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Abstract. In this paper, we investigate the external effects of the parent's decisions on the number of newly born children and the firm's decisions on the amount of polluting emissions that occur in industrial production. We employ an optimal control model which comprises three stock variables representing population, the economic capital stock and the pollutant immissions in the natural environment. We distinguish two different types of households, in which the decision on the number of births takes place. These two types may be regarded as two extremes: dynastic households, in which the family sticks together forever

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and micro-households, in which children leave their parent's household immediately after birth. We conclude that in both cases the decentralized outcome is not optimal due to two externalities: one occurs in the individual decision on polluting emissions, the other one in the individual decision on the number of births.

It turns out that whereas the environmental externality is of the same form in both cases, the type of external effect from the household's decision on fertility is qualitatively different. The different types of population externalities require different policy instruments in order to internalize them. We discuss a Pigouvian tax on emissions as well as taxes on population: if an appropriate tax on the household size is applied in the case of dynastic households and an appropriate tax on children is applied in the case of small households a first best development of the economy is obtained.

JEL-Classification: J18, O13, Q25

Keywords: sustainability, endogenous fertility, externalities

1 Introduction

During the past five decades world population has more than doubled to almost 6.5 billion (UNDP 2005), while global gross domestic product has increased by a factor 4.4 between 1960 and 2001 (World Bank 2003). At the same time the use of resources and the emissions from producing goods have also increased substantially. E.g., total anthropogenic CO_2-emissions have increased by more than 330 percent. For many people these observations show that population growth and the necessary expansion of the production of goods in order to fulfill the needs of a larger population is the main source of the ongoing environmental degradation (e.g. Ehrlich and Ehrlich 2002). However, even if there is a consensus that population and the environment are closely related, the links between them are complex and ambiguous (e.g. Robinson and Srinivasan 1997). In many circumstances the use of the environment as a sink for wastes is determined by population rather indirectly. The amount and type of emissions are not only determined by the number of people living on earth, but depend also on production technologies and consumption patterns. Furthermore, if we accept that people decide on the number of births, one could also expect that per-capita consumption and environmental quality play a role in this decision.

Because of these complex interdependencies between demographic change, economic development and the use of the environment, it is not surprising that one cannot definitely answer the question whether present population growth is too high in order to protect the environment. There exists a broad range of results from studies which try to calculate the carrying capacity of our planet in terms of total population size. The results of these studies crucially depend on assumptions e.g. concerning agricultural productivity, economic and social conditions and potential environmental restrictions (cf. Heilig 1996:196)

It is the aim of our paper to contribute to a better understanding of the interactions between demographic change, economic development and environmental deterioration within the framework of the economic theory of externalities. Even if this economic point of view is a rather narrow and particular perspective on the complex interactions between population and the environment it helps to clarify, in how far the individual decisions concerning the number of births, consumption, and production lead to a undesirable outcome for the society. And it gives insights into the direction of interventions in the individual decisions and appropriate policy instruments. In particular we want to answer the following two questions:

1. Do individual decisions concerning consumption, the amount of emissions, and the number of children lead to overpopulation and environmental deterioration?

2. If this is the case, which policy instruments are suitable to solve these problems?

The problem of environmental externalities is well understood in economic literature. However, there are only few contributions in population and environmental economics dealing with environmental externalities and external effects of the decisions concerning fertility simultaneously. The endogenous fertility literature mainly analyzes the decision of individuals concerning their number of children in a setting without environmental problems (Becker 1960; Becker et al. 1999; Galor and Weil 1999; Raut and Srinivasan 1994; Yip and Zhang 1997). The bulk of environmental economics papers assumes that population is given and constant over time. One of the few contributions dealing with endogenous fertility and environmental problems at the same time is the monograph of Shah (1998). He shows in a static framework that the individual decisions can lead to an inferior outcome for the society, because individuals do not take into account that additional children contribute to negative effects due to a higher population density or due to an overall decrease in the marginal product of labor. Hence there is a difference between the outcome of individual decisions and the social optimum. A similar result is derived by Dasgupta (2000). Nerlove (1991), and Nerlove and Meyer (1997) derive a similar conclusion in a dynamic framework. These models are coined on regional environmental problems of rural areas in developing countries. In contrast, the approach by Cronshaw and Requate (1997) is adequate to investigate the interlinkages between population size and environmental damage due to emissions that occur in industrial production. They discuss the effects of variations in population size on environmental quality and consumption possibilities in a comparative static analysis. However, the static framework restricts the possibilities to analyze the dynamic effects of population growth.

Harford (1997, 1998), analyzes environmental externalities caused by consumption in a dynamic model, in which he considers two generations. He shows that in such a setting without production the problem of externalities has two dimensions: the emissions per capita as well as the number of children are too high in comparison to the social optimum. Therefore one needs two policy instruments in order to achieve a social optimum: a Pigouvian tax to reduce per-capita emissions, and a tax on the number of children to regulate population growth.

We aim to extend the existing literature in the following way: we investigate the external effects of the parent's decisions on the number of births and consumption and the firm's decisions on the amount of polluting emissions that occur from industrial production. We employ an optimal control model which comprises three stock variables representing population, the economic capital stock and the pollutant immissions in the natural environment. We distinguish two different types of households, in which the decision on the number of children takes place, which may be regarded as two extremes: dynastic households, in which the family sticks together forever and micro-households, in which children leave their parent's household immediately after birth. In the case of dynastic households, the number of households is constant over time, but the size of the household changes. In contrast to this, in the case of micro-households the size of the household is constant, but the number of households changes over time. Both extreme cases are not observable in reality. However there may be some relations to different ways of living together found in different cultures. While the life in rural areas in developing countries might be similar to the case of dynastic households; the situation in urban areas of modern industrialized countries is more similar to the case of micro-households.

We conclude that in both cases the decentralized outcome is inefficient due to two externalities: one occurs in the individual decision on polluting emissions, the other one in the individual decision on the number of births. It turns out that while the environmental externality is of the same type in both cases, the type of external effect from the household's decision on fertility is qualitatively different.

The different types of population externalities require different policy instruments to internalize them. We discuss a Pigouvian tax on emissions as well as taxes to regulate the development of the population. If an appropriate tax on the household size is applied in the case of dynastic households and an appropriate tax on children is applied in the case of small households, respectively, a first best development of the model economy is obtained.

The paper is organized as follows: in section 2 we develop an optimal growth model of an economy with endogenous fertility and environmental deterioration and derive conditions for the socially optimal development. In section 3 we analyze two different models for decentralized economies, where households decide about the number of births and consumption and where firms choose the amount of environmentally harmful emissions. The two models differ with respect to the two types of households described above, in which the decisions take place. We compare the results of these two different decision settings with the results of the central planned economy in order to show how the decentralized decisions lead to a suboptimal outcome. Furthermore, we derive policy instruments that allow to achieve the social optimum in a decentralized economy. Finally, we summarize our results in section 4 and discuss the policy implications of our model in a broader perspective.

2 Endogenous fertility, optimal population and the environment

2.1 The model

In the following, we consider a model economy with an endogenous population of N identical individuals.[1] The intertemporal welfare function U of a social planner is as follows:

$$U = \int_0^\infty u(c(t), n(t), S(t)) \cdot \exp(-\rho t) dt. \tag{1}$$

Here, $c(t)$ denotes the consumption per capita, $n(t)$ is the per-capita birth rate, and $S(t)$ is the stock of pollutant in the environment at time t. $u(c, n, S)$ denotes the instantaneous utility of each individual with $u_c > 0$ and $u_n > 0$ reflecting non-satiation in consumption and births.[2] To model the influence of environmental quality on welfare, we assume that production generates emissions $e(t)$ which accumulate to a pollutant stock $S(t)$ in the environment. This stock generates

[1] Therefore, we extend approaches of Yip and Zhang (1997) and Barro and Sala-I-Martin (1995) by introducing environmental quality as an argument in the welfare function and by taking into account that production causes polluting emissions.

[2] In the following, we omit the time dependency of the variables for reasons of more convinient notation. Subscripts denote the partial derivatives with respect to the corresponding variables.

disutility, i.e. $u_S < 0$. $\rho > 0$ is the constant discounting rate.

The dynamics of the three subsystems population, environment, and economy are described by three stock variables and the corresponding control variables. As outlined above, $N(t)$ denotes the absolute population size with $n(t)$, the per-capita birth rate, being the corresponding control variable. The stock of pollutant in the environment, $S(t)$, is controlled by per-capita emissions $e(t)$. Per-capita physical capital, $k(t)$, is controlled by per-capita consumption $c(t)$. In the following, we formulate the three differential equations determining the dynamics of these subsystems.

The following equation (3) expresses the dynamics of population growth, where we neglect the age structure and define an exogenous and constant death rate d.[3]

$$\dot{N} = (n - d)N. \tag{2}$$

Per-capita capital accumulation $\dot{k}$ is governed by

$$\dot{k} = f(k, e, t) - c - (n - d)k - bnk - b_0 n. \tag{3}$$

The per-capita output $f(\cdot)$ in equation (3) is divided into consumption c, investment $\dot{k}$, and costs to raise children. Interpreting $(n - d)$ as net per-capita birth rate, the term $-(n - d)k$ expresses the fact that each new population member has to be provided with the per-capita amount of capital. Hence, this amount has to

[3]In the model, we treat N and n as continuous variables. This approximation is valid because we exclusively regard large numbers for population size N. Hence, n denotes an average birth rate. With the same rational, instead of regarding an individual's probability to die, we employ an average death rate d for the whole population.

be subtracted from production output.

The term $-bnk-b_0n$, with b and b_0 being positive constants, denotes the costs of raising children. Here, average per-capita capital is used as a proxy for the relative size of the opportunity costs of raising children. This is an easy way to model the fact that opportunity costs for raising children are higher in developed countries, especially for women. The higher per-capita capital stock in developed countries in comparison to developing regions corresponds to better working opportunities for women. These better opportunities increase their opportunity costs of raising children (Barro and Sala-I-Martin 1995:313). The second part, b_0n, represents a share of opportunity costs for raising children that is independent of the women's working opportunities.[4]

With regard to production we assume constant returns to scale, i.e. that there exists a linear homogeneous production function $F(N, Nk, Ne, t)$. The production inputs are labor, capital and emissions.[5] We assume that every person supplies one unit of labor inelastically. Hence, the total labor input is equal to the population size N. We further assume that there exists an abatement technology which allows emissions arising in the production process to be cleaned up. This is integrated into the production function by assuming that emissions are inputs into the production process and can be substituted by other input factors. Furthermore we assume

[4]Here, we assume that the costs of raising children are covered by a single, large outlay at the time of their birth. Therefore, only the birth rate n occurs in equation (3).

[5]In physical terms, emissions are unwanted outputs of production. However, for purposes of analysis they are formally treated as production inputs (Pethig 2006).

that the production function is time-dependent, i.e. that technical progress occurs.

Finally, equation (4) describes the dynamics of the stock of pollutant in the natural environment in a much simplified way.

$$\dot{S} = Ne - \delta S. \tag{4}$$

Here, we assume that the pollutant is equally distributed throughout the environment. Pollution degradation is proportional to the concentration S of the pollutant in the environmental system with δ being the natural degradation rate of the pollutant.[6]

2.2 The optimal development of the coupled system

The optimal development of the coupled demographic-economic-environmental system is derived from the maximization of the intertemporal welfare function (1) with respect to the three restrictions (2), (3), and (4). In order to solve the

[6]For CO_2 this assumption is reasonable if one exclusively considers the anthropogenic CO_2 excess above the natural level. Furthermore, this excess has to be comparatively small and timescales regarded must not be too long. For a critical comment on the use of a single differential equation for the description of the accumulation of greenhouse gases in the environment see Joos and Stephan (1999) and Moslener and Requate (2001).

maximization problem we define the following present-value Hamiltonian

$$
\begin{aligned}
H = \quad & u(c,n,S)\exp(-\rho t) \\
& + \lambda^k \left[f(k,e,t) - c - (n-d)k - bnk - b_0 n\right] \\
& + \lambda^N (n-d)N \\
& + \lambda^S \left[Ne - \delta S\right]
\end{aligned} \tag{5}
$$

We get the first order conditions for a maximum by taking the derivatives with respect to control (i.e. c, n, e) and state (i.e. k, N, S) variables. Denoting a derivative with respect to one of the control or state variables by the corresponding subscript, we get the following equations:

$$H_c = 0 \qquad u_c \exp(-\rho t) - \lambda^k = 0 \tag{6}$$

$$H_n = 0 \qquad u_n \exp(-\rho t) - \lambda^k \left[k + bk + b_0\right] + \lambda^N N = 0 \tag{7}$$

$$H_e = 0 \qquad \lambda^k f_e + \lambda^S N = 0 \tag{8}$$

$$H_k = -\dot{\lambda}^k \qquad \lambda^k \left[f_k - (n-d) - bn\right] = -\dot{\lambda}^k \tag{9}$$

$$H_N = -\dot{\lambda}^N \qquad \lambda^N (n-d) + \lambda^S e = -\dot{\lambda}^N \tag{10}$$

$$H_S = -\dot{\lambda}^S \qquad u_S - \lambda^S \delta = -\dot{\lambda}^S \tag{11}$$

The transversality condition (Michel 1982)

$$\lim_{t\to\infty} H^0 = 0,^{7} \tag{12}$$

allows us to derive values for the control variable for $t \to \infty$. The solution of the set of equations (2)–(11) together with the transversality condition (12) yields

[7] The *maximized* Hamiltonian H^0 is the function H after we have substituted the control variables by (6)-(8).

the optimal time path for every set of initial conditions $N(t = 0)$, $k(t = 0)$ and $S(t = 0)$.[8]

3 Household decisions, endogenous fertility and externalities

In the following we analyze the above given economy by assuming that households decide on the number of children and consumption, and that firms decide on production plans. By comparing the results of the private decisions of households and firms with the optimal solution of the social planner we can identify externalities of private decisions. In particular, we expect that individual decisions fail to reach the social optimum, due to the public-good problem of environmental pollution. From a welfare-theoretic point of view such an inefficiency requires regulating the markets. Therefore we investigate how the identified externalities could be internalized by Pigouvian taxes.

We look at two different types of households: in section 3.2 we assume "dynastic households", were the founder of a dynasty decides taking into account the utility of all succeeding generations. In section 3.3 we assume "micro-households". Their characteristic feature is that children leave their home very early, such that households consist only of the parents. For both types of households we derive

[8] The first-order conditions are also sufficient for a welfare maximum, if the maximized Hamiltonian H^0 is concave in the state variables (Arrow and Kurz 1970). For the sake of this analysis, we assume that this is the case.

policy measures which guarantee that the individual decisions are efficient. We restrict our analysis on taxes imposed by a central regulator. Tax revenues are redistributed lump sum to the households such that the governmental budget is balanced.

Every individual inelastically supplies one unit of labor on a competitive labor market and earns a wage w, receives the rent r on capital and the profit π of shares of firms it owns.[9] The firms produce a homogeneous output using labor, capital and emissions with identical production technologies given by the production function $f(k, e)$. The entrepreneurs decide on labor and capital inputs and on the amount of emissions.

Within this framework we analyze the intertemporal allocation which results from the profit-maximizing behavior of firms and the utility maximizing behavior of households. We identify externalities according to individual decisions and we make policy recommendations how to implement a social optimal development of the economy. In the first step we consider the production side, which we model identically for both types of households.

3.1 Production

All N firms produce the homogenous output using the same technology described by the production function $f(k_i, e_i)$. The index i indicates that the capital and emission inputs may differ between firms. We assume perfect competition on

[9]We assume that each individual owns one firm, which is not a restrictive assumption, as we assume constant returns to scale in production.

labor and capital markets and on the market for the consumption good. The owners of the firm decide on per-capita capital k_i and per-capita emissions e_i. The government imposes a tax τ on emissions, which will be discussed below.

Each firm i maximizes its profit π_i, which is given as the difference between revenues $f(k_i, e_i)$ and costs $rk_i + w + \tau e_i$.

$$\max_{k_i,e_i} \pi_i = \max_{k_i,e_i} f(k_i, e_i) - rk_i - w - \tau e_i. \tag{13}$$

The first order conditions of this problem are given by:

$$f_{k_i} = r \tag{14}$$

$$f_{e_i} = \tau. \tag{15}$$

Hence, the firms choose per-capita inputs such that the value of the marginal product of the inputs equals the price of the inputs.[10]

Since the production functions of the firms fulfill the usual neoclassical assumptions the necessary conditions for a profit maximum are sufficient as well. As all firms are identical, their factor inputs and profits are the same in equilibrium:

$$k_i = k, \; e_i = e, \; \text{und } \pi_i = \pi,$$

i.e., we may drop the index i, which decides between firms.

We now turn to the decision problem of a representative household. In the following section 3.2 we consider the case of dynastic households; in section 3.3 we consider micro-households.

[10]Because of the constant returns to scale technology the marginal product of the per-capita inputs is equal to the marginal product in absolute terms, i.e. $F_{K_i} = f_{k_i}$ and $F_{E_i} = f_{e_i}$. The optimal labor input is given by $w = \frac{d(N_i \cdot f(k_i,e_i))}{dN_i} = f(k_i, e_i) - k_i f_{k_i} - e_i f_{e_i}$.

3.2 Dynastic households

In this section, we consider dynastic households, in which the decision on consumption and the number of children takes place. This relates to the common assumption in optimal control models that a founder of a dynasty maximizes the utility stream of all succeeding generations. Since in the setting of dynastic households all children stay in the household, the number I of housholds is constant. As households are identical, they are of the same size in equilibrium. Thus, the number N_j of members of the household j equals the I^{th} part of total population:

$$N_j = \frac{N}{I}.$$

The budget constraint of household j in per capita terms reads:[11]

$$\pi + rk_j + w + T = \dot{k}_j + c_j + (n_j - d)k_j + bn_jk_j + b_0n_j + \Theta N_j. \tag{16}$$

The left hand side of the budget constraint describes the income of the household.[12] It is the sum of wage and capital earnings $w + rk_j$, the profits π_j, and a lump-sum transfer T from the government (we discuss this transfer below). The right hand side of (16) contains the expenditures of the household. They consist of the investment $\dot{k}_j$, consumption c_j, the costs of children, which are the result from a endowment of a growing household with capital $(n_j - d)k_j$, and the costs of raising children $bn_jk_j + b_0n_j$. Furthermore, the household has to pay a tax Θ, which

[11]In general the per capita capital k_j owned by each member of household j is different from the capital k_i used by firm i. In equilibrium, however, $k_j = k_i = k$ holds.

[12]Because of non-satiation the budget constraint holds as equality.

depends on its size, and which has to be paid for each member of the household. This tax will be discussed below (proposition 1).

The regulator chooses the lump-sum transfer T such that tax revenues – the taxes $N_j\,\Theta$, from each of the N_j members of the I households and the emission taxes $e_i\,\tau$, from the N firms – are fully redistributed. In equilibrium, the transfer is

$$T = \Theta \cdot \frac{N}{I} + \tau \cdot e,$$

as in equilibrium the emissions $e_i = e$ of all firms and the sizes $N_j = N/I$ of all households are equal by symmetry.

The second constraint taken into account by the dynasty's founder is the change of the household size N_j resulting from the birth rate n_j. This constraint reads (parallel to the development of total population, equation (2)):

$$\dot{N}_j = (n_j - d)N_j. \tag{17}$$

The dynasty's founder decides on two variables: per capita consumption c_j and birth rate n_j. We assume that he has no influence on the firm's decision on the amount of polluting emissions in production. Hence, he does not take into account a constraint concerning the environment, as the environment is not affected by his decisions.[13] Thus, the dynasty's founder maximizes the stream of utility of his dynasty subject to the budget constraint (16) and the change in household size (17).

[13] A similar problem with constant population size is analyzed in Aronsson et al. (1997).

The optimization problem of the founder of a dynasty has the following form:

$$\max_{c_j(t),n_j(t)} \int_0^{\infty} u\left(c_j(t), n_j(t), S(t)\right) \cdot \exp(-\rho t)dt \text{ s.t. (16) and (17).} \tag{18}$$

The present value Hamiltonian is:

$$\begin{aligned} H_j^d &= u\left(c_j, n_j, S\right) \exp(-\rho t) \\ &\quad + \lambda_j^k \left(\pi + rk_j + w + T - c_j - (n_j - d)k_j - bn_jk_j - b_0n_j - \Theta N_j\right) \\ &\quad + \lambda_j^N \left(n_j - d\right) N_j. \end{aligned}$$

And we get the following necessary conditions for an optimum:[14]

$$\frac{\partial H_j^d}{\partial c_j} = 0 \qquad u_{c_j} \exp(-\rho t) - \lambda_j^k = 0 \tag{19}$$

$$\frac{\partial H_j^d}{\partial n_j} = 0 \qquad u_{n_j} \exp(-\rho t) - \lambda_j^k \left[k_j + bk_j + b_0\right] + \lambda_j^N N_j = 0 \tag{20}$$

$$\frac{\partial H_j^d}{\partial k_j} = -\dot{\lambda}_j^k \qquad \lambda_j^k \left[r - (n_j - d) - bn_j\right] = -\dot{\lambda}_j^k \tag{21}$$

$$\frac{\partial H_j^d}{\partial N_j} = -\dot{\lambda}_j^N \qquad \lambda_j^N (n_j - d) - \lambda_j^k \Theta = -\dot{\lambda}_j^N. \tag{22}$$

Comparing these conditions as well as conditions (14) and (15) for the firm's profit maximum with the with the corresponding equations (6)–(11) determining the social optimum yields the following proposition:

Proposition 1

The market equilibrium in the case of dynastic symmetric households is a social optimum if we impose a tax on emissions

$$\tau = -\frac{\lambda^S N}{\lambda^k}, \tag{23}$$

[14]Under the appropriate taxation system, these conditions are identical to the conditions (6)–(11) for the optimal solution. Thus, they are also sufficient, provided the first-order conditions for the social optimum are also sufficient.

and a tax on the household size with rate

$$\Theta = -I\, e\, \frac{\lambda^S}{\lambda^k} = fracI\, eN\, \tau, \tag{24}$$

where λ^S and λ^k are the shadow prices of the pollutant and capital stock in the social optimum.

Proof: see appendix A.1.

The intuition for the result that two policy instruments are necessary in order to achieve the social optimum is as follows. In the market equilibrium two decisions generate external effects: the decision concerning emissions and the decision concerning fertility. On the one hand the emissions are too high for a given size of the population. This externality can be internalized by the Pigouvian tax τ on emissions. In addition, there is a further externality due to the decision on the birth rate: the dynasty's founder does not take into account the social costs of additional children that arise, because each additional child increases total emissions given the level of per capita emissions. It will do so for its whole lifetime. Therefore, the regulative tax has to address the stock variable and correct the shadow price λ^N of the population size (cf. equation 22). A tax on the birth rate would not solve the problem, because the household's valuation of the population stock is incorrect from a social point of view, not the valuation of the number of births. Thus, the environmental problem in our model economy leads to two different externalities.

If there was no environmental deterioration, the individual decisions concerning consumption and the number of children were optimal: the shadow price λ^S

of the pollutant stock would be zero. Thus, according to proposition 1, both taxes τ and Θ were equal to zero. From this we obtain as a special case without environmental problems a result from Razin and Sadka (1995), who show in a model without emissions that children should not be taxed, because the individual decisions concerning the number of children leads to a social optimum.

The tax on the size of the household is a linear argument of the per-capita budget constraint. This means, the tax-burden per household i is equal to ΘN_i^2, and hence, increasing quadratic in N_i. A linear increasing tax would be equivalent to a constant detection from wage income. Because of the perfect labor market, such a tax would not influence the household decision. If the tax is quadratic in N_i, each member of the household do not only pay taxes for itself, but for all other members too. By this, the effect of the population growth is taken into account by the private decisions in an appropriate way.

If we know the solution for the social planner's optimization problem, i.e. the optimal paths for the control variables and shadow prices, we can express the tax rates τ and Θ in real terms.[15] They read:

$$\tau = f_e(k, e) \tag{25}$$

$$\Theta = \frac{I \cdot e}{N}\,\tau, \tag{26}$$

where the values of the optimal paths have to be inserted for k, e and N, respectively. From this we see immediately that both taxes are positive. The result of a

[15]This is done by rearranging the necessary conditions for a optimum with respect to the shadow prices and using these conditions in equations (23) and (24) for the taxes.

positive tax on the population size means that the private costs of having a large household in the case without taxation are less than social costs. In this sense, there exists a problem of "overpopulation", unless the household size is taxed in the appropriate way which is given by proposition 1.

3.3 Micro-households

In the following we discuss a different setting of households which decide on consumption and fertility. Instead of dynasties, where all children stay in the household, we consider the opposite theoretical extreme case that children leave their parent's household immediately after birth. The size of such "micro-households" is fixed, as they consist only of the parents, but the number N of households varies, as children found their own households. This is in contrast to the case of dynastic households, where the size of households varies whereas the number of households in the economy is fixed.

The budget constraint of micro-household j in per capita terms is:

$$\pi + rk_j + w + T = \dot{k}_j + c_j + (n_j - d)k_j + bn_jk_j + b_0n_j + \theta n_j. \tag{27}$$

The income is described by the left hand side of the constraint and is equal to the sum of capital and wage incomes, profits and the lump-sum transfer. The expenditures are determined by the right-hand side. They consist of investments $\dot{k}_i$, consumption c_i, and costs of children, which equal the change in endowment with capital $(n_i - d)k_i$ plus the direct costs of raising children $bn_ik_i + b_0n_i$. Instead of a tax Θ on the household size we have introduced a tax θ on the number of children in equation (16). This type of taxation is justified in proposition 2 below.

The micro-household j decides on two variables: per capita consumption $c_j(t)$ and the birth rate $n_j(t)$. The micro-household consists only of the parents, so the decision on the birth rate does not affect the size of the household. As the decision makers care only for their own household, they do not take into account any change in population size resulting from their decision on the birth rate. Similar to the case of dynastic households, we assume that the micro-household has no influence on the firm's decision on the amount of polluting emissions generated in production. Therefore, the only side condition for the optimization of the micro-household j is his budget constraint. Consequently, his decision problem is as follows:

$$\max_{c_j(t),n_j(t)} \int_0^\infty u\left(c_j(t), n_j(t), S(t)\right) \cdot \exp(-\rho t) dt \text{ s.t. } (27) \tag{28}$$

The present value Hamiltonian for this problem is given by:

$$\begin{aligned} H_j^m =& \, u\left(c_j, n_j, S\right) \exp(-\rho t) \\ & + \lambda_j^k \left(\pi + r k_j + w + T - c_j - (n_j - d) k_j - b n_j k - b_0 n_j - \theta n_j\right). \end{aligned}$$

The necessary conditions for the private optimum are the following:

$$\frac{\partial H_j^m}{\partial c_j} = 0 \qquad u_{c_j} \exp(-\rho t) - \lambda_j^k = 0 \tag{29}$$

$$\frac{\partial H_j^m}{\partial n_j} = 0 \qquad u_{n_j} \exp(-\rho t) - \lambda_j^k \left[k + bk + b_0\right] - \lambda_j^k \theta = 0 \tag{30}$$

$$\frac{\partial H_j^m}{\partial k_j} = -\dot{\lambda}_j^k \qquad \lambda_j^k \left[r - (n_j - d) - b n_j\right] = -\dot{\lambda}_j^k. \tag{31}$$

Equations (29) and (31) together with (14) are the same as the corresponding optimality conditions (6) and (9). The remaining equations are identical to the corresponding optimality conditions, if the tax rates τ and θ are set in the appropriate way:

Proposition 2

A market equilibrium with micro-households is a social optimum in the symmetric case if an emission tax is introduced such that

$$\tau = -\frac{\lambda^S N}{\lambda^k}, \tag{32}$$

and a tax or subsidy on the number of children is implemented such that

$$\theta = -\frac{\lambda^N N}{\lambda^k}. \tag{33}$$

Proof: see Appendix A.2.

Our analysis shows that in the case of small households an environmental problem exists because the emissions are higher than in the social optimum. This externality may be internalized by the Pigouvian tax τ on emissions. However, in contrast to the case of dynastic households in section 3.2 the economy of micro-households suffers from externalities even without any environmental problem, i.e. $\lambda^S = 0$. The decision maker does not take into account that his private decision on the birth rate influences the future population size. This leads to a suboptimal population size, as can be seen by comparing condition (30) for the private optimum with the corresponding condition for the social optimum (7), which do not coincide even if $\lambda^S = 0$. The inefficient population size influences the labor supply and by this the marginal productivity of capital and emissions given by the equations (14) and (15). Hence, the choice of emissions – given the level of the emission tax – is suboptimal: a profit maximizing firm chooses its emissions such that the marginal product of emissions is equal to the emission tax (cf. equation (25)). Therefore, investment in capital and the amount of emissions in production are inefficient as well, unless the appropriate tax θ on the birth rate is levied.

We come back to the case with environmental pollution now. Let's suppose for the moment that the population size resulting from the unrestricted decisions of micro-households is higher than socially optimal. The labor supply is higher then compared to the social optimum. Thus, emissions are substituted by labor in production. This effect leads to less emissions in the laissez-faire-outcome than in the optimum, given a level of the emission tax. On the other hand, however, total output is higher with the higher population. Given per capita emissions, this effect yields higher total emissions compared to the optimum.

These two opposite effects are the reason that we can a priori draw no conclusions if the birth rate is higher or less than socially optimal. That means, in contrast to the situation of dynastic household we do a priori not know whether the number of children should be taxed or subsidized, i.e. the sign of θ is ambiguous, and can even change over time if the economy is not in a steady state.

To summarize, if parents decide on their number of children in the setting of micro-households, they do not take into account that children influence future production possibilities. This leads to an inefficient development of the economy. Our result is similar to the results of Dasgupta (2000) and Shah (1998), which show in a static context with a fixed resource that individual decisions concerning the numbers children can lead to inferior results.

4 Conclusions

We have analyzed externalities which occur if parents decide on the number of births and consumption in an economy, in which the production of consumption goods causes environmentally harmful emissions. Our analysis shows that the private decisions of households concerning consumption and fertility and the decisions of firms concerning production lead to an inefficient development of the economy indeed. The types of households turned out to be relevant for the type of externality. In both settings, where the decision is made by dynastic households or by micro-households, an externality occurs in the decision of the entrepreneurs concerning the amount of emissions in production. This leads to a situation in which the per-capita emissions of every firm are too high compared to the social optimum. But the externalities resulting from the private decision on the number of births depends on the household structure in which this decision takes place.

In the case of dynastic households the resulting size of the population is higher than in the social optimum. Total emissions are higher than optimal, because the decision makers do not take into account the additional pollutant emissions, which are caused by each additional child during its whole lifetime. In this setting the decision of the dynastic household would be socially optimal, if there was no enivronmental problem. Therefore one can say that the environmental problem causes an additional population problem, as it generates two distinct external effects.

This is different in the setting with micro-households. Here, the decision makers does not take into account that additional children have an effect on the production system, because they change labour supply and hence the marginal product of capital and emissions. Therefore, even without environmental problems, there exists an external effect caused by the unregulated decision on the birth rate.

From a welfare-theoretic point of view these results give reason for a governmental intervention into the individual decisions, in order to achieve a social optimum. We have shown that with an appropriate choice of regulative taxes, a regulator is able to achieve the social optimum. According to the different nature of the externalities, the form of the regulation of the population size depends on the decision setting: different instruments are needed in the case of dynastic households than in the case of micro-households. On dynastic households, the social planner has to impose a tax on the household's size. In the case of micro-households the social planner has to tax the birth rate number. To internalize the environmental externality from the firm's decisions on the amount of emissions occuring in production, tax on per-capita emissions is necessary which is of the same form in both settings.

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A Appendix

A.1 Proof of proposition 1

The optimal emission tax can be derived by comparison of condition (15) for the amount of emissions in the firm's profit maximum and the corresponding condition (8) for the socially optimal amount of emissions. As, by symmetry, $e_i = e$ and $k_i = k$, we may insert equation (15) in (8) and find the optimal tax rate by rearranging

$$\tau = -N \, \frac{\lambda^S}{\lambda^k}.$$

Now, we derive the optimal tax rate Θ on the household size. This has to be chosen such that condition (22) is equal to the corresponding condition (10) for the social optimum. This is the case, if $-I \, \lambda_j^k \, \Theta = \lambda^S \, e$, i.e., if

$$\Theta = \; - I \, e \, \frac{\lambda^S}{\lambda_j^k} = -I \, e \, \frac{\lambda^S}{\lambda^k},$$

where the last requirement $\lambda_j^k = \lambda^k$ holds by symmetry. Inserting Θ in (22) yields

$$\begin{aligned} \lambda_j^N \, (n_j - d) + I \, e \, \lambda^S &= \; - \, \dot{\lambda}_j^N \\ \tilde{\lambda}^N \, (n_j - d) + e \, \lambda^S &= \; - \, \dot{\tilde{\lambda}}^N, \end{aligned}$$

where $\tilde{\lambda}^N := \lambda_j^N / I$ is the same in equilibrium for all households. Furthermore, $n_j = n$ holds by symmetry. As I is constant, we have $\dot{\tilde{\lambda}}^N = \dot{\lambda}_j^N / I$.

For Θ given by equation (24), the conditions for the dynasty's founder's private optimum and for the social optimum are identical.

A.2 Proof of proposition 2

The optimal rate of the emission tax results from the comparison of the equation (15) with the optimality condition (8). Completely analogous to the proof of proposition 1, we get

$$\tau = -\frac{\lambda^S N}{\lambda^k}.$$

The optimal tax rate θ on the birth rate results from comparing condition (30) for the household's optimal birth rate – using the symmetry conditions $k_j = k$, $e_i = e$ and $\lambda_j^k = \lambda^k$ – with condition (7) for the socially optimal birth rate. Both equations are equal, if

$$\theta = -N\,\frac{\lambda^N}{\lambda^k}.$$