rt}\left[q_1^* - \frac{d}{2}S_1^2\right]dt - e^{-rt_2}R \\ &+ \int_{t_2}^{t_3} e^{-rt}\left[\sum_{i=1}^{2}\left(q_i^* - \frac{d}{2}S_i^2\right)\right]dt - e^{-rt_3}R \\ &+ \dots \\ &+ \int_{t_N}^{\infty} e^{-rt}\left[\sum_{i=1}^{N}\left(q_i^* - \frac{d}{2}S_i^2\right)\right]dt \qquad (36)\end{aligned}$$

where asterisks indicate optimal values, subject to (1), (4) and (8).

The corresponding Hamiltonian for each stage, where n technologies already exist and given the optimal pollution policy, is

$$H_n^* \equiv \sum_{i=1}^{n}\left[e^{-rt}\left(q_i^* - \frac{d}{2}S_i^2\right) + \mu_i(\alpha q_i^* - \delta S_i)\right] \quad , n = 1, ..., N. \qquad (37)$$

Asterisks indicate optimal values. We use the necessary conditions for multiple stage dynamic optimization problems established by [11]. Given the optimal pollution policies two additional conditions[4] determine the optimal instant t^*_{n+1} to undertake R&D in order to develop the n+1st technology. The first condition is that

$$\mu^*_i(t^*_{n+1}) = \breve{\mu}^*_i(t^*_{n+1}) \quad , i = 1, ..., n. \tag{38}$$

where $\mu^*_i(t^*_{n+1})$ is the shadow price of stock i at t^*_{n+1} with n technologies while $\breve{\mu}^*_i(t^*_{n+1})$ is the shadow price of the same stock with $n+1$ technologies. The second condition is that

$$\sum_{n=1}^{N-1} \left\{ \left[H^*_n\left(t^*_{n+1}\right) + e^{-rt^*_{n+1}} rR - H^*_{n+1}\left(t^*_{n+1}\right) \right] \delta t_{n+1} \right\} \leq 0 \tag{39}$$

for any admissible perturbation δt_{n+1} in the innovation time t^*_{n+1}. This yields an R&D arbitrage equation of the form (proof see appendix)

$$rR = \alpha \left[\breve{\mu}^*_{n+1}\left(t^*_{n+1}\right) \breve{q}^*_{n+1}\left(t^*_{n+1}\right) - \sum_{i=1}^{n} \mu^*_i\left(t^*_{n+1}\right) q^*_i\left(t^*_{n+1}\right) \right] e^{rt^*_{n+1}}. \tag{40}$$

Hence, the optimal time to innovate is when the marginal gain of waiting (the left hand side) equals the marginal cost of doing so (the right hand side). Shadow prices and optimal quantities differ contingent on whether innovation occurs during the transition period or when all technologies are used simultaneously. In the former case $q^*_1 = ... = q^*_{n-k} = 0$, $q^*_{n-k+1} = ... = q^*_n = 1/k$ and $\mu^*_{n-k+1} = ... = \mu^*_n$, where k is the number of technologies developed at t^*_n, while in the latter $q^*_1 = ... = q^*_n = 1/n$ and $\mu^*_1 = ... = \mu^*_n$. In both cases (40) can be rewritten as

$$rR = \alpha \left[\breve{\mu}^*_{n+1}\left(t^*_{n+1}\right) - \mu^*_n\left(t^*_{n+1}\right) \right] e^{rt^*_{n+1}} \quad , n = 1, ..., N-1 \tag{41}$$

[4] Condition (38) in our paper is a simplified version of (21) in [11]. This is possible because the costs of R&D are independent of the pollution stock in our model.

using (26) which states that after innovation of a single technology at t_{n+1}^* its output is at full capacity ($\check{q}_{n+1}^* = 1$). Hence, the optimal timing of R&D depends only on the shadow prices of the new (μ_{n+1}^*) and of the most recent active (μ_n^*) technology. The latter has a pollution stock that is a lower bound to all stocks of technologies available prior to innovation.

We now establish the first four key propositions regarding the optimal innovation and pollution policy.

Proposition 1

Innovation is sequential. At most one technology is developed at any point in time.

If more than one technology is developed at the same instant, (41) would also have to hold for technologies $n+1$ and $n+2$. However, by symmetry their shadow prices have to be the same at the instant they are developed. Hence, $\mu_{n+2}^*\left(t_{n+1}^*\right) - \mu_{n+1}^*\left(t_{n+1}^*\right) = 0$ and therefore (41) can not hold for more than one new technology at each point in time unless innovation is costless ($R = 0$) or discounting nonexistent ($r = 0$).

The shadow prices can be calculated given the optimal dynamics of pollution stocks derived above (see appendix). Substituting them into (41) yields the following condition for the optimal time of innovation

$$rR = \frac{\alpha d}{r+2\delta}\left(S_n^*\left(t_{n+1}^*\right)+\frac{\alpha}{\delta}\right)\left[1-\left(\frac{\delta}{\alpha}S_n^*\left(t_{n+1}^*\right)+1\right)^{-\frac{r+2\delta}{\delta}}\right]$$
$$-\frac{\alpha^2 d}{\delta(r+\delta)}\left[1-\left(\frac{\delta}{\alpha}S_n^*\left(t_{n+1}^*\right)+1\right)^{-\frac{r+\delta}{\delta}}\right]. \tag{42}$$

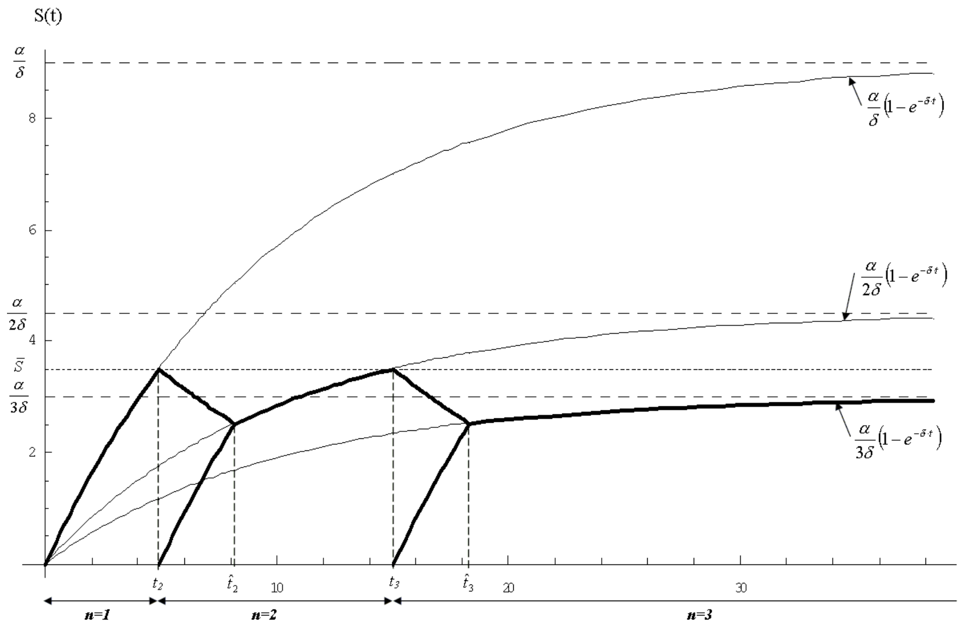

Figure 1: Optimal pollution policy and R&D sequence (*N=3*)

Note, that time enters only via the pollution stock of the most recent technology n. Hence, there is a constant threshold level $\bar{S} = S_n^*\left(t_{n+1}^*\right)$ that triggers innovation. Such an $\bar{S}$ exists for all relevant parameter values, since the right hand side is monotonic in S_n^*, is zero for $S_n^* = 0$ and approaches infinity for $S_n^* \to +\infty$, while the right hand side has a finite and non-negative value. Some-what surprisingly, only the pollution stock of the most recent technology is relevant for the timing of innovation. Aggregate pollution in the economy and all other stocks do not appear in (42). The intuition is the following. Stocks are only relevant as far as they indicate marginal changes in future damage of additional emissions (expressed by shadow prices). Hence, only stocks of technologies actually in use prior to innovation are relevant. Due to symmetry, technologies are only used simultaneously, when their stocks are also symmetric. Additive separability of damages across technologies (2) together with symmetry ensures that shadow prices are equal across all technologies used at the same time. Hence, the policy maker can restrict attention to the stock of the most recent technology for the purpose of determining R&D (see the step from (40) to (41)). Moreover, the threshold pollution stock $\bar{S}$ that triggers R&D is constant. All technologies in use prior to innovation have the same shadow prices. Since the aggregate output constraint is always binding prior to innovation, the number of technologies engaged in production does not affect the weighted sum of marginal future damages $\sum_{i=1}^{n} \mu_i^* \left(t_{n+1}^*\right) q_i^* \left(t_{n+1}^*\right)$. Furthermore, all new technologies are identical and equally costly. Only the shadow price of the technologies is dependent on time. Since the problem is a Markov-process, this dependence is perfectly explained by the corresponding pollution stocks. Hence, neither time nor the number of technologies already available directly affect the optimal timing of R&D.

In Figure 1 the threshold $\bar{S}$ is given by the dotted horizontal line. The dashed horizontal lines indicate the steady state levels for $n = 1$, $n = 2$ and

$n = 3$. While the actual equilibrium stocks are represented by bold lines, the fine (solid) lines are the approach paths to the steady states given 1, 2 or 3 technologies, respectively. Note that the approach path for the first technology ($S_1 = \frac{\alpha}{\delta}(1 - e^{-\delta t})$) also indicates the evolution of the aggregate stock of pollution.

Proposition 2

Innovation occurs only when all available technologies are used simultaneously.

The threshold level is the same for innovation to occur on or off the singular path (see above). Hence, innovation off the singular path is impossible with fixed costs of R&D. The reason is that the system is off the singular path only during the process of stock convergence following an innovation and then stocks are by definition below the trigger level. This has the interesting implication that the aggregate pollution stock of the economy is twice (n-times) as high when the third ($n + 1^{st}$) technology is developed than it was when the second arrived. However, what matters for the optimal timing are changes in marginal future damages rather than aggregate stocks. Due to the additive separability of damages across technologies, the two concepts are not the same.

Proposition 3

The number of technologies developed is finite and the entire portfolio is used in the long run. Innovation is spread over time with increasing intervals between *successive innovations.*

By comparing $\bar{S}$ to the pollution stock in the complete steady state (18) it is apparent that eventually innovation ceases as the complete steady state level drops below the trigger level of innovation (see Figure 1). Hence, for any set of exogenous parameters there is a finite number of technologies that are developed. Moreover, once the pollution stock of the N^{th} technology and those of the other technologies have converged, the entire technology portfolio is used simultaneously for the infinite future.

After a new technology is developed pollution stocks converge along the most rapid approach path. This takes $\hat{t}_{n+1} - t_{n+1}$. According to (29) the length of this period is independent of the number of technologies already available. Moreover, the next innovation is triggered if all pollution stocks simultaneously reach the constant threshold level. Since after convergence is completed all technologies are used at a rate of $1/(n+1)$ the time that passes between successive innovations increases.

Innovation occurs only if both types of steady state stocks (the complete and the incomplete) are above $\bar{S}$. Moreover, since the incomplete steady state is independent of the number of technologies it will never be reached if innovation occurs at least once.

Proposition 4

Pollution stocks overshoot. Each time innovation occurs, pollution stocks are *above their long run steady state level.*

A direct consequence of the constant threshold and a steady state pollution stock that decreases in n is an overshooting of the pollution stock if innovation occurs at least once (see Figure 1). This effect has also been found by [6] in a model involving only a dirty and a clean technology.

While clearly stylized, key elements of the predicted pattern generated by this model are empirically observable phenomena, in particular the temporary displacement of established technologies by new substitutes, the simultaneous use of different technologies, and a sequential increase in the portfolio of technologies. These phenomena will be most easily observed in settings where users are essentially indifferent about the production technology, justifying the assumption of perfect substitutability, and the technology-specificity of capital is low, thus justifying the assumption of insignificant investment constraints. A suitable setting for this is the context of refrigeration. Consumers are arguably indifferent about the technological basis of the refrigeration services they consume; and the rate of product replacement for smaller devices is sufficiently high and retrofitting is economical for most existing larger installations [12]. From the 1890s, when refrigeration became commercially viable, several technologies based on different refrigerants competed in this market. The three main competitors were technologies based on ammonia, carbon dioxide and sulphur dioxide, each with specific health and environmental drawbacks. In the mid-20^{th}

century, they were substituted by CFCs on a large scale. After the ozone depleting effect of CFCs was discovered, three things happened. First, production of CFCs was phased out. Second, the available alternative technologies based on ammonia, carbon dioxide and sulphur dioxide were revived [13]. Third, research in and subsequently production of new substitutes such as perfluorocarbons (PFCs) and HCFCs increased. Both PFCs and HCFCs have a considerably lower ozone depleting potential than CFCs. However, both have stock pollution problems of their own: HCFCs decay into trifluoroacetate (TFA) which is toxic and accumulates in harmful amounts in soil and vegetation, necessitating policy intervention in time [10]. PFCs result in the release of greenhouse gases and therefore contribute to an existing stock pollutant problem. As a result, PFC production is included as a regulatory target in the context of the Kyoto Protocol [12]. Hence, despite the highly stylized nature of the model, core features of the predicted pattern arise in in suitable real world settings.

5 Extensions

In the following two sub-sections we consider generalizations to the model specifications. In section 5.1., we consider alternatives to the assumption of time-invariant R&D costs. In section 5.2., we study the effects of allowing for generalized welfare and stock accumulation functions. It will turn out that the extensions in the two sections differ with respect to their impact on the optimal R&D and pollution policy. Both extensions affect the general properties of the

R&D arbitrage equation (40), with the extensions of the first section affecting its left-hand side and those of the second affecting its right-hand side. The general properties of the optimal pollution policy, on the other hand, are unaffected by changes to the assumption on R&D costs. A precise characterization of the effects of generalized R&D processes on the innovation and pollution dynamics in section 5.1. is therefore possible. Alternative welfare and stock accumulation functions, by contrast, can have a profound impact on the optimal pollution policy. As a result, a complete characterization in section 5.2. is not possible within the limits of this paper. Instead, we offer several partial results as building blocks for future research.

5.1 Alternative R&D Processes

In this section, we relax the assumption of time-invariant R&D costs to study cases such as an exogenous reduction in research costs over time as well as increasing and decreasing returns to R&D. All have in common that they affect only the left hand side of condition (40). With the optimal pollution policy between innovations unaffected, optimal timing is as before determined by (42) and hence, only the specific timing of innovation changes.

Exogenous Efficiency Improvements in Research

We now assume that the costs to develop a new technology exogenously decrease over time

$$R = R(t) \quad \text{with } \dot{R} < 0.$$

This can be due to technological progress realized outside of the economy or industry under concern. The cost to acquire a new technology decreases over time and so does the innovation trigger level, $\bar{S}(t_{n+1}) > \bar{S}(t_{n+2})$. Hence, the time between successive innovations does no longer necessarily increase and is certainly shorter than under constant research costs at the same initial level. The steeper the slope of the research cost function the more likely are decreasing intervals between innovations. If the cost decline is sufficiently steep, the trigger level might be reached before technologies have completely converged. In this case Proposition 2 ceases to hold. Moreover, if $R(t)$ converges sufficiently fast toward zero, there might be no finite $N \leq M$ where innovation stops. If we relax the assumption of a finite upper bound M on the number of potential innovations, the first order condition (41) is no longer a necessary condition and theory offers no guidance on alternative necessary conditions [11]. While the optimal timing of R&D cannot be established, it is certain that innovation proceeds *ad infinitum*.

Increasing Returns to R&D

Assume, e.g. due to learning by doing, that the costs of R&D decrease with the number of technologies already developed

$$R = R(n) \quad \text{with } \frac{\partial R}{\partial n} < 0. \tag{43}$$

According to the same logic as in the previous specification with exogenous cost reductions, innovation occurs earlier than with constant research costs and potentially more technologies are developed. The former is in line with findings by [18]. Propositions 1 and 4 hold while 2 and 3 do not. Again, the formal analysis is restricted by the lack of a theoretical proof of necessary conditions for optimal control problems with infinite regime switches and an infinite time horizon.

Proposition 5

If the costs of research decrease over time, at least as many technologies are developed than in a situation with similar initial but constant research costs. Innovation might not cease. If it does, research occurs earlier than in a situation with similar initial but constant research costs.

Decreasing Returns to R&D

Assume the costs of R&D increase with the number of technologies already developed. For example, it may become more and more difficult to find new solutions to the same problem.

$$R = R(n) \quad \text{with } \frac{\partial R}{\partial n} > 0 \tag{44}$$

Proposition 6

If the costs of research increase in the number of already developed technologies, research occurs later and at most as many technologies are developed than in a *situation with similar initial but constant research costs. Innovation neither guarantees overshooting nor production at full capacity in the long run.*

The innovation trigger level increases in the number of technologies already developed, since R is increasing in n. Hence, the time between successive innovation increases compared to the case with similar initial but constant research costs. Propositions 1 to 3 hold while 5 does not. Overshooting does not occur if the long run steady state is above the threshold level of the last innovation (otherwise it wouldn't have occurred) but below the new, increased trigger level of the next (not developed) technology. Hence, Proposition 4 does not hold. In contrast to the original set-up it is possible that after innovation has occurred the incomplete steady state is reached.

5.2 Generalized functional specifications

Here we consider a generalization of the social welfare function from (6) to (3) allowing for asymmetry between technologies. As a result, additional R&D motives that are determinants of empirically observable innovation and pollution activities will now enter into the analysis. In contrast to the previous section, both the R&D and the pollution policy are now directly affected.

A first step in the analysis is to consider the social planner's problem (7) now based on the general instantaneous welfare function (3) while retaining all linearity assumptions such that $\beta = 1$ and $c_i(q_i, t) = c_i q_i$. With the symmetry assumption regarding technologies removed, the $n+1$st technology can improve on the nth technology in the form of a lower accumulation rate per unit of output, $\alpha_{n+1} < \alpha_n$, a faster rate of stock decay, $\delta_{n+1} > \delta_n$, a lower marginal damage of pollution, $d_{n+1} < d_n$, or a lower cost of production, $c_{n+1} < c_n$. The long-run properties of the pollution stocks now take into account the heterogeneity of pollutants such that the long-run equilibrium stock of pollutant i given n technologies is

$$S_i^*(n) = \frac{(\delta_i + r)(1 - c_i - \kappa_n)}{\alpha_i d_i} \tag{45}$$

where $\kappa_n = \frac{\sum_{i=1}^n \frac{\delta_i(\delta_i+r)(1-c_i)}{\alpha_i d_i} - 1}{\sum_{i=1}^n \frac{\delta_i(\delta_i+r)}{\alpha_i d_i}}$ is the steady-state shadow price of the output constraint given n technologies. Since the $n+1$st technology unambiguously improves on the nth technology, $\kappa_{n+1} - \kappa_n > 0$ and the difference increases with the magnitude of the improvement. The long-run pollution stocks of all previous technologies therefore decrease with the number of technologies used and they decrease by more than in the case of symmetric technologies. While the long-run steady-states targeted by pollution policy therefore reflect the heterogeneity in technologies, the fundamental properties of the approach paths remain unchanged on account of the linearity of the pollution control problem. As before, the optimal pollution policy involves a sequence of at most (a) a most-rapid approach (of the singular solution), (b) a convergent singular solution path, and (c) a stationary singular solution (the steady state). With the optimal pollution policy qualitatively unchanged, the first part of Proposition 2 (existence of a period of exclusive use of the most recent technology) remains therefore intact.

An important implication of (45) is that heterogeneity in all parameters other than cost c_i has no qualitative impact on the optimal pollution policy:

With $c_i = c < 1$, $\alpha_i > 0$ and $d_i > 0$ for all i, all long-run stocks will be positive, implying that all technologies will be used simultaneously in the steady state. If - on the other hand - R&D delivers improvements in the cost of production such that $c_{n+1} < c_n$ for all $n > 1$ then there exist numbers of technologies $n^1, n^2, ...$ for which the long run stock of the first, second and so on technology will be zero and the technology will be permanently discontinued in the steady-state. This implies that while the long-run steady-state will feature the use of several technologies at once, the steady state is no longer guaranteed to include *all* available technologies (see Proposition 3).

Even under the retention of the linearity assumptions, the optimal R&D policy remains inconclusive without the imposition of considerable structure on the characteristics of new technologies. On the one hand, technological improvements in subsequent technologies provide greater initial incentives for R&D. In the present set-up, these additional incentives are reflected in the optimal innovation point t^*_{n+1} determined by (40). Improvements in technological characteristics of the $n+1$st technology enter into (40) via a lower shadow price μ^*_{n+1}, thus making it optimal *ceteris paribus* to engage in R&D earlier. On the other hand, (40) also implies that greater initial incentives for R&D do not necessarily translate into more cumulative R&D overall: Compared with a setting of symmetric technologies, returns from investing in the $n+1$st technology are *ceteris paribus* lower the better the portfolio of the previously developed n technologies. This 'competitive pressure of the past' is reflected in the weighted shadow prices of previous technologies $\sum_{i=1}^{n} \mu^*_i q^*_i$ and a result of the substitutability of technologies in production. The net effect can be fully derived for specific R&D production functions only (in terms of expected properties of novel technologies) and is the subject of future research.

Other possible generalizations of the model include non-linearities in the social welfare function. We comment on the case of $\beta < 1$ and $c(q,t) = c(q)$ with $\frac{dc}{dq} > 0$. As discussed in section 2, in the case of $\beta < 1$, the policy-maker faces decreasing marginal returns from production in each single technology and R&D incentives exist for reasons of product differentiation. Similarly, with increasing marginal cost of production in each technology, diversification of production allows escaping from decreasing net returns, leading to similar R&D incentives as in the case of $\beta < 1$. With the general direction clear, considering the specific impact of these generalizations on the results of this paper requires a restatement of both the optimal R&D and the optimal pollution policy. The reason is that with the linearity in the optimal pollution policy removed, the results change not only quantitatively, but also qualitatively. The result will be pollution policies that are characterized (a) by the absence of discontinuities in production shares by different technologies on account of the concavity of the net benefit function and (b) more cumulative R&D on account of the additional rents from technology differentiation [7].

6 Conclusion

In this paper, we studied the optimal pollution and R&D policy in a setting in which new technologies are never perfect backstops. In such a situation, pollution policy and R&D are interlinked on account of past R&D determining current degrees of freedom in allocating production shares to different technologies. In this paper, the pollution-R&D link is tight in the sense that typically the entire technology portfolio developed up to this point is used, except for transitional adjustment periods. This contrasts with previous papers where the use of a technology portfolio is a transitional phenomenon.

The characterization of the problem of 'green' R&D as one involving no 'magic bullets' also allows us to study the optimal timing of R&D decisions involving more than one technology. We find that innovation will be sequential rather than simultaneous and that under certain conditions there is a technological endpoint in the form of a highly diversified technological portfolio. Confirming results elsewhere in the literature, pollution stocks overshoot the long run steady state levels. The pattern of pollution and R&D policies derived in this paper is a new result and we illustrate its empirical relevance in the context of stock pollutants emanating from refrigeration technologies.

Extending the modeling framework to include (a) alternative R&D processes, (b) asymmetries across technologies, and (c) a generalized instantaneous welfare function provides additional insights. In case of (a), only the timing of R&D is affected through the R&D arbitrage equation while the optimal pollution policy remains essentially unchanged. Situations characterized by decreasing R&D costs involve both accelerated timing of R&D investments and an expansion of the amount of R&D carried out. Under certain circumstances, this leads to a situation in which R&D never ceases. In the case of (b), both R&D timing and the pollution policy are affected. The changes in the pollution policy are merely quantitative, however, with the exception of cost heterogeneities since these can cause technologies to be excluded permanently from the long run production portfolio. The timing and cumulative amount of R&D are ambiguous under (b), with the net effect hinging on the expectations regarding the evolution of the properties of new technologies. In the case of (c), decreasing marginal net returns in each technology lead to a pollution policy that will be characterized by an absence of discontinuities in production shares accorded to individual technologies and more R&D overall.

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A Proofs of Section 4

A.1 Condition for the Optimal Timing of R&D: (39) and (38) to (40)

Condition (39) requires that $G(t^*_{n+1}) = H^*_n\left(t^*_{n+1}\right) + e^{-rt^*_{n+1}}rR - H^*_{n+1}\left(t^*_{n+1}\right)$ is non-negative for all $\delta t_{n+1} < 0$ and non-positive for all $\delta t_{n+1} > 0$. Otherwise, there exist perturbations for which (39) becomes positive. $G(t^*_{n+1}) = 0$ is therefore a necessary condition for all $t^*_{n+1} > 0$. For $t^*_{n+1} = 0$, G is allowed to be negative. The intuition is that the optimal instant where $G = 0$ is at some negative point in time that is not feasible. However, in our case where technologies start with a zero initial stock this situation does not occur and all R&D is carried out at strictly positive points in time. Hence,

$$H^*_n\left(t^*_{n+1}\right) + e^{-rt^*_{n+1}}rR = H^*_{n+1}\left(t^*_{n+1}\right) \tag{A.1}$$

is a necessary condition for all t^*_{n+1}. Substituting (37) into (A.1) yields

$$\begin{aligned} e^{-rt^*_{n+1}}\left[\sum_{i=1}^{n}\left(q^*_i - \frac{d}{2}S^{*2}_i\right) + rR\right] + \sum_{i=1}^{n}\mu^*_i\left(\alpha q^*_i - \delta S^*_i\right) &= \\ e^{-rt^*_{n+1}}\sum_{i=1}^{n+1}\left(\check{q}^*_i - \frac{d}{2}\check{S}^{*2}_i\right) + \sum_{i=1}^{n+1}\check{\mu}^*_i\left(\alpha\check{q}^*_i - \delta\check{S}^*_i\right) & \end{aligned} \tag{A.2}$$

where a ˇ indicates post-innovation values. Using $\sum_{i=1}^{n} q_i^* = \sum_{i=1}^{n+1} \check{q}_i^* = 1$, $S_{n+1}(t_{n+1}^*) = 0$ this reduces to

$$e^{-rt^*_{n+1}} \left[1 - \sum_{i=1}^{n} \frac{d}{2} S_i^{*2} + rR \right] + \sum_{i=1}^{n} \mu_i^* \left(\alpha q_i^* - \delta S_i^* \right) = \tag{A.3}$$
$$e^{-rt^*_{n+1}} \left[1 - \sum_{i=1}^{n} \frac{d}{2} \check{S}_i^{*2} \right] + \sum_{i=1}^{n} \check{\mu}_i \left(\alpha \check{q}_i^* - \delta \check{S}_i^* \right) + \check{\mu}_{n+1}^* \alpha \check{q}_{n+1}^*.$$

After innovation has occurred the output of the new technology is one and that of all other technologies zero (see (25) and (26)). Moreover, the pollution stock of the new technology is zero and the stocks of all other technologies are the same as at the instant before innovation occurred, i.e. $S_i^* = \check{S}_i^*$ for all $i = 1, ..., n$. According to (38) it also holds that the shadow prices of the pollution stocks of all incumbent technologies remain unchanged, i.e. $\mu_i^* = \check{\mu}_i^*$ for all $i = 1, ..., n$. Hence, (A.3) simplifies to (40).

A.2 Shadow Prices and Optimal Timing of R&D: (41) to (42)

The shadow price of the new technology at the time of innovation can be derived by the following procedure. Given that prior to innovation all technologies are used simultaneously, their shadow prices are equal and unaffected by innovation (38). Their post-innovation movement is completely determined by the respective stock dynamics (see (10)) which are given by (27).

$$\mu_n^*(t) = e^{\delta(t - t^*_{n+1})} \cdot \left[\mu_n^* \left(t^*_{n+1} \right) + dS_n^* \left(t^*_{n+1} \right) e^{2\delta t^*_{n+1}} \left(\frac{e^{-(r+2\delta)t^*_{n+1}} - e^{-(r+2\delta)t}}{r + 2\delta} \right) \right] \tag{A.4}$$

Analogously, using (28) one can determine $\check{\mu}_{n+1}^{*}(t)$ as a function of $\check{\mu}_{n+1}^{*}\left(t_{n+1}^{*}\right)$

$$\mu_{n+1}^{*}(t) = e^{\delta(t-t_{n+1}^{*})}\left[\mu_{n+1}^{*}\left(t_{n+1}^{*}\right)+\frac{\alpha d}{\delta}e^{\delta t_{n+1}^{*}} \cdot\left(\frac{e^{-(r+\delta)t_{n+1}^{*}}-e^{-(r+\delta)t}}{r+\delta}-\frac{e^{-(r+\delta)t_{n+1}^{*}}-e^{-(r+2\delta)t+\delta t_{n+1}^{*}}}{r+2\delta}\right)\right] \tag{A.5}$$

At $\hat{t}_{n+1}$ stocks and hence the shadow prices of incumbent and new technologies converge. Hence, from $\mu_{n}^{*}\left(\hat{t}_{n+1}^{*}\right)=\mu_{n+1}^{*}\left(\hat{t}_{n+1}^{*}\right)$ and (A.4), (A.5) and (29) it follows that

$$\begin{aligned}\mu_{n+1}^{*}(t_{n+1}^{*}) = {} & \mu_{n}^{*}\left(t_{n+1}^{*}\right) \\ & +\frac{d}{r+2\delta}e^{-rt_{n+1}^{*}}\left[S_{n}^{*}\left(t_{n+1}^{*}\right)+\frac{\alpha}{\delta}\right]\left[1-\left(\frac{\delta}{\alpha}S_{n}^{*}\left(t_{n+1}^{*}\right)+1\right)^{-\frac{r+2\delta}{\delta}}\right] \\ & -\frac{\alpha d}{\delta(r+\delta)}e^{-rt_{n+1}^{*}}\left[1-\left(\frac{\delta}{\alpha}S_{n}^{*}\left(t_{n+1}^{*}\right)+1\right)^{-\frac{r+\delta}{\delta}}\right].\end{aligned} \tag{A.6}$$

By substituting this into (41) it simplifies to (42). For the case where innovation occurs during the post-innovation transition period the shadow price of the new technology at t_{n+1}^{*} can be calculated analogously. The result is exactly the same.