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Uncertainty and Sustainability in the Management of Semi-Arid Rangelands

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Abstract. We analyze a dynamic and stochastic ecological-economic model of grazing management in semi-arid rangelands. The non-equilibrium ecosystem is driven by stochastic precipitation. A risk averse farmer chooses a grazing management strategy under uncertainty such as to maximize expected utility from farming income. Grazing management strategies are rules about which share of the rangeland is given rest depending on the actual rainfall in that year. In a first step we determine the farmer's short-term optimal grazing management strategy. We show that a risk-averse farmer chooses a strategy such as to obtain insurance from the ecosystem: the optimal strategy reduces income variability, but yields less mean income than possible. In a second step we analyze the long-run ecological and economic impact of different strategies. We conclude that the more risk-averse a farmer is, the more conservative and sustainable is his short-term optimal grazing management strategy, even if he has no specific preference for the distant future.

JEL-Classification: Q57, Q12, Q24

Keywords: Ecological-economic model, semi-arid rangeland, grazing management, risk-aversion, uncertainty, sustainability

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1 Introduction

There is a widely held belief that individual short-term optimization is at odds with long-term sustainability of an ecological-economic system. In this paper, we want to take a fresh look at this position. We show that for typical ecosystems and under plausible and standard assumptions about individual decision making, short-term optimization leads to sustainable outcomes. In particular, in order to explain the sustainable use of ecosystems, it is not necessary to assume preferences for sustainability – or any special concern for the distant future – on the part of the decision maker.

The ecological-economic system under study here is grazing in semi-arid rangelands. Semi-arid regions cover two thirds of the Earth's land surface. They are characterized by low and highly variable precipitation. Their utilization in livestock grazing provides the livelihood for a large part of the local populations. But semi-arid ecosystems are extremely sensitive: over-utilization and non-adapted grazing strategies lead to environmental problems such as desertification.

Grazing in semi-arid rangelands is a prime object of study for ecological economics, as the ecological and economic systems are tightly coupled (e.g. Beukes et al. 2002, Heady 1999, Janssen et al. 2004, Perrings 1997, Perrings and Walker 1997, 2004, Westoby et al. 1989). The grass biomass is directly used as forage for livestock, which is the main source of income; and the grazing pressure directly influences the ecological dynamics. The crucial link is the grazing management.

The ecological dynamics, and thus, a farmer's income, essentially depend on the low and highly variable rainfall. The choice of a properly adapted grazing management strategy is crucial in two respects: first, to maintain the rangeland system as an income base, that is, to prevent desertification; and second, to smooth out income fluctuations, in particular, to avoid high losses in the face of droughts.

Assuming that the farmer is non-satiated in income and risk-averse, we analyze the choice of a grazing management strategy from two perspectives. On the one hand, we determine the farmer's short-term optimal grazing management strategy. We show that a risk-averse farmer chooses a strategy in order to obtain 'insurance' from the ecosystem (Baumgärtner and Quaas 2004). That is, the optimal strategy reduces income variability at the expense of yielding less mean income than possible.

On the other hand, we analyze the long-term ecological and economic impact of different strategies. We conclude that the more risk-averse a farmer is, the more conservative and the more sustainable is his short-term optimal grazing management strategy. In short, in the context of grazing in semi-arid regions, risk-aversion implies sustainability.

The literature on grazing management under uncertainty mainly analyzes the choice of a *stocking rate* of livestock, as this is the most important aspect of rangeland management (e.g. Hein and Weikard 2004, Karp and Pope 1984, McArthur and Dillon 1971, Perrings 1997, Rodriguez and Taylor 1988, Torell et al. 1991, Westoby et al. 1989). The innovative analytical approach taken here is to consider the choice of a *grazing management strategy*, which is a *rule* about the stocking rate to apply in any given year depending on the rainfall in that year. This is inspired by empirical observations in Southern Africa. Rule-based grazing management has the twofold advantage that a farmer has to make a choice (concerning the rule) only once, and yet, keeps a certain flexibility and scope for adaptive management (concerning the stocking rate). The flexibility thus obtained is the decisive advantage of choosing a constant rule over choosing a constant stocking rate.

The paper is organized as follows. In Section 2, we discuss grazing management in semi-arid rangelands in more detail and describe one particular 'good practice'-example: the Gamis Farm, Namibia. In Section 3, we develop a dynamic and stochastic ecological-economic model, which captures the essential aspects and principles of grazing management in semi-arid rangelands, and features the key aspect of the Gamis-strategy. Our results are presented in Section 4, with all derivations and proofs given in the Appendix. Section 5 concludes.

2 Grazing management in semi-arid rangelands: The Gamis Farm, Namibia

The ecological dynamics of semi-arid regions are essentially driven by low and highly variable precipitation (Westoby et al. 1989, Behnke et al. 1993, Sullivan and Rhode 2002).[1]

[1] Another important driver of ecological dynamics in semi-arid rangelands is the stochastic occurrence of fire (Janssen et al. 2004, Perrings and Walker 1997, 2004). In our case, fire plays only a minor role, but the stochasticity of rainfall is crucial (Müller et al. 2004).

Sustainable economic use of these ecosystems requires an adequate adaption to this environment. The only sensible economic use, which is indeed predominant (Mendelsohn et al. 2002), is by extensive livestock grazing. For this purpose, sophisticated grazing management strategies have been developed. For example, the 'opportunistic' grazing management strategy (e.g. Beukes et al. 2002: 238) simply matches the herd size with available forage in every year. This is done by de-stocking when there is little forage in the dry years, and restocking when more forage is available in years with sufficient rainfall.

One example of a more sophisticated and particularly successful management system has been employed for forty years at the Gamis Farm, Namibia (Stephan et al. 1996, 1998a,b; Müller et al. 2004). The Gamis Farm is located 250 km southwest of Windhoek in Namibia (2405′S 1630′E) close to the Naukluft mountains at an altitude of 1,250 m. The climate of this arid region is characterized by low mean annual precipitation (177 mm/y) and high variability (variation coefficient: 56 percent). The vegetation type is dwarf shrub savanna (Giess 1998); the grass layer is dominated by the perennial grasses *Stipagrostis uniplumis*, *Eragrostis nindensis* and *Triraphis ramosissima* (Maurer 1995).

Karakul sheep (race Swakara) are bred on an area of 30,000 hectares. The primary source of revenue is from the sale of lamb pelts. Additionally, the wool of the sheep is sold. In good years, up to 3,000 sheep are kept on the farm. An adaptive grazing management strategy is employed to cope with the variability in forage. The basis of the strategy is a rotational grazing scheme: the pasture land is divided into 98 paddocks, each of which is grazed for a short period (about 14 days) until the palatable biomass on that paddock is used up completely, and then is rested for a minimum of two months. This system puts high pressure on the vegetation for a short time to prevent selective grazing (Heady 1999). While such a rotational grazing scheme is fairly standard throughout semi-arid regions, the farmer on the Gamis Farm has introduced an additional resting: in years with sufficient precipitation one third of the paddocks are given a rest during the growth period (September - May). In years with insufficient rainfall this rest period is reduced or completely omitted. Once a year, at the end of the rainy season (April), the farmer determines – based on actual rainfall and available forage – how many paddocks will be rested and, thus, how many lambs can be reared. This strategy is a particular example of what has been called 'rotational resting' (Heady 1970, 1999; Stuth and Maraschin 2000;

Quirk 2002).

The grazing management system employed at the Gamis Farm has been successful over decades, both in ecological and economic terms. It, therefore, represents a model for commercial farming in semi-arid rangelands.

3 The model

Our analysis is based on an integrated dynamic and stochastic ecological-economic model, which captures essential aspects and principles of grazing management in semi-arid regions. It represents a non-equilibrium dynamic ecosystem, which is driven by stochastic precipitation, and a risk averse farmer, who rationally chooses a grazing management strategy under uncertainty.

3.1 Precipitation

Uncertainty is introduced into the model by the stochasticity of rainfall, which is assumed to be an independent and identically distributed (iid) random variable. For semi-arid areas, a log-normal distribution of rainfall $r(t)$ is an adequate description (Sandford 1982).[2] The log-normal distribution, with probability density function $f(r)$ (Equation A.17), is determined by the mean μ_r and standard deviation σ_r of precipitation. Here, we measure precipitation in terms of 'ecologically effective rain events', i.e. the number of rain events during rainy season with a sufficient amount of rainfall to be ecologically productive (Müller et al. 2004).

3.2 Grazing management strategies

The farm is divided into a number $I \in \mathbb{N}$ of identical paddocks, numbered by $i \in \{1, \ldots, I\}$. In modeling grazing management strategies, we focus on the aspect of additional resting during the growth period, which is the innovative element in the Gamis grazing system. The strategy is applied in each year, after observing the actual rainfall at the end of the rainy season. Its key feature is that in dry years all paddocks are

[2] While the distribution of rainfall $r(t)$ is exogenous, all other random variables in the model follow an induced distribution.

used, while in years with sufficient rainfall a pre-specified fraction of paddocks is rested. Whether resting takes place, and to what extent, are the defining elements of what we call the farmer's grazing management strategy:

Definition 1

A *grazing management strategy* $(\alpha, \underline{r})$ is a rule of how many paddocks are not grazed in a particular year given the actual rainfall in that year, where $\alpha \in [0, 1]$ is the fraction of paddocks rested if rainfall exceeds the threshold value $\underline{r} \in [0, \infty)$.[3]

Thus, when deciding on the grazing management strategy, the farmer decides on two variables: the rain threshold $\underline{r}$ and the fraction α of rested paddocks. While the rule is constant (i.e. α =const., $\underline{r}$ =const.) its application may yield a different stocking with livestock in any given year depending on actual rainfall in that year. Note that the 'opportunistic' strategy (e.g. Beukes et al. 2002: 238) is the special case without resting, i.e. $\alpha = 0$.

3.3 Ecosystem dynamics

Both the stochastic rainfall and grazing pressure are major determinants of the ecological dynamics. Following Stephan et al. (1998a), we consider two quantities to describe the state of the vegetation in each paddock i at time t: the green biomass $G^i(t)$ and the reserve biomass $R^i(t)$ of a representative grass species,[4] both of which are random variables, since they depend on the random variable rainfall. The green biomass captures all photosynthetic ('green') parts of the plants, while the reserve biomass captures the non-photosynthetic reserve organs ('brown' parts) of the plants below or above ground (Noy-Meir 1982). The green biomass grows during the growing season in each year and dies almost completely in the course of the dry season. The amount $G^i(t)$ of green biomass available on paddock i in year t after the end of the growing season depends on rainfall $r(t)$ in the current year, on the reserve biomass $R^i(t)$ on that paddock, and on a growth

[3]We assume that the number I of paddocks is so large that we can treat α as a real number.

[4]We assume that a rotational grazing scheme is employed, such that selective grazing is completely prevented, i.e. there is no competitive disadvantage for more palatable grasses (see e.g. Beukes et al. 2002). Hence, we consider a single, representative species of grass.

parameter w_G:

$$G^i(t) = w_G \cdot r(t) \cdot R^i(t). \tag{1}$$

As the green biomass in the current year does not directly depend on the green biomass in past years, it is a flow variable rather than a stock.

In contrast, the reserve biomass $R^i(t)$ on paddock i in year t is a stock variable. That is, the reserve biomass parts of the grass survive several years ('perennial grass'). Thereby, the dynamics of the vegetation is not only influenced by the current precipitation, but also depends on the precipitation of preceding years (O'Connor and Everson 1998). Growth of the reserve biomass from the current year to the next one is

$$R^i(t+1) - R^i(t) = -d \cdot R^i(t) \cdot \left(1 + \frac{R^i(t)}{K}\right) + w_R \cdot (1 - c \cdot x^i) \cdot G^i(t) \cdot \left(1 - \frac{R^i(t)}{K}\right), \tag{2}$$

where d is a constant death rate of the reserve biomass, and w_R is a growth parameter. A density dependence of reserve biomass growth is captured by the factors containing the capacity limits K: The higher the reserve biomass on paddock i, the slower it grows. The status variable x^i captures the impact of grazing on the reserve biomass of paddock i. If paddock i is grazed in the current year, we set $x^i = 1$, if it is rested, we set $x^i = 0$. The parameter c (with $0 \leq c \leq 1$) describes the amount by which reserve biomass growth is reduced due to grazing pressure. For simplicity, we assume that the initial ($t = 1$) stock of reserve biomass of all paddocks is equal,

$$R^i(1) = R \quad \text{for all } i = 1, \ldots, I. \tag{3}$$

3.4 Livestock and income

As for the dynamics of livestock, the herd size $S(t)$ at time t is given by[5]

$$S(t) = \sum_{i=1}^{I} x^i \cdot G^i(t), \tag{4}$$

i.e. the herd size is limited by total available forage, which equals the green biomass G^i of all grazed paddocks i with $x^i = 1$. Here, we assume that the farmer can (and

[5]We normalize the units of green biomass in such a way that one unit of green biomass equals the need of one sheep per year.

will) adapt the herd size to the available forage and to his chosen grazing management strategy without any cost or benefit. That is, de-stocking does not generate revenue, and re-stocking is possible at no cost.[6]

The herd size $S(t)$ determines the farmer's income $y(t)$. We assume that the quantity of marketable products from livestock is identical to the herd size $S(t)$ at time t.[7] The farmer sells his products on a world market at a given price p, which is constant over time. Thus, the farmer's income $y(t)$ is

$$y(t) = p \cdot S(t). \tag{5}$$

Since the herd size $S(t)$ is a random variable, income $y(t)$ is a random variable, too. In order to simplify the notation in the subsequent analysis, we standardize the product price to

$$p \equiv (w_G \cdot I \cdot R)^{-1}. \tag{6}$$

This means, from now on, we measure product value in units of total forage per unit of precipitation.

Given the actual rainfall r in the first grazing period, the initial reserve biomass (Equation 3) and a grazing management rule $(\alpha, \underline{r})$, the herd size $S \equiv S(1)$ is determined by Equation (4). Inserting Equation (1) and using Assumption (3), as well as standardization (6), the farmer's income $y \equiv y(1)$ at the end of the first grazing period is given by Equation (5) as

$$y = \frac{1}{I} \sum_{i=1}^{I} x^i \cdot r = r \cdot \begin{cases} 1 & \text{if} \quad r \leq \underline{r} \\ 1 - \alpha & \text{if} \quad r > \underline{r} \end{cases} . \tag{7}$$

Given the probability density distribution $f(r)$ of rainfall, the mean $\mu_y(\alpha, \underline{r})$ and the

[6] Assuming costs of de- and restocking would not fundamentally alter our results, but potentially re-enforce our central Result 3.

[7] That is, the quantity of marketable products is proportional to the herd's biomass and units are normalized in an appropriate way.

standard deviation $\sigma_y(\alpha, \underline{r})$ of income are (see Appendix A.1)

$$\mu_y(\alpha, \underline{r}) = \mu_r - \alpha \int_{\underline{r}}^{\infty} r\, f(r)\, dr \tag{8}$$

$$\sigma_y(\alpha, \underline{r}) = \sqrt{\sigma_r^2 + 2\,\alpha\,\mu_r \int_{\underline{r}}^{\infty} r\, f(r)\, dr - \alpha^2 \left[\int_{\underline{r}}^{\infty} r\, f(r)\, dr \right]^2 - \alpha\,(2-\alpha) \int_{\underline{r}}^{\infty} r^2\, f(r)\, dr}, \tag{9}$$

where μ_r and σ_r are the mean and the standard deviation of rainfall.

The model, as it has been specified so far, implies that a conservative strategy has a positive long-term impact on reserve biomass and, thus, on future income. In addition, there is a positive immediate effect of resting on income, which has not been captured so far. Income losses in the face of drought are smaller with resting than under a strategy without resting, since forage is available on rested paddocks. This effect will be captured indirectly by Assumption 1 below.

3.5 Farmer's choice of grazing management strategy

We assume that the farmer's utility only depends on income y, and that he is a non-satiated and risk averse expected utility maximizer. Let

$$U = \sum_{t=1}^{T} \frac{\mathcal{E}_t\,[v(y(t))]}{(1+\delta)^{t-1}} \tag{10}$$

be his von Neumann-Morgenstern expected utility function, where δ is the discount rate, the Bernoulli utility function $v(\cdot)$ is a strictly concave function of income y, and $\mathcal{E}_t$ is the expectancy operator at time t. In particular, we will employ a utility function with constant relative risk-aversion,

$$v(y) = \frac{y^{1-\rho} - 1}{1-\rho}, \tag{11}$$

where $\rho > 0$ is the constant parameter which measures the degree of relative risk-aversion (Mas-Colell et al. 1995: 194).

The farmer will choose the grazing management strategy which maximizes his von Neumann-Morgenstern expected utility function (10). In order to analyze this choice, the basic idea is to regard the choice of a grazing management strategy as the choice of a 'lottery' (Baumgärtner and Quaas 2004). Each possible lottery is characterized by the probability distribution of pay-off, where the pay-off is given by the farmer's income $y(t) \in \mathbb{R}_+$, and the (log-normal) probability distribution is characterized by the mean income $\mu_y(t)$ and the standard deviation $\sigma_y(t)$ of income. Given the ecological dynamics, both the mean income and the standard deviation solely depend on the grazing management strategy applied. Thus, choosing a grazing management strategy implies choosing a particular distribution of income.

We assume that the farmer initially, i.e. at $t = 0$ prior to the first grazing period, chooses a grazing management strategy $(\alpha, \underline{r})$, which is then applied in all subsequent years. This is a simple form of adaptive management. However, it is more sophisticated than the 'opportunistic' strategy. Thereby, when choosing the strategy, the farmer does not know which amount of rainfall will actually occur, but he knows the probability distribution of rainfall and he has full knowledge of the ecosystem. As a result, he knows the probability distribution of his income for any possible grazing management strategy. Formally, the farmer's decision problem is

$$\max_{(\alpha,\underline{r})} \sum_{t=1}^{T} \frac{\mathcal{E}_t\left[v(y(t))\right]}{(1+\delta)^{t-1}} \quad \text{s.t.} \quad (1),(2),(3),(4),(5). \tag{12}$$

In order to focus on short-term optimization, we consider the extreme case of a farmer with a planning horizon of $T = 1$. This means that prior to the first grazing period, the farmer chooses the grazing management strategy which maximizes his expected utility for this period only. Hence, the farmer's decision problem (12) reduces to optimizing the first-period expected utility. Concerning the probability distribution of income, we assume the following.

Assumption 1

The farmer's income is log-normally distributed with mean $\mu_y(\alpha, \underline{r})$ *as given by* (8) *and standard deviation* $\sigma_y(\alpha, \underline{r})$ *as given by* (9).

That is, we replace the probability density function of the farmer's income (7) by a log-normal distribution with the same mean and standard deviation. The reason is twofold. (i) The log-normal distribution is analytically convenient, as it allows us to completely specify the problem in terms of mean μ_y and standard deviation σ_y, and to derive analytical results. (ii) It is an elegant and simple way of including short-term positive effects

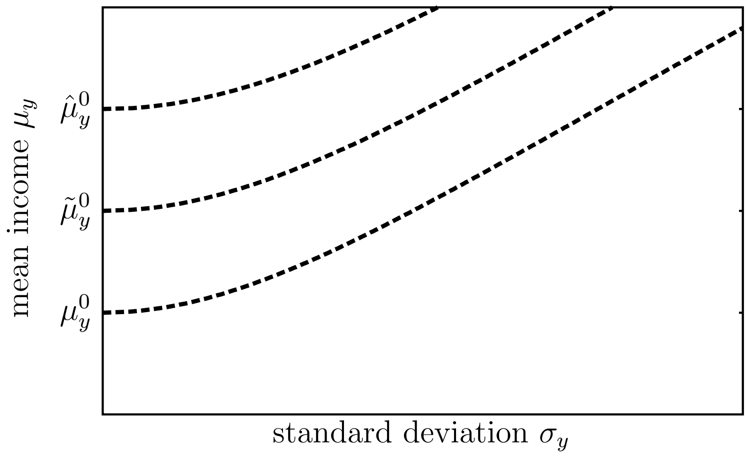

Figure 1: A set of indifference curves of the risk-averse farmer in the mean-standard deviation space for log-normally distributed incomes and constant relative risk aversion $\rho = 1$.

of resting on income because the occurrence of very low incomes, under the probability distribution of income as given by Assumption 1, is less likely if resting is applied than if no resting takes place (see Appendix A.1).

Using Assumption 1, the farmer's expected utility for $T = 1$ can be calculated from Equation (10) with the specification (11) of the Bernoulli utility function $v(y)$. It is given by the following explicit expression (see Appendix A.2):

$$\mathcal{E}\left[\frac{y^{1-\rho}-1}{1-\rho}\right] = \frac{\mu_y^{1-\rho}\left(1+\sigma_y^2/\mu_y^2\right)^{-\rho(1-\rho)/2}-1}{1-\rho}. \tag{13}$$

The indifference curves of the farmer's utility function U can be drawn in the mean – standard deviation space. Figure 1 shows such a set of indifference curves for a given degree ρ of relative risk-aversion. The indifference curves are increasing and convex if the standard deviation is sufficiently small compared to the mean, i.e. for $(\mu_y/\sigma_y)^2 > 1+\rho$ (see Appendix A.3). The slope of the indifference curves is increasing in the degree of relative risk aversion ρ (see Appendix A.3). In particular, the indifference curves are horizontal lines for risk-neutral farmers, i.e. for $\rho = 0$.

With Assumption 1, the farmer's optimization problem is (using a monotonic transformation of utility function 13)

$$\max_{(\alpha,\underline{r})} \frac{\mu_y}{\left(1+\sigma_y^2/\mu_y^2\right)^{\rho/2}} \quad \text{s.t.} \quad (8) \text{ and } (9). \tag{14}$$

4 Results

The analysis proceeds in three steps (Results 1, 2 and 3 below): First, we analyze the short-term optimization of the farmer. By choosing a grazing management strategy $(\alpha, \underline{r}) \in [0,1] \times [0,\infty)$, the farmer determines the mean and the standard deviation $(\mu_y, \sigma_y) = (\mu_y(\alpha, \underline{r}), \sigma_y(\alpha, \underline{r}))$ of his income at the end of the first grazing period. Thereby, the farmer faces a trade-off between strategies which yield a high mean income at a high standard deviation, and strategies which yield a low mean income at a low standard deviation. The farmer in our model does not consider the distant future at all. This is for the sake of analytical clarity: intertemporal effects of the grazing management strategies on the ecosystem dynamics are not taken into consideration in the farmer's decision.

Second, we analyze the long-term consequences of different grazing management strategies on the ecological-economic system. In particular, we study how the intertemporal development of the mean reserve biomass and the mean income depend on the strategy.

Finally, we put the two parts of the analysis together and derive conclusions about how the long-term sustainability of the short-term optimal strategy depends on the farmer's degree of risk-aversion.

4.1 Feasible strategies and income possibility set

To start with, we define the *income possibility set* as the set of all mean incomes and standard deviations of income $(\mu_y(\alpha, \underline{r}), \sigma_y(\alpha, \underline{r})) \in (0,\infty) \times [0,\infty)$, which are attainable in the first grazing period by applying a feasible management rule $(\alpha, \underline{r}) \in [0,1] \times [0,\infty)$. These are given by Equations (8) and (9). Figure 2 shows the income possibility set for particular parameter values.

The figure provides one important observation: there exist inefficient strategies, i.e. feasible strategies that yield the same mean income, but with a higher standard deviation (or: the same standard deviation, but with a lower mean) than others. These strategies can be excluded from the set of strategies from which the optimum is chosen by a risk-averse and non-satiated decision maker. In the following, we thus focus on the efficient

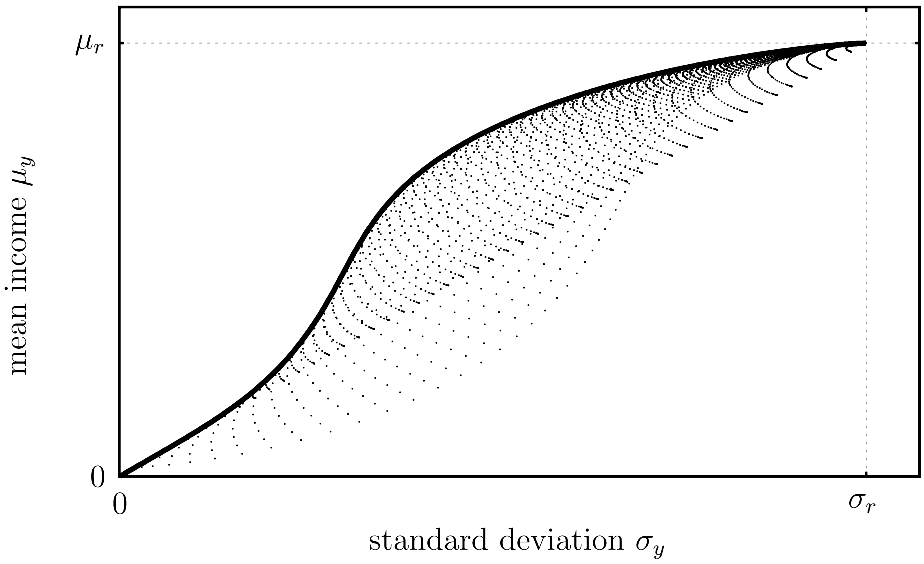

Figure 2: The set of all means μ_y and standard deviations σ_y of the farmer's income y, each point denoting a separate strategy, as well as the income possibility frontier (thick line). Parameter values are $\mu_r = 1.2$ and $\sigma_r = 0.7$.

strategies, which generate the *income possibility frontier* (Figure 2, thick line):

Definition 2

The *income possibility frontier* is the set of expected values μ_y and standard deviations σ_y of income for which the following conditions hold:

1. (μ_y, σ_y) is in the income possibility set, i.e. it is feasible.

2. There is no $(\mu'_y, \sigma'_y) \neq (\mu_y, \sigma_y)$ in the income possibility set with $\mu'_y \geq \mu_y$ and $\sigma'_y \leq \sigma_y$.

The question at this point is, 'What are the grazing management strategies $(\alpha, \underline{r})$ that generate the income possibility frontier?' We call these strategies *efficient*.

Lemma 1

The set of efficient strategies has the following properties.

- *Each point on the income possibility frontier is generated by exactly one (efficient) strategy.*

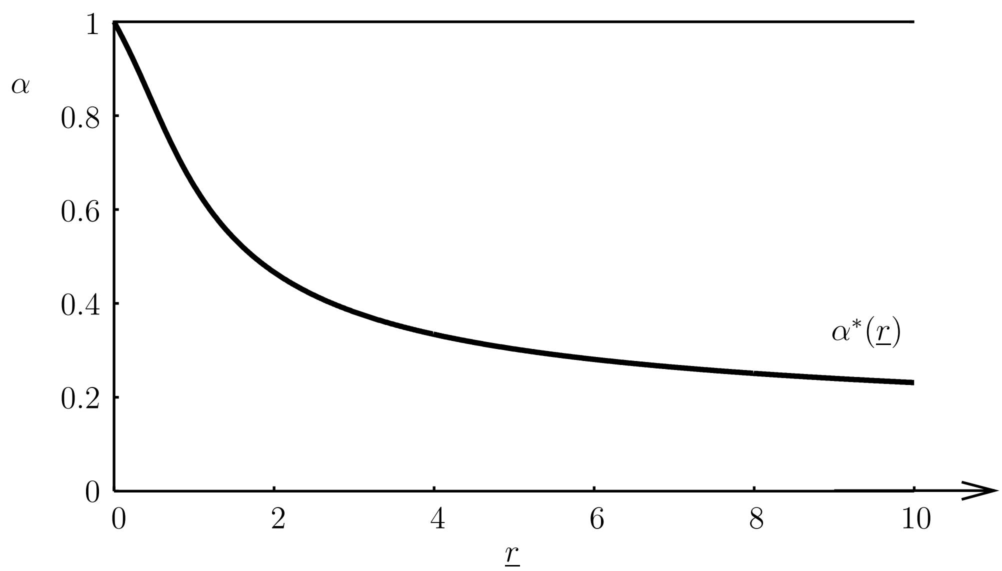

Figure 3: The set of feasible strategies is given by the whole area $\alpha \in [0,1]$, $\underline{r} \in [0,\infty)$. The set of efficient strategies for parameters $\mu_r = 1.2$ and $\sigma_r = 0.7$ is the curve.

- *There exists* $\Omega \subseteq [0,\infty)$, *such that the set of efficient strategies is given by* $(\alpha^*(\underline{r}), \underline{r})$ *with*

$$\alpha^*(\underline{r}) = \frac{\int_{\underline{r}}^{\infty} r\,(r-\underline{r})\,f(r)\,dr}{\int_{\underline{r}}^{\infty} r\,(r-\underline{r}/2)\,f(r)\,dr} \quad \textit{for all} \quad \underline{r} \in \Omega. \tag{15}$$

- $\alpha^*(\underline{r})$ *has the following properties:*

$$\alpha^*(0) = 1, \quad \lim_{\underline{r}\to\infty} \alpha^*(\underline{r}) = 0, \quad \textit{and} \quad \frac{d\alpha^*(\underline{r})}{d\underline{r}} < 0 \quad \textit{for all} \quad \underline{r} \in \Omega.$$

Proof: see Appendix A.4.

Figure 3 illustrates the lemma. Whereas the set of feasible strategies is the two-dimensional area bounded by $\underline{r} = 0$, $\alpha = 0$, $\alpha = 1$, the set of efficient strategies, as given by Equation (15), is a one-dimensional curve. Thus, the efficient strategies are described by only one parameter, $\underline{r}$, while the other parameter α is determined by $\alpha = \alpha^*(\underline{r})$ (Equation 15). The curve $\alpha^*(\underline{r})$ is downward sloping: With a higher rain threshold $\underline{r}$, i.e. if resting only takes place in years with higher precipitation, the efficient share $\alpha^*(\underline{r})$ of rested paddocks is smaller. In other words, for efficient strategies, a higher rain threshold

$\underline{r}$ does not only mean that the condition for resting is less likely to be fulfilled, but also that a smaller share α^* of paddocks is rested if resting takes place. Hence, if an efficient strategy is characterized by a smaller $\underline{r}$, we call it *more conservative*.

Knowledge of the efficient strategies allows us to characterize the income possibility frontier, and to establish a relationship between efficient grazing management strategies and the resulting means and standard deviations of income.

Lemma 2

The farmer's expected income in the first grazing period, $\mu_y(\alpha, \underline{r})$ (Equation 8), is increasing in $\underline{r}$ for all efficient strategies:

$$\frac{d\,\mu_y(\alpha^*(\underline{r}), \underline{r})}{d\underline{r}} > 0 \quad \textit{for all } \underline{r} \in \Omega.$$

The extreme strategies, $\underline{r} = 0$ and $\underline{r} \to \infty$, lead to expected incomes of $\mu_y(\alpha^*(0), 0) = 0$ and $\lim_{\underline{r}\to\infty} \mu_y(\alpha^*(\underline{r}), \underline{r}) = \mu_r$.

Proof: see Appendix A.5.

For all efficient strategies a higher rain threshold $\underline{r}$ for resting, i.e. a less conservative strategy, implies a higher mean income. Whereas no resting, $\underline{r} = 0$ (opportunistic strategy), leads to the maximum possible mean income of μ_r, the opposite extreme strategy, $\underline{r} \to \infty$ (no grazing at all), leads to the minimum possible income of zero. Overall, a change in the grazing management strategy affects both, the mean income and the standard deviation of income.

Lemma 3

The income possibility frontier has the following properties:

- *The income possibility frontier has two corners:*
  - *The southwest corner is at $\sigma_y = 0$ and $\mu_y = 0$. At this point, the income possibility frontier is increasing with slope μ_r/σ_r.*
  - *The northeast corner is at $\sigma_y = \sigma_r$ and $\mu_y = \mu_r$. At this point, the income possibility frontier has a maximum and its slope is zero.*
- *In between the two corners, the income possibility frontier is increasing and located above the straight line from one corner to the other. It is S-shaped, i.e. from southwest to northeast there is first a convex segment and then a concave segment.*

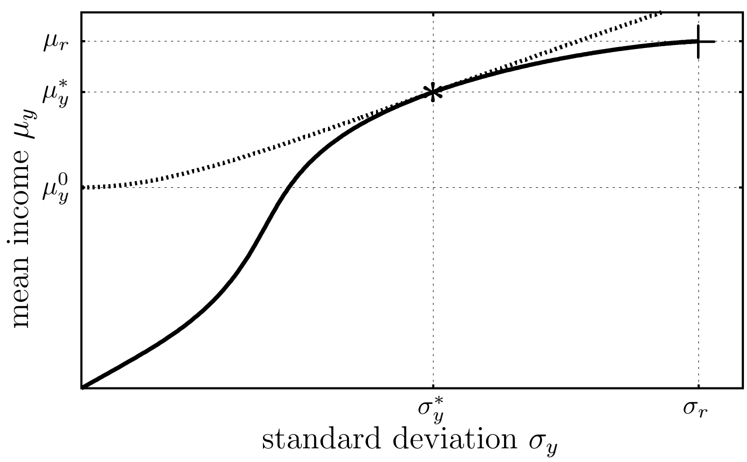

Figure 4: The optimum for a risk-averse farmer ($\rho = 5.5$, denoted by $*$) and a risk-neutral farmer ($\rho = 0$, denoted by $+$).

Proof: see Appendix A.6.

Figure 2 illustrates the lemma. The property, that the income possibility frontier is increasing, suggests that resting acts like an insurance for the farmer. This means, by choosing a more conservative grazing management strategy, the farmer can decrease his risk (standard deviation) of income, but only at the price of a decreased mean income. Thus, there is an insurance value associated with choosing a more conservative strategy (Baumgärtner and Quaas 2004).

4.2 Optimal strategy in the short-run

The optimal strategy results from both the farmer's preferences (Figure 1) and the income possibility frontier (Figure 2). It is determined by the mean μ_y^* and the standard deviation σ_y^*, at which the indifference curve is tangential to the income possibility frontier (Figure 4). It turns out that the optimal strategy is uniquely determined.

Lemma 4

(i) If $(\mu_r/\sigma_r)^2 > 1 + \rho$, *the optimum* (μ_y^*, σ_y^*) *is unique.*[8]

(ii) For $\rho > 0$, *the optimum is an interior solution with* $0 < \mu_y^* < \mu_r$ *and* $0 < \sigma_y^* < \sigma_r$.

[8] This is a sufficient condition which is quite restrictive. A unique optimum exists for a much larger range of parameter values.

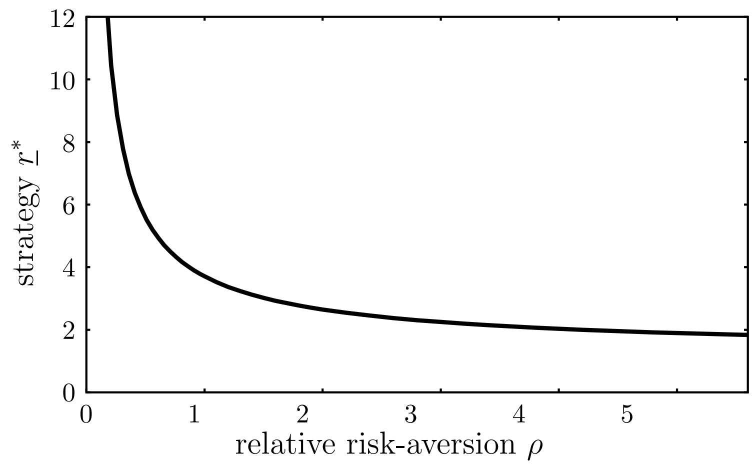

Figure 5: The rain threshold $\underline{r}^*$ of the optimal strategy as a function of the farmer's degree of risk-aversion ρ. Parameter values are the same as in Figure 2.

For $\rho = 0$, the optimum is a corner solution with $\mu_y^* = \mu_r$ and $\sigma_y^* = \sigma_r$.

Proof: see Appendix A.7.

The optimal strategy crucially depends on the degree of risk-aversion. In the particular case of a risk-neutral farmer ($\rho = 0$), the strategy that yields the maximum mean, irrespective of the standard deviation associated with it, is chosen. The optimal grazing management strategy of such a risk-neutral farmer is the strategy without resting, i.e. with $\underline{r} = \infty$ (and, therefore, $\alpha = 0$). That is, he employs an opportunistic strategy.

If the farmer is risk-averse, he faces a trade-off between expected income and variability of the income, because strategies that yield a higher mean income also display a higher variability of income. This leads to the following result, which is illustrated in Figures 4 and 5.

Result 1

A unique interior solution $(\alpha^*(\underline{r}^*), \underline{r}^*)$ to the farmer's decision problem (14), if it exists (see Lemma 4), has the following properties:

(i) The more risk-averse the farmer, the smaller are the mean μ_y^* and the standard deviation σ_y^* of his income.

(ii) The more risk-averse the farmer, the more conservative is his grazing management

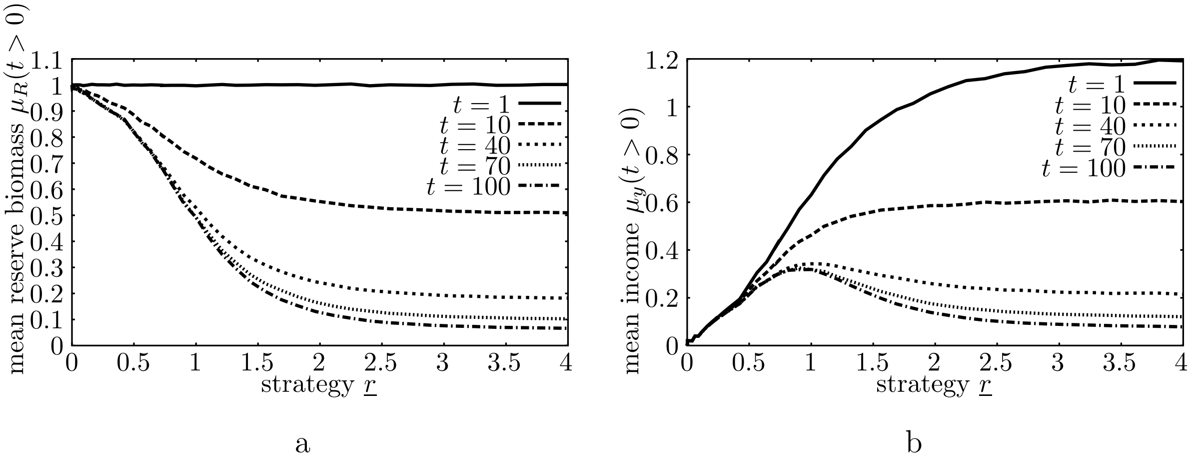

Figure 6: Relation between the grazing management strategy (given by the rain threshold $\underline{r}$) and future mean reserve biomass $\mu_R(t > 0)$ (in units of initial reserve biomass), as well as future mean income $\mu_y(t > 0)$ for different strategies on the income possibility frontier. Parameter values are $\mu_r = 1.2$, $\sigma_r = 0.7$, $I \cdot K = 8000$, $d = 0.15$, $w_G = 1.2$, $w_R = 0.2$, $c = 0.5$, $I \cdot R = 2400$.

strategy:

$$\frac{d\underline{r}^*}{d\rho} < 0. \tag{16}$$

Proof: see Appendix A.8.

4.3 Intertemporal impact of grazing management strategies

To study the intertemporal ecological and economic impact of the grazing management strategy chosen on the basis of short-term optimization (Problem 14), we assume that the farmer continues to apply this strategy in every subsequent period. Under this assumption, we compute the resulting probability distribution of income and reserve biomass over several decades in the future.[9] This calculation covers all efficient strategies $(\alpha^*(\underline{r}), \underline{r})$. The results of the numerical computation are shown in Figure 6, which enables the comparison of the long-term impacts, both in ecological and economic terms, of the different strategies that are efficient in the short-run. Their interpretation leads to the following result.

[9]The numerical details of the computation are described in Müller et al. (2004).

Result 2

For parameter values which characterize typical semi-arid rangelands (i.e. w_G, w_R, μ_r are small and c, σ_r are large) the long-term ecological impact is as stated in (i), and the long-term economic impact is as stated in (ii):

(i) The more conservative the strategy, i.e. the lower $\underline{r}$, the higher the mean reserve biomass $\mu_R(t)$ in the future,

$$\frac{d\,\mu_R(t)}{d\underline{r}} < 0 \quad \textit{for all} \quad t > 1 \quad \textit{and} \quad \underline{r} \in \Omega.$$

(ii) For high rain thresholds $\underline{r} \geq \underline{\hat{r}}$, the following holds: The more conservative the strategy, i.e. the lower $\underline{r}$, the higher the mean income in the long-term future for $t > \hat{t}$,

$$\frac{d\,\mu_y(t)}{d\underline{r}} < 0 \quad \textit{for all} \quad t > \hat{t} \quad \textit{and} \quad \underline{r} \geq \underline{\hat{r}}.$$

Proof: see Appendix A.9.

Result 2 states that the slope of the curves in Figure 6 is negative throughout, as far as reserve biomass is concerned; and is negative for high $\underline{r} \geq \underline{\hat{r}}$ and $t > \hat{t}$, as far as income is concerned. The smaller the rain threshold $\underline{r}$ in this domain, i.e. the more conservative the strategy, the higher are the mean reserve biomass and the mean income in future years, if the same strategy is applied over the whole period. This effect is in line with intuition for reserve biomass: the more conservative the strategy, the better is the state of the rangeland in the future.

As far as income is concerned, the argument is less straightforward. In particular, the mean income in the first period is increasing in $\underline{r}$ (Lemma 2). A less conservative strategy yields a higher mean income in this period, since more livestock is kept on the rangeland. This holds for several periods in the near future (cf. the line for $t = 10$ in Figure 6b). However, in the long run (for $t \geq \hat{t} \approx 40$), the strong grazing pressure on the pasture leads to reduced reserve biomass growth and less forage production in the long-term future, compared to a more conservative strategy. As a result, mean income is smaller. This can be seen in Figure 6b: the curves are downward-sloping for sufficiently high $t \geq \hat{t}$ and sufficiently high $\underline{r}$.[10]

[10] As can be seen in the figure, this effect becomes stronger in the long-term future: the curves are steeper for higher t.

The result states that the relationships between the strategy and the long-term impact on the mean reserve biomass and mean income, depend on the parameters. As shown in Appendix A.9 the assertions are true if the growth rates of the green and reserve biomass are low, the impact of grazing on the growth of the reserve biomass is high, and rainfall is low and highly variable. This is just the range of parameter values which is adequate for semi-arid rangelands, because these are fragile ecosystems which are highly susceptible to degradation if grazing pressure is high. For very robust ecosystems or very low stochasticity of rainfall, however, the result is not valid.

For small rain thresholds $\underline{r} < \hat{r}$, a more conservative strategy (i.e. a smaller $\underline{r}$) leads to a lower mean income, not only in the first period (Lemma 2), but also in the future. In this domain of strategies, resting is already so high that the future gains in reserve biomass from additional resting do not outweigh the losses from lower stocking.

Overall, the more conservative the strategy, i.e. the lower $\underline{r}$, the higher the mean reserve biomass and mean income in the long-term. In that sense, more conservative strategies are more sustainable; whereby we understand *sustainability*, for the sake of this analysis, in the following way.

Definition 3
A strategy A is called *more sustainable* than another strategy B, if and only if there exists some point in time t' such that for all $t > t'$ both the expected income and the expected reserve biomass under strategy A are higher than under B.

While this definition may seem peculiar, in the framework of our model it captures essential aspects of what has been called strong sustainability (Pearce et al. 1990, Neumayer 2003). It comprises an ecological as well as an economic dimension, with mean reserve biomass as an ecological indicator and mean income as an economic indicator. It expresses the aspect of long-term conservation of an ecological-economic system in the following sense. If we compare two strategies, A and B, where A is more sustainable than B, then both the reserve biomass and the income are better conserved until time $t > t'$ under A than under B. That is, both have declined less on average between the first period and period t under A than under B.[11]

[11] Furthermore, as far as the income criterion is concerned, Definition 3 includes an aspect of intertemporal efficiency. The formal criterion employed in Definition 3 is essentially Weizsäcker's (1965)

Combining Results 1 and 2, we can now make a statement about the relation between the farmer's short-term optimization and its long-term implications. From Result 1, we know that the more risk-averse a farmer is, the more conservative is his short-term optimal strategy. From Result 2, we know that a more conservative strategy is also more sustainable. This leads to the following result.

Result 3

The more risk-averse the farmer, the more sustainable is his short-term optimal grazing management strategy.

Result 3 sheds new light on the question 'How can one explain that people do behave in a sustainable way?' For, Result 3 suggests the following potential explanation. That a farmer A manages an ecosystem in a more sustainable manner than another farmer B, may be explained simply by a higher risk-aversion of farmer A. In particular, it is not necessary to assume that farmer A has any kind of stronger preferences for future income or sustainability than farmer B.

5 Conclusions

We have developed an integrated dynamic and stochastic ecological-economic model of grazing management in semi-arid rangelands. Within this, we have analyzed the choice of grazing management strategies of a risk-averse farmer, and the long-term ecological and economic impact of different strategies. We have shown that the more risk-averse a farmer is, the more conservative and sustainable is the short-term optimal strategy, although the distant future is neglected in his optimization.

A more conservative use of the ecosystem generates less expected income in the present than a less conservative use, but has benefits in two respects: First, it may be regarded as an investment in ecosystem quality, which enables higher future incomes from ecosystem use. This is the common understanding of the purpose of a conservative ecosystem management. We have shown that there is a second benefit, in so far as a conservative use of the rangeland reduces the variability of present income. In other words, conservative

overtaking criterion, which can be seen as an attempt to define intertemporal efficiency without making recourse to a discount rate.

ecosystem management provides insurance.

In our model the description of the farmer's decision focuses on the second aspect. It turns out that, in the face of uncertainty, higher risk-aversion is sufficient to induce the farmer to employ a more conservative and, thus, more sustainable ecosystem management. However, one should not conclude from our analysis that risk-aversion is sufficient to ensure a sustainable development in semi-arid areas. This issue requires a variety of further considerations.

In this analysis, we have focused on the environmental risk resulting from the uncertainty of rainfall. Other forms of risk, e.g. uncertainty concerning property-rights, or the stability of social and economic relations in general, might generate a tendency in the opposite direction, and promote a less conservative and less sustainable management of the ecosystem. Hence, in the face of different uncertainties, the net effect is not clear and has to be analyzed in detail.

Additional sources of income (say from tourism) or the availability of financial services (such as savings, credits, or commercial insurance), constitute possibilities for hedging income risk. For farmers, all these are substitutes for obtaining 'insurance' by conservative ecosystem management and, thus, may induce farmers to choose less conservative and less sustainable grazing management strategies. This becomes relevant as farmers in semi-arid regions are more and more embedded in world trade and have better access to global commodity and financial markets.

Our analysis was aimed at the specific context of grazing management in semi-arid rangelands. This system is characterized by a strong interrelation between ecology and economic use, which drives the results. While this is a very specific ecological-economic system, the underlying principles and mechanisms of ecosystem functioning and economic management are fairly general. Hence, we believe that there are similar types of economically used ecosystems, e.g. fisheries or other agro-ecosystems, to which our results should essentially carry over.

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A Appendix

A.1 Probability distribution of income

The rainfall r is log-normally distributed, i.e. the probability density function is

$$f(r) = \frac{1}{r\sqrt{2\pi s_r^2}} \exp\left(-\frac{(\ln r - m_r)^2}{2s_r^2}\right). \tag{A.17}$$

The two parameters m_r and s_r can be expressed in terms of the mean μ_r and standard deviation σ_r, $m_r = \ln \mu_r - \frac{1}{2}\ln(1 + \sigma_r^2/\mu_r^2)$ and $s_r^2 = \ln(1 + \sigma_r^2/\mu_r^2)$.

The probability density function (pdf) of income (Equation 7) is

$$\tilde{f}(y) = \begin{cases} f(y) & \text{if } y \le (1-\alpha)\,\underline{r} \\ f(y) + \frac{1}{1-\alpha} f\left(\frac{y}{1-\alpha}\right) & \text{if } (1-\alpha)\,\underline{r} < y < \underline{r} \\ \frac{1}{1-\alpha} f\left(\frac{y}{1-\alpha}\right) & \text{if } \underline{r} \le y \end{cases}. \tag{A.18}$$

Note that in the case without resting, i.e. $\alpha = 0$ or $\underline{r} = \infty$, the distribution of income equals the distribution of rainfall, $\tilde{f}(y) = f(y)$.

Proof: If rainfall is low, $r \le \underline{r}$, income equals rainfall, $y = r$. If rainfall is high, resting is applied and income is $y = (1-\alpha)\,r$. Hence, an income $y \in [y, y+dy]$, where $y \le (1-\alpha)\,\underline{r}$, arises with probability $f(y)\,dy$. An income $y \in [y, y+dy]$, where $(1-\alpha)\,\underline{r} < y < \underline{r}$, arises if $r \in [y, y+dy]$ *or* if $r \in [\frac{y}{1-\alpha}, \frac{y+dy}{1-\alpha}]$, i.e. with probability $f(y)\,dy + f\left(\frac{y}{1-\alpha}\right)\frac{dy}{1-\alpha}$. An even higher income $y \in [y, y+dy]$, where $y > \underline{r}$, arises only if $r \in [\frac{y}{1-\alpha}, \frac{y+dy}{1-\alpha}]$, i.e. with probability $f\left(\frac{y}{1-\alpha}\right)\frac{dy}{1-\alpha}$. $\square$

The expected value of the first year's income, Equation (7), is

$$\mu_y(\alpha, \underline{r}) = \int_0^\infty y\, f(r)\, dr = \int_0^{\underline{r}} r\, f(r)\, dr + (1-\alpha)\int_{\underline{r}}^\infty r\, f(r)\, dr = \mu_r - \alpha \int_{\underline{r}}^\infty r\, f(r)\, dr.$$

The variance is

$$\begin{aligned} \sigma_y^2(\alpha, \underline{r}) &= \int_0^\infty (y - \mu_y)^2\, f(r)\, dr = -\mu_y^2 + \int_0^{\underline{r}} r^2\, f(r)\, dr + (1-\alpha)^2 \int_{\underline{r}}^\infty r^2\, f(r)\, dr \\ &= \sigma_r^2 + 2\,\alpha\,\mu_r \int_{\underline{r}}^\infty r\, f(r)\, dr - \alpha^2 \left[\int_{\underline{r}}^\infty r\, f(r)\, dr\right]^2 - \alpha\,(2-\alpha)\int_{\underline{r}}^\infty r^2\, f(r)\, dr. \end{aligned}$$

To illustrate how Assumption 1 introduces (small) immediate positive effects of resting on income, consider Figure 7. The blue line depicts the probability density function (A.18)

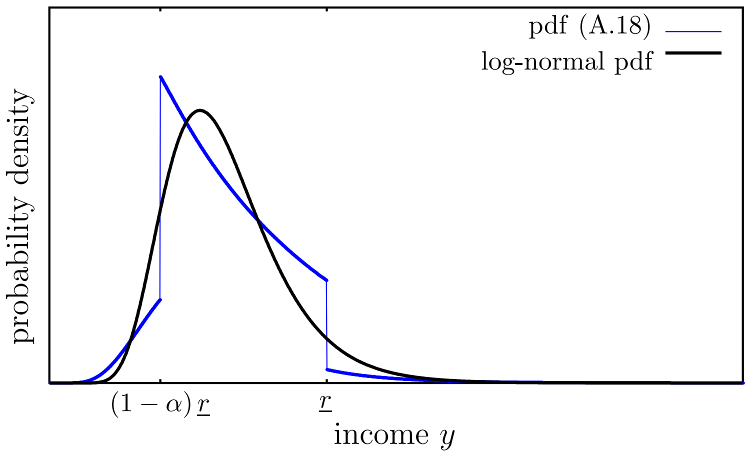

Figure 7: The probability density function (A.18) and the log-normal probability density function with the same mean and standard deviation.

of income for a strategy $(\alpha, \underline{r})$ with resting, the black line the log-normal pdf with the same mean and standard deviation. For small incomes $y \leq (1-\alpha)\,\underline{r}$, the pdf (A.18) of income equals the log-normal distribution of rainfall (A.17). Hence, for small incomes the pdf (A.18) of income resulting from a strategy with resting is equal to the pdf resulting from the strategy without resting, which is (A.17).

This is different for the log-normal distribution of income from Assumption 1, i.e. the black line in Figure 7. Here, a range of very small incomes exists, for which the log-normal distribution of income resulting from a strategy with resting is below the pdf resulting from the strategy without resting. The reason is that both the mean and the standard deviation are the same for the pdf (A.18) and the log-normal distribution of income, but the jump of the pdf (A.18) at $y = (1-\alpha)\,\underline{r}$ is smoothed out by the log-normal distribution.

Hence, very low incomes are less likely under the log-normal distribution of income corresponding to strategy $(\alpha, \underline{r})$ than under a strategy without resting. This is the reason why replacing the pdf (A.18) of income by the log-normal distribution is a convenient way of including short-term positive income-effects of resting in the model.

A.2 Expected utility function

With the specification (11) of the farmer's Bernoulli utility function $v(y)$, and Assumption 1 we get (using the notation $m_y = \ln\mu_y - \frac{1}{2}\ln\left(1+\sigma_y^2/\mu_y^2\right)$ and $s_y^2 = \ln\left(1+\sigma_y^2/\mu_y^2\right)$):

$$\begin{aligned}
\mathcal{E}[v(y)] &= \int_0^\infty \frac{y^{1-\rho}-1}{1-\rho}\,\frac{1}{y\sqrt{2\pi s_y^2)}}\exp\left(-\frac{(\ln y - m_y)^2}{2s_y^2}\right)dy \\
&\overset{z=\ln y}{=} \frac{1}{1-\rho}\left[\frac{1}{\sqrt{2\pi s_y^2}}\int_{-\infty}^{\infty}\exp\left((1-\rho)\,z\right)\,\exp\left(-\frac{(z-m_y)^2}{2s_y^2}\right)dz - 1\right] \\
&= \frac{\exp\left((1-\rho)\,\left(m_y + \frac{1-\rho}{2}\,s_y^2\right)\right)-1}{1-\rho} = \frac{\mu_y^{1-\rho}\,\left(1+\sigma_y^2/\mu_y^2\right)^{-\rho\,(1-\rho)/2}-1}{1-\rho}.
\end{aligned}$$

A simple monotonic transformation leads to the equivalent expected utility function

$$U = \frac{\mu_y}{\left(1+\sigma_y^2/\mu_y^2\right)^{\rho/2}}.$$

A.3 Properties of the indifference curves

Each indifference curve intersects the μ_y-axis at $\sigma_y = 0$. The point of intersection, μ_0, is the certainty equivalent of all lotteries on that indifference curve. Hence, the indifference curve is the set of all $(\mu_y, \sigma_y) \in \mathbb{R}_+ \times \mathbb{R}_+$ for which

$$\frac{\mu_y}{\left(1+\sigma_y^2/\mu_y^2\right)^{\rho/2}} = \mu_0. \tag{A.19}$$

The slope of the indifference curve is obtained by differentiating Equation (A.19) with respect to σ_y (considering μ_y as a function of σ_y) and rearranging:

$$\frac{d\mu_y}{d\sigma_y} = \frac{\rho\,\sigma_y\,\mu_y}{(1+\rho)\,\sigma_y^2+\mu_y^2} > 0. \tag{A.20}$$

The curvature is obtained by differentiating this equation with respect to σ_y, inserting $d\mu_y/d\sigma_y$ again and rearranging

$$\frac{d^2\mu_y}{d\sigma_y^2} = \frac{d}{d\sigma_y}\frac{d\mu_y}{d\sigma_y} = \frac{\rho\,\mu_y\,(\mu_y^2-(1+\rho)\,\sigma_y^2)(\sigma_y^2+\mu_y^2)}{\left((1+\rho)\,\sigma_y^2+\mu_y^2\right)^3}, \tag{A.21}$$

which is positive, if and only if $\mu_y^2 > (1+\rho)\,\sigma_y^2$. Furthermore, the slope of the indifference curves increases with rising risk aversion,

$$\frac{d}{d\rho}\frac{d\mu_y}{d\sigma_y} = \frac{\sigma_y\,\mu_y\,\left(\sigma_y^2+\mu_y^2\right)}{\left((1+\rho)\,\sigma_y^2+\mu_y^2\right)^2} > 0.$$

A.4 Proof of Lemma 1

To find the efficient strategies, we first determine the strategies which minimize the standard deviation of income given the mean income. Out of these strategies those are efficient which maximize the mean income for a given standard deviation. Each point on the income possibility frontier is generated by exactly one efficient strategy, since the solution of the corresponding minimization problem is unique.

Equivalent to minimizing the standard deviation, we minimize the variance for a given mean income,

$$\min_{\alpha,\underline{r}} \sigma_y^2 \quad \text{s.t.} \quad \mu_y \geq \bar{\mu}_y,\ \alpha \in [0,1],\ \underline{r} \in [0,\infty). \tag{A.22}$$

For more convenient notation, we use the abbreviations

$$R_1(\underline{r}) = \int_{\underline{r}}^{\infty} r\, f(r)\, dr \quad \text{and} \quad R_2(\underline{r}) = \int_{\underline{r}}^{\infty} r^2\, f(r)\, dr. \tag{A.23}$$

The Lagrangian for the minimization problem (A.22) is

$$\begin{aligned} \mathcal{L} &= \sigma_y^2(\alpha,\underline{r}) + \lambda\,[\mu_y(\alpha,\underline{r}) - \bar{\mu}_y] \\ &= \sigma_r^2 + 2\,\alpha\,\mu_r\,R_1(\underline{r}) - \alpha^2\,R_1^2(\underline{r}) - \alpha\,(2-\alpha)\,R_2(\underline{r}) + \lambda\,[\mu_r - \alpha\,R_1(\underline{r}) - \bar{\mu}_y]\,. \end{aligned}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = 0 \quad \Leftrightarrow \quad \alpha\,\underline{r}\,f(\underline{r})\,\left[-2\,(\mu_r - \alpha\,R_1(\underline{r})) + (2-\alpha)\,\underline{r}\right] = -\,\lambda\,\alpha\,\underline{r}\,f(\underline{r}) \tag{A.24}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \quad \Leftrightarrow \quad 2\,(\mu_r - \alpha\,R_1(\underline{r}))\,R_1(\underline{r}) - 2\,(1-\alpha)\,R_2(\underline{r}) = -\,\lambda\,[-R_1(\underline{r})]\,. \tag{A.25}$$

Dividing (A.24) by (A.25) and rearranging leads to

$$R_1(\underline{r})\,(2-\alpha)\,\underline{r} = 2\,(1-\alpha)\,R_2(\underline{r}) \quad \Leftrightarrow \quad \alpha^*(\underline{r}) = \frac{R_2(\underline{r}) - \underline{r}\,R_1(\underline{r})}{R_2(\underline{r}) - \frac{1}{2}\,\underline{r}\,R_1(\underline{r})}.$$

Re-inserting (A.23) leads to (15), which is the unique solution of the first order conditions. $\sigma_y(\alpha^*(\underline{r}),\underline{r})$ is the minimum, since $\sigma_y(\alpha,\underline{r})$ is maximum at the corners $\alpha = 1$ (with $\underline{\rho} > 0$), or $\underline{\rho} = 0$ (with $\alpha < 1$): For $\alpha = 1$, we have $\partial\,\sigma_y^2(1,\underline{r})/\partial\alpha = 2\,(\mu_r - R_1(\underline{r}))\,R_1(\underline{r}) > 0$, i.e. if α is decreased from $\alpha = 1$, the variance decreases. For the $\underline{r} = 0$, we have $\lim_{\underline{r}\to 0} \partial\,\sigma_y^2(\alpha,\underline{r})/\partial\underline{r} = \lim_{\underline{r}\to 0} \alpha\,\underline{r}\,f(\underline{r})\,\left[-2\,\mu_r\,(1-\alpha)\right] \nearrow 0$, i.e. if $\underline{r}$ is increased from $\underline{r} = 0$, the variance decreases.

Equation (15) determines the set of strategies, which generate the minimum standard deviation for any given mean income. This set may include strategies for which a higher mean income is attainable with the same standard deviation. These are excluded in the set of efficient strategies, which is determined by $\alpha^*(\underline{r}, \underline{r})$, where $\underline{r}$ is chosen from the appropriate subset $\Omega \subseteq [0, \infty)$ of feasible rain thresholds.[12]

Turning to the properties of $\alpha^*(\underline{r})$, for $\underline{r} = 0$ the numerator and denominator of (15) are equal, hence $\alpha^*(0) = 1$. For $\underline{r} \to \infty$, we have, using L'Hospital's rule repeatedly, $\lim_{\underline{r}\to\infty} \alpha^*(\underline{r}) = 0$. Numerical computations for a wide range of parameters (μ_r, σ_r) resulted in qualitatively the same curves $\alpha^*(\underline{r})$ as shown in Figure 3.

A.5 Proof of Lemma 2

Inserting equation (15) and (A.23) into (8) and differentiating with respect to $\underline{r}$ yields:

$$\begin{aligned}
\frac{d\,\mu_y(\alpha^*(\underline{r}), \underline{r})}{d\underline{r}} &= -\frac{d\alpha^*(\underline{r})}{\underline{r}}\, R_1(\underline{r}) + \alpha^*(\underline{r})\,\underline{r}\, f(\underline{r}) \\
&= \frac{2\, R_1^2(\underline{r})\, R_2(\underline{r})}{(2\, R_2(\underline{r}) - \underline{r}\, R_1(\underline{r}))^2} + {\alpha^*}^2(\underline{r})\,\underline{r}\, f(\underline{r}) > 0.
\end{aligned}$$

For $\underline{r} \to 0$, we have $\lim_{\underline{r}\to 0} R_1(\underline{r}) = \mu_r$ and $\lim_{\underline{r}\to 0} R_2(\underline{r}) = \sigma_2^2 + \mu_r^2$, and $\alpha^*(0) = 1$. Inserting into equations (8) and (9) yields $\lim_{\underline{r}\to 0} \mu_y(\alpha^*(\underline{r}), \underline{r}) = 0$ and $\lim_{\underline{r}\to 0} \sigma_y(\alpha^*(\underline{r}), \underline{r}) = 0$.

For $\underline{r} \to \infty$, we have $\lim_{\underline{r}\to\infty} R_1(\underline{r}) = 0$ and $\lim_{\underline{r}\to\infty} R_2(\underline{r}) = 0$, and $\lim_{\underline{r}\to\infty} \alpha^*(\underline{r}) = 0$. Inserting into equations (8) and (9) yields $\lim_{\underline{r}\to\infty} \mu_y(\alpha^*(\underline{r}), \underline{r}) = \mu_r$ and $\lim_{\underline{r}\to\infty} \sigma_y(\alpha^*(\underline{r}), \underline{r}) = \sigma_r$.

A.6 Proof of Lemma 3

As shown in Appendix A.5, $\lim_{\underline{r}\to\infty} \mu_y(\alpha^*(\underline{r}), \underline{r}) = \mu_r$ and $\lim_{\underline{r}\to\infty} \sigma_y(\alpha^*(\underline{r}), \underline{r}) = \sigma_r$. This is the northeast corner of the income possibility frontier, since $\mu_y = \mu_r$ is the maximum possible mean income (cf. Lemma 2).

The slope of the income possibility frontier is

$$\frac{d\mu_y^{\text{ipf}}}{d\sigma_y} = \frac{d\mu_y(\alpha^*(\underline{r}), \underline{r})/d\underline{r}}{d\sigma_y(\alpha^*(\underline{r}), \underline{r})/d\underline{r}} = 2\,\sigma_y(\alpha^*(\underline{r}), \underline{r})\, \frac{d\mu_y(\alpha^*(\underline{r}), \underline{r})/d\underline{r}}{d\sigma_y^2(\alpha^*(\underline{r}), \underline{r})/d\underline{r}}.$$

Differentiating $\sigma_y^2(\alpha^*(\underline{r}), \underline{r}) = -\mu_y^2 + \int_0^\infty r^2\, f(r)\, dr - \alpha^*(\underline{r})\,(2 - \alpha^*(\underline{r}))\, R_2(\underline{r})$ with respect to $\underline{r}$ and inserting the expressions for $d\alpha^*(\underline{r})/d\underline{r}$ and $\mu_y^2(\alpha^*(\underline{r}), \underline{r})/d\underline{r}$ leads with some

[12] In the example shown in figure 2, however, we have $\Omega = [0, \infty)$.

rearrangement to $d\sigma_y^2(\alpha^*(\underline{r}),\underline{r})/d\underline{r} = [-2\,\mu_y + \underline{r}\,(2-\alpha^*(\underline{r}))]\; d\mu_y/d\underline{r}$. Thus, we have

$$\frac{d\mu_y^{\mathrm{ipf}}}{d\sigma_y} = \frac{2\,\sigma_y(\alpha^*(\underline{r}),\underline{r})}{-2\mu_y(\alpha^*(\underline{r}),\underline{r}) + \underline{r}\,(2-\alpha^*(\underline{r}))}. \tag{A.26}$$

In particular for $\underline{r}\to\infty$, the slope of the income possibility frontier is

$$\lim_{\underline{r}\to\infty}\frac{d\mu_y^{\mathrm{ipf}}}{d\sigma_y} = \lim_{\underline{r}\to\infty}\frac{2\,\sigma_y(\alpha^*(\underline{r}),\underline{r})}{-2\mu_y(\alpha^*(\underline{r}),\underline{r}) + \underline{r}\,(2-\alpha^*(\underline{r}))} = \lim_{\underline{r}\to\infty}\frac{2\,\sigma_r}{-2\mu_r + 2\,\underline{r}} = 0.$$

For $\underline{r}\to 0$ both the mean income $\mu_y(\alpha^*(\underline{r},\underline{r})$ and the standard deviation of income $\sigma_y(\alpha^*(\underline{r},\underline{r})$ vanish (cf. Appendix A.5). Since both cannot be negative, this is the southwest corner of the income possibility frontier. At this point, the slope of the income possibility frontier is

$$\lim_{\underline{r}\to 0}\frac{d\mu_y^{\mathrm{ipf}}}{d\sigma_y} = \lim_{\underline{r}\to 0}\frac{\mu_y(\alpha^*(\underline{r},\underline{r}))}{\sigma_y(\alpha^*(\underline{r}),\underline{r})} = \lim_{\underline{r}\to 0}\frac{(1-\alpha^*(\underline{r}))\,\mu_r}{\sqrt{\sigma_r^2\,(1-\alpha^*(\underline{r}))^2}} = \frac{\mu_r}{\sigma_r}.$$

For $\underline{r}=0$, and any given α, we have

$$\begin{aligned}\mu_y(\alpha,0) &= \mu_r - \alpha R_1(0) = (1-\alpha)\,\mu_r\\ \sigma_y^2(\alpha,0) &= \sigma_r^2 + 2\,\alpha\,\mu_r\,R_1(0) - \alpha^2\,R_1^2(0) - \alpha\,(2-\alpha)\,R_2(0) = (1-\alpha)^2\,\sigma_r^2,\end{aligned}$$

i.e. the straight line between $(\mu_y,\sigma_y)=(0,0)$ $(\alpha=1)$ and $(\mu_y,\sigma_y)=(\mu_r,\sigma_r)$ $(\alpha=0)$ is always within the income possibility set. Since for $\underline{r}=0$ the standard deviation is maximum for given mean income (cf. Appendix A.6), the income possibility frontier is located above this straight line.

To show numerically that the income possibility frontier may be divided into two domains – the convex domain for small σ_y and the concave domain for large σ_y –, we computed $\sigma_y(\alpha^*(\underline{r}),\underline{r})$ and $\mu_y(\alpha^*(\underline{r}),\underline{r})$ for parameters $(\mu_r,\sigma_r)=(\phi\cdot\xi,\xi)$, where ξ varies between 0.1 and 7.1 in steps of 1 and $\phi\in\{0.5,1,2,8\}$.[13] In order to get the data points equally distributed in the σ_y-μ_y-space, we calculated them for $\underline{r}=\psi\cdot(\xi+0.5)$, ψ varying between 0.01 and 20 in steps of 0.2. The results, which provide evidence for the assertion of the lemma, are shown in Figure 8. Note that the left borders of the respective income possibility sets are shown, whereas the income possibility frontiers are the upper parts of these curves. That is, the income possibility frontier has a jump, where the left border of the income possibility set is inwardly curved to the right (e.g. the curves at the bottom right of Figure 8).

[13]The idea is to scan the parameter space by varying μ_r and σ_r along straight lines with different slope through the origin of the (μ_r,σ_r)-space.

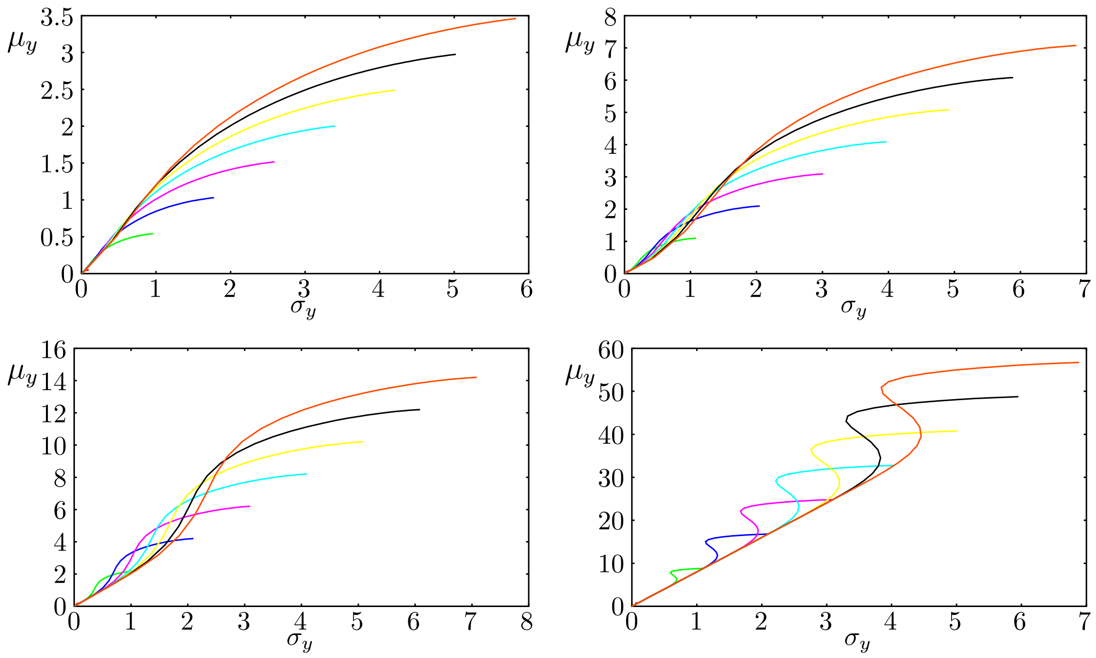

Figure 8: $\sigma_y(\alpha^*(\underline{r}), \underline{r})$ and $\mu_y(\alpha^*(\underline{r}), \underline{r})$ computed for parameters $(\mu_r, \sigma_r) = (\phi \cdot \xi, \xi)$. ξ varies between 0.1 and 7.1 in steps of 1, $\xi = 7.1$ being the curve on top in each case. ϕ is 0.5 for figure 8 on the top left, 1 on the top right, and 2 and 8 on the bottom left and right, respectively. Details about the procedure in the text.

A.7 Proof of Lemma 4

To prove part (i), we show that (a) the optimal indifference curve is convex over the whole range $\sigma_y \in [0, \sigma_r]$, and (b) the optimum is within the concave domain of the income possibility frontier.

Ad (a). Rearranging Equation (A.19) yields the following expression for the optimal indifference curve (where μ_0^* is the certainty equivalent for the optimum)

$$\left(\frac{\sigma_y}{\mu_y}\right)^2 = \left(\frac{\mu_y}{\mu_0^*}\right)^{2/\rho} - 1. \tag{A.27}$$

Inserting in the condition for the convexity of the indifference curve yields

$$\left(\frac{\mu_y}{\sigma_y}\right)^2 > 1 + \rho \quad \Leftrightarrow \quad \frac{\mu_y}{\mu_0^*} < \left(\frac{2+\rho}{1+\rho}\right)^{2/\rho}. \tag{A.28}$$

By assumption, this condition is fulfilled for $\mu_y = \mu_r$ on the indifference curve which intersects (μ_r, σ_r), i.e. which is below the optimal one. Since $\mu_y \leq \mu_r$ for all efficient strategies, this condition is fulfilled for all μ_y on the optimal indifference curve.

Ad (b). The minimum slope of the income possibility frontier in the convex domain (i.e. at the southwest border) is μ_r/σ_r (Lemma 3). The slope of the indifference curve at the optimum (μ_y^*, σ_y^*), however, is smaller,

$$1+\rho < \left(\frac{\mu_r}{\sigma_r}\right)^2 < \frac{\mu_r}{\sigma_r}\frac{\mu_y^*}{\sigma_y^*} \quad\Rightarrow\quad \frac{\rho}{1+(1+\rho)\,\frac{\sigma_y^{*2}}{\mu_y^{*2}}} < \frac{\mu_r}{\sigma_r}\frac{\mu_y^*}{\sigma_y^*} \quad\Leftrightarrow\quad \frac{\rho\,\sigma_y^*\,\mu_y^*}{\mu_y^{*2}+(1+\rho)\,\sigma_y^{*2}} < \frac{\mu_r}{\sigma_r},$$

where the inequality $\mu_r/\sigma_r < \mu_y^*/\sigma_y^*$ holds as a consequence of Lemma 3, and the expression on the left hand side of the last inequality is the slope of the indifference curve at the optimum (cf. Equation A.20). Hence, the optimum cannot be in the convex domain of the income possibility frontier.

Ad (ii). For $\rho = 0$, the indifference curves are horizontal lines. Hence, the maximum of the income possibility frontier, which is at the corner $(\mu_y, \sigma_y) = (\mu_r, \sigma_r)$, is the optimum.

For $\rho > 0$ corner solutions are excluded. At the corner $(\mu_y, \sigma_y) = (\mu_r, \sigma_r)$ the slope of the income possibility frontier is zero (Lemma 3), whereas the indifference curves have a positive slope, provided $\rho > 0$. At the corner $(\mu_y, \sigma_y) = (0, 0)$, the income possibility frontier is increasing with a slope μ_r/σ_r (Lemma 3), but the slope of the indifference curves is zero for $\sigma_r = 0$ (cf. Appendix A.3).

A.8 Proof of Result 1

We have shown that the unique optimum is in the concave domain of the income possibility frontier (Appendix A.7), and that the slope of the farmer's indifference curves increases with ρ (Appendix A.3). Thus, the optimal mean income μ_y^* decreases if ρ increases. Since for efficient strategies the mean μ_y^* is increasing in $\underline{r}$, the rain threshold $\underline{r}^*$ of the optimal strategy decreases if ρ increases.

A.9 Proof of Result 2

The aim of this section is to show in a sensitivity analysis how the qualitative results shown in Figure 6 and stated in Result 2 depend on the parameters of the model. The sensitivity analysis was performed using a Monte Carlo approach, repeating the computations with multiple randomly selected parameter sets. We focussed on three parameters, namely the growth parameter of green biomass w_G, the influence c of grazing on the growth of reserve biomass, and the standard deviation σ_r of rainfall. The other parameters either affect the outcomes in the same direction as the selected parameters (this is the case for the growth parameter of the reserve biomass w_R and the expected value of rainfall μ_r), or in the inverse direction (this is the case for the death rate of the reserve biomass d).[14] Hence their variation enables no further insights.

A sample size of $N = 20$ parameter sets was created according to the Latin Hypercube sampling method (Saltelli et al. 2000).[15] The three parameters were assumed to be independent uniformly distributed, with $0 \leq w_G \leq 5$, $0 \leq \sigma_r \leq 2.4$ and $0 \leq c \leq 1$, the upper bounds for w_G and σ_r are guesses which proved to be suitable. The respective simulation results were compared to the results shown in Figure 6. The following types of long-term dynamics of mean reserve biomass and mean income (distinct from those stated in Result 2) were found:[16]

(i) If the growth parameter of the green biomass w_G is very low, i.e. if $w_G \cdot w_R < d$, the reserve biomass is not able to persist at all. Keeping livestock is not possible, independent of the chosen grazing management strategy.

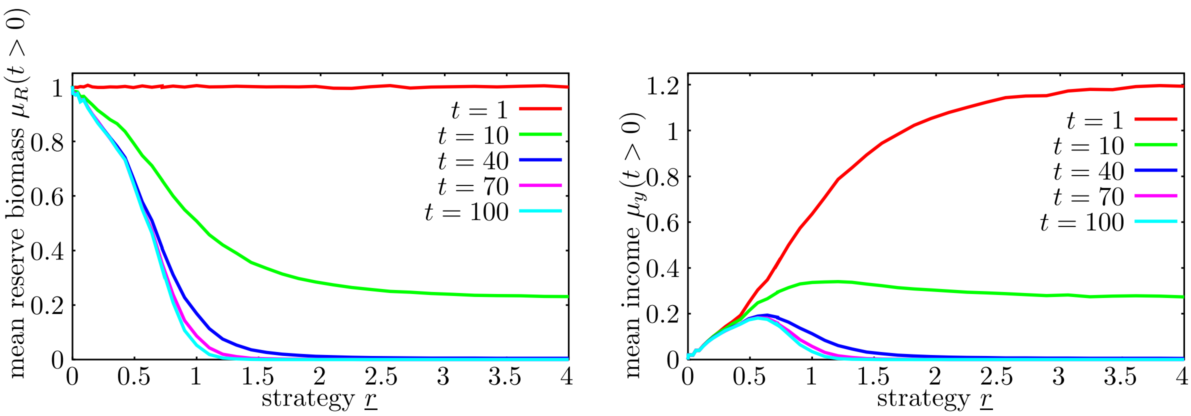

Figure 9: Parameter values are as in Figure 6, except for $c = 0.9$.

(ii) If the impact c of grazing on the growth of the reserve biomass is very high, the mean reserve biomass declines to zero in finite time, unless the grazing management

[14]For the two parameters K and R, no substancial influence is to be expected: they just rescale the problem.

[15]This method, by stratifying the parameter space into N strata, ensures that each parameter has all proportions of its distribution represented in the sample parameter sets.

[16]To illustrate them, additional calculations were done, where one parameter was chosen differently from the original parameter set of Figure 6 in each case.

strategy is very conservative. This is illustrated in Figure 9, where we have chosen $c = 0.9$.

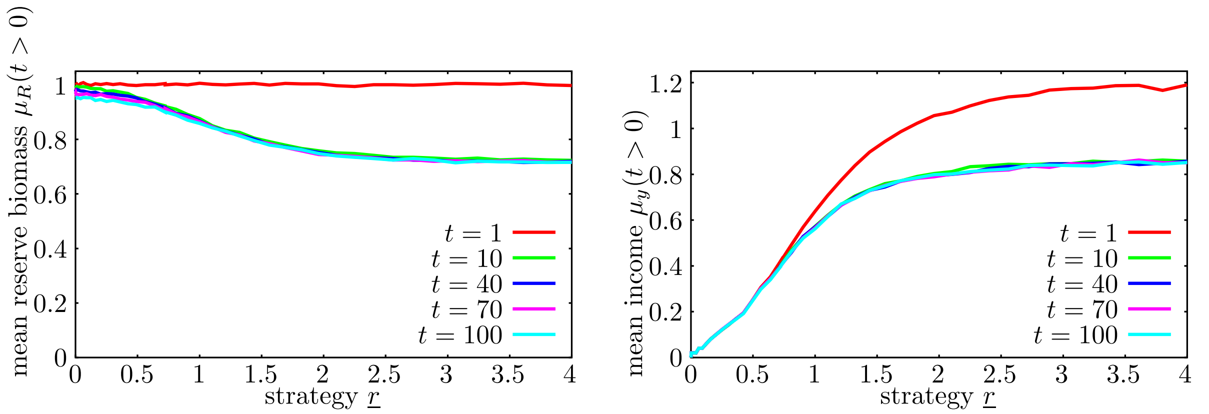

Figure 10: Parameter values are as in Figure 6, except for $w_G = 4$.

(iii) If the growth parameter of the green biomass is very high or the impact of grazing on the growth of the reserve biomass is very low, the future mean income is the higher the less conservative the strategy is, i.e. resting is not required to preserve the ecosystem. This is illustrated in Figure 10 for a very high growth rate of the biomass, $w_G = 4$. Qualitatively the same outcome arises for very low c (see also Müller et al. 2004).

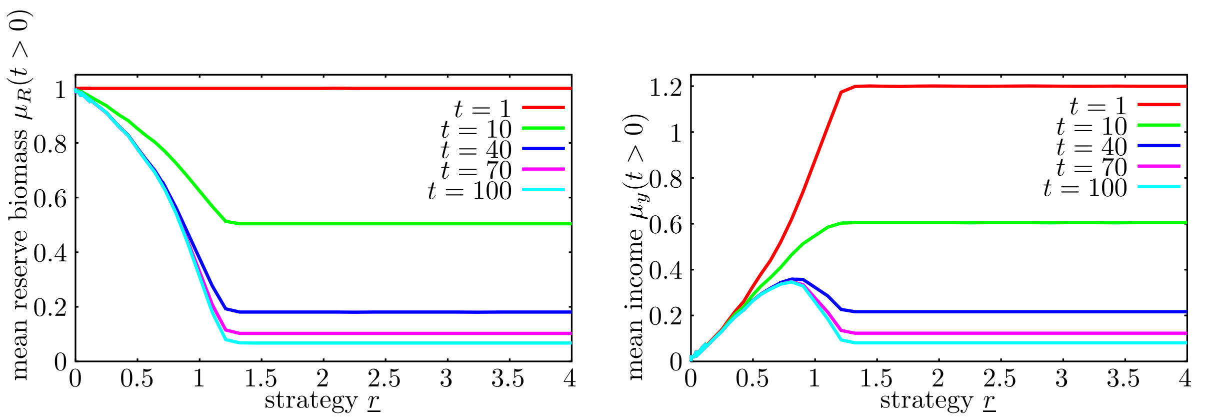

Figure 11: Parameter values are as in Figure 6, except for $\sigma_r = 0.05$.

(iv) If the standard deviation of rainfall σ_r is very small, resting is almost deterministic: for $\underline{r} > \mu_r$, resting will take place in hardly any year, such that μ_R and μ_y are independent of the strategy. For $\underline{r} < \mu_r$, resting will take place in almost every year, i.e. the share $\alpha^*(\underline{r})$ of rested paddocks determines the outcome, as illustrated in Figure 11 for $\sigma_r = 0.05$.