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Abstract

In this paper we present a two-period model where a left-wing and a right-wing political party are solely interested in the politics they pursue. We assume that voters are fully rational but show reciprocal behavior. By contrast, political parties are not motivated by reciprocity. We show that reciprocity may have dramatic consequences for models of voting behavior. The incentive to be kind to the median voter may guarantee that a position closer to the the median voter's position is adopted even when political parties are not directly interested in being elected and cannot commit to a political stance during an election campaign. Moreover, reciprocity increases incumbency advantages.

Keywords: Reciprocity, Voting, Median Voter Theorem.

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1 Introduction

Reciprocity seems to be an important factor in determining human behavior under some circumstances. Experiments have provided evidence that people are not solely motivated by material payoffs, but also have a desire to reward the kindness of others and to punish behavior which is perceived as unkind. This behavior has been shown to have important consequences in several areas, e.g., for employer-employee relationships and for the price-setting of a monopolist. Reciprocity may lead to efficiency wages above the market-clearing level, because wage-cuts would be considered an unfriendly act and would result in less effort exerted by workers.[1] Reciprocity may also lead to less aggressive price-setting by a monopolist, because consumers may punish "greedy" behavior.[2]

Although, to our knowledge, there have been no applications of models with reciprocal behavior to the political process so far, it seems reasonable that the decision of voters may not be solely dependent on their expected future payoffs but may also be the consequence of psychological factors. If the policy conducted by the government has been favorable to some voters, they may feel grateful and may want to reward such behavior. On the other hand, voters may also be angry with politicians who have made a decision unfavorable to them. Consequently, voters may want to punish these politicians by voting for an alternative party although the alternative party's policy may not yield higher material payoffs to voters.

Reciprocity (or fairness) is formalized by, e.g., Rabin (1993) for two-player normal form games. Since we want to consider a dynamic model where voters may punish or reward past behavior of politicians, we use the concept of Dufwenberg and Kirchsteiger (2004), which is an extension of Rabin's concept to multi-player games in extensive form.

It is well-known that political parties may adopt the median voter position if they can commit to future policy in advance. This strategy maximizes their chances of being

[1] Cf. e.g. Akerlof (1982) and Akerlof and Yellen (1990).
[2] Cf. Kahneman et al. (1986).

elected. This observation is known as the "Median Voter Theorem".[3] If, however, political parties cannot commit to following a particular policy in the future and voters are standard rational utility-maximizing agents, then parties have no incentive to pursue the median voter's preferred policy.[4] In our model we show that reciprocal voters may reward party behavior which is perceived as kind. This, in turn, may induce political parties to implement policies which are closer to the one preferred by the median voter. We also demonstrate that reciprocity may lead to a substantial advantage of the incumbent party when it comes to an election.

The paper is organized as follows. In the next section, we present the basic model with heterogeneous preferences of voters and parties. Then we assume that voters are merely interested in material payoffs as a benchmark case. Thus we present the subgame-perfect Nash equilibrium for this game. In section 4, we review the concept of reciprocity by Dufwenberg and Kirchsteiger (2004) which we apply in this paper. In section 5, we examine the situation where voters may be motivated by reciprocity. In the next section we show that the incumbent party would be re-elected even if it is well-known that its policy yields lower material payoffs for a majority of voters. We summarize our results in section 7. In the appendix, we present a different version of our model with homogeneous preferences of voters, but where politicians may find working on socially beneficial projects tedious and thus may usually supply a level of effort on these projects which is too low. We also illustrate that the effect identified in this paper continues to hold if we modify the definition of the equitable payoff. The equitable payoff describes the payoff level that is perceived as neither mean nor kind but neutral.

2 Model

We assume that there is a continuum of potential policies in each period, represented by the interval $[-1; +1]$. There is an odd number N of voters who are characterized by

[3]Cf. Downs (1957).

[4]In a multi-period framework, there are incentives to pursue announced policies due to reputation effects. Cf. Alesina (1988).

their bliss point τ. The ideal positions of voters τ are equally spaced on the interval $[-1;+1]$, i.e., the set of voters is $\mathcal{N} := \left\{-1, -1+\frac{2}{N-1}, ..., 0, ...1-\frac{2}{N-1}, 1\right\}$. The material payoffs of voters are $u_\tau^{(t)} = -\left(\tau - p^{(t)}\right)^2$ in each period t. The discount factors are identical across voters and given by δ $(0 < \delta < 1)$. $p^{(t)}$ denotes the policy conducted by the government in period t. We assume that there are two political parties, a left-wing party, L, and a right-wing party, R. The parties are not interested in holding office per se, but have utility functions u_{τ_L} and u_{τ_R} with $\tau_L = -P$ and $\tau_R = +P$ respectively ($P \in]0,1]$). Their discount factors also amount to δ. Hence, parties are only interested in the type of policies ultimately implemented.

While we assume that reciprocity may be important for the decision of voters, we assume that such considerations play no role for political parties. Since they face a large anonymous electorate, they are not likely to feel gratitude or the desire to take revenge on individual voters.

There are two periods. At the beginning of the first period, each citizen votes for a political party. The party which gets the majority of votes forms the new government. Afterwards, the newly elected government chooses a policy $p^{(1)}$ from the interval of possible policies $[-1;+1]$. At the beginning of the second period, voters can either re-elect the government or choose the alternative party instead. Then the government may again choose a policy $p^{(2)}$ ($p^{(2)} \in [-1;+1]$).

3 Solution if Reciprocal Behavior Does Not Matter

It is easy to show that the following subgame-perfect Nash equilibrium exists if the electorate does not show reciprocal behavior:

Proposition 1

Without reciprocity, the government will always pursue its preferred policy, i.e. in both periods, a left-wing government chooses $\tau = -P$ and a right-wing government chooses $\tau = +P$. Voters with $\tau < 0$ always vote for the left-wing party, whereas voters with $\tau > 0$ always vote for the right-wing party. The median voter with $\tau = 0$ is indifferent between both parties and can be assumed to vote for each party with probability $\frac{1}{2}$.

Thus, in the first period, no party has an incentive to deviate from its preferred policy by adopting a policy which is closer to the median position $\tau = 0$. Because voters are purely forward-looking when casting votes at the beginning of the second period, adopting a more moderate policy will not improve the party's chances of being re-elected.

4 The Solution Concept of Dufwenberg and Kirchsteiger

Since the reader may not be familiar with the concept of sequential reciprocity equilibrium introduced by Dufwenberg and Kirchsteiger (2004), we will start with their conceptual framework.

Suppose there is a game in extensive form with n players. Then we can define $\mathbb{N} = \{1, ..., n\}$ as the set of players. Let the set of histories, H, be the set of choice profiles that lead to subgames. We use A_i to denote the set of (possibly mixed) strategies for player $i \in \mathbb{N}$. The set of strategy profiles is given by $A = \prod_{i \in \mathbb{N}} A_i$. The material payoff function is denoted by π_i, $\pi_i : A \to \mathbb{R}$ where $\mathbb{R}$ denotes the set of real numbers.

In addition to material payoffs reciprocity payoffs are introduced. For this purpose, we define $B_{ij} = A_j$ as the set of possible beliefs of player i about player j's strategy. We also need to define second order beliefs. Formally, we can write the set of possible beliefs of player i about player j's beliefs about player k's behavior as $C_{ijk} = B_{jk} = A_k$. As the game evolves, players must update their beliefs about the strategies of other players. Thus we use $a_i(h)$ with $h \in H$ to denote the strategy a_i updated for history h. It is identical to a_i except for the choices which define history h; these choices are made with probability 1. For $b_{ij} \in B_{ij}$ and $c_{ijk} \in C_{ijk}$, the beliefs $b_{ij}(h)$ and $c_{ijk}(h)$ are updated over time accordingly.

In order to have a standard by which to compare whether player i is treating player j kindly, we have to define the equitable payoff:

$$\pi_j^{e_i}\left((b_{ik})_{k \neq i}\right) = \frac{1}{2}\left[\max_{a_i \in A_i} \pi_j\left(a_i, (b_{ik})_{k \neq i}\right) + \min_{a_i \in A_i} \pi_j\left(a_i, (b_{ik})_{k \neq i}\right)\right]$$

We have simplified this definition slightly. In the original concept, maximization and minimization are restricted to “efficient” strategies. Since in our model all strategies are efficient, we can follow the simple definition here.

Now we can define the kindness of player i to another player j at history $h \in H$ as:

$$\kappa_{ij}\left(a_i(h),(b_{ik}(h))_{k\neq i}\right)=\pi_j\left(a_i(h),(b_{ik}(h))_{k\neq i}\right)-\pi_j^{e_i}\left((b_{ik}(h))_{k\neq i}\right)$$

Thus whenever the material payoff player i is offering to player j exceeds the equitable payoff, player i thinks she is kind to player j, otherwise she thinks she is unkind.

Similarly, we have to consider how kind player i thinks player j is to her. The respective expression is defined analogously as:

$$\lambda_{iji}\left(b_{ij}(h),(c_{ijk}(h))_{k\neq j}\right)=\pi_i\left(b_{ij}(h),(c_{ijk}(h))_{k\neq j}\right)-\pi_i^{e_j}\left((c_{ijk}(h))_{k\neq j}\right)$$

The utility of player i is now defined as the sum of material payoffs and reciprocity payoffs:

$$\begin{aligned}&U_i\left(a_i(h),\left(b_{ij}(h),(c_{ijk}(h))_{k\neq j}\right)_{j\neq i}\right)\\=\;&\pi_i\left(a_i(h),(b_{ij}(h))_{j\neq i}\right)+\sum_{j\in\mathcal{N}\setminus\{i\}}Y_{ij}\kappa_{ij}\left(a_i(h),(b_{ik}(h))_{k\neq i}\right)\cdot\lambda_{iji}\left(b_{ij}(h),(c_{ijk}(h))_{k\neq j}\right)\end{aligned}$$

where $Y_{ij}\geq 0$ are exogenous parameters which measure how important reciprocity is between players i and j. Player i derives utility from treating player j kindly ($\kappa_{ij}>0$) if she believes that player j is nice to her ($\lambda_{iji}>0$). By contrast, if player i thinks that player j is treating her unfairly ($\lambda_{iji}<0$), then she derives utility from retaliation, i.e. from being unkind.

Let $A_i(h,a)\subseteq A_i$ be the set of strategies that prescribe the same behavior as implied by $a_i(h)$ for all histories other than h. With these definitions we can now introduce the concept of sequential reciprocity equilibrium.

Definition 1

The strategy profile $a^*\in A$ is a sequential reciprocity equilibrium if for all $i\in\mathbb{N}$ and for each history $h\in H$ it holds that:

1. $a_i^*(h) \in \arg\max_{a_i \in A_i(h,a^*)} U_i\left(a_i(h), \left(b_{ij}(h), (c_{ijk}(h))_{k \neq j}\right)_{j \neq i}\right)$,

2. $b_{ij} = a_j^* \ \forall j \neq i$,

3. $c_{ijk} = a_k^* \ \forall j \neq i, k \neq j$.

Roughly speaking, the first condition ensures the optimality of players' choices, whereas the second and third conditions guarantee that their beliefs are correct.

5 The Case with Reciprocal Voters

Having defined the concept of sequential reciprocity equilibrium, we assume that voters' behavior is influenced by their material payoffs and by fairness considerations. The significance of reciprocal motives amounts to $Y_{\tau i} > 0$ (with $\tau \in \mathcal{N}$ and $i \in \{L, R\}$). By contrast, we assume that reciprocity is not important for political parties, i.e., $Y_{i\tau} = 0$ $\forall \tau \in \mathcal{N}, i \in \{L, R\}$ and $Y_{LR} = Y_{RL} = 0$. In addition, we assume that the reciprocal motives to the left-wing party and the right-wing party are equally strong for the median voter $\tau = 0$, i.e. $Y_{0L} = Y_{0R}$. We also neglect reciprocity of voters towards other voters which seems to be a plausible assumption in our context.

Let us assume that the left-wing party and the right-wing party are elected with probability $\frac{1}{2}$ in the first period. This is plausible because of the symmetry of the problem. Any different assumption would arbitrarily favor one party over the other. It is obvious that parties will always adopt their favorite policies in the second period. Thus, we will take this behavior as given. It remains to be shown how parties behave in the first period of the game and how voters react at the election stage of the second period.

The probability of an individual voter τ voting for party i ($i \in \{L, R\}$) at the beginning of period t is denoted by $a_{\tau i}^{(t)}$. We use $b_{\tau L}^{(t)}$ and $b_{\tau R}^{(t)}$ to denote the beliefs of individual τ about the policy that a left-wing and a right-wing party chooses in period t. $c_{\tau i \tau'}^{(t)}$ denotes the beliefs of voter τ about the beliefs of party i about the probability of voter τ' voting for party i in period t.

From now on we examine the decision of the median voter, i.e., voter $\tau = 0$ under the assumption that all voters with $\tau < 0$ vote for the left-wing party while all voters with $\tau > 0$ vote for the right-wing party. We will argue later that this assumed behavior is indeed optimal for players $\tau \neq 0$.

The material payoffs of party i are given by:

$$\begin{aligned} & \pi_i \left(q_i^{(1)}, q_i^{(2)}, p_i^{(1)}, p_j^{(1)}, p_i^{(2)}, p_j^{(2)} \right) \\ = \; & -q_i^{(1)} \left(\tau_i - p_i^{(1)} \right)^2 - \left(1 - q_i^{(1)} \right) \left(\tau_i - p_j^{(1)} \right)^2 \\ & \qquad + \delta \left[-q_i^{(2)} \left(\tau_i - p_i^{(2)} \right)^2 - \left(1 - q_i^{(2)} \right) \left(\tau_i - p_j^{(2)} \right)^2 \right] \end{aligned}$$

where j denotes the other party, $q_i^{(t)}$ denotes the probability of party i being elected in period t, and $p_i^{(t)}$ denotes the policy implemented by party i in period t.

The equitable payoffs which serve as a reference point when the median voter $\tau = 0$ decides how to behave towards party i are given by:

$$\begin{aligned} \pi_i^{e_0} \left(b_{0L}^{(1)}, b_{0R}^{(1)} \right) & = \frac{1}{2} \Bigg[\max_{a_{0i}^{(1)}, a_{0i}^{(2)}} \pi_i \left(a_{0i}^{(1)}, a_{0i}^{(2)}, b_{0i}^{(1)}, b_{0j}^{(1)}, -P, +P \right) \\ & \qquad + \min_{a_{0i}^{(1)}, a_{0i}^{(2)}} \pi_i \left(a_{0i}^{(1)}, a_{0i}^{(2)}, b_{0i}^{(1)}, p_{0j}^{(1)}, -P, +P \right) \Bigg] \\ & = -\frac{1}{2} \left[\left(\tau_i - b_{0i}^{(1)} \right)^2 + \left(\tau_i - b_{0j}^{(1)} \right)^2 \right] - 2\delta P^2 \end{aligned}$$

where we have used the fact that each party will always adopt its favorite policy in the second period which implies $b_{0L}^{(2)} = -P$ and $b_{0R}^{(2)} = +P$ in equilibrium.[5]

Let us now compute the kindness of the median voter $\tau = 0$ towards party i ($i \in \{L, R\}$). Intuitively this represents the size of the cake that the median voter is offering to party i compared to the reference point which is given by the equitable payoff.

[5] Note that in our model all strategies are efficient as defined in Dufwenberg and Kirchsteiger (2004), because switching from one policy to another or from voting for one party to the other party always makes one player worse off. Thus, when forming the expressions for the equitable payoffs we have to maximize payoffs over all possible strategies.

$$
\begin{aligned}
&\kappa_{0i}\left(a_{0i}^{(1)},a_{0i}^{(2)},b_{0i}^{(1)},b_{0j}^{(1)}\right)\\
=\;&\pi_i\left(a_{0i}^{(1)},a_{0i}^{(2)},b_{0L}^{(1)},b_{0R}^{(1)},-P,+P\right)-\pi_i^{e_0}\left(b_{0L}^{(1)},b_{0R}^{(1)}\right)\\
=\;&-a_{0i}^{(1)}\left(\tau_i-b_{0i}^{(1)}\right)^2-\left(1-a_{0i}^{(1)}\right)\left(\tau_i-b_{0j}^{(1)}\right)^2-4\delta\left(1-a_{0i}^{(2)}\right)P^2\\
&\qquad+\frac{1}{2}\left[\left(\tau_i-b_{0i}^{(1)}\right)^2+\left(\tau_i-b_{0j}^{(1)}\right)^2\right]+2\delta P^2\\
=\;&\left[\frac{1}{2}-a_{0i}^{(1)}\right]\left[\left(\tau_i-b_{0i}^{(1)}\right)^2-\left(\tau_i-b_{0j}^{(1)}\right)^2\right]+4\delta\left(a_{0i}^{(2)}-\frac{1}{2}\right)P^2
\end{aligned}
$$

Now that we have determined how kind the median voter thinks she is towards party i, we have to compute how kind the median voter believes party i to be towards her.

The payoffs of voter 0 are given by:

$$
\begin{aligned}
&\pi_0\left(q_i^{(1)},q_i^{(2)},p_i^{(1)},p_j^{(1)},p_i^{(2)},p_j^{(2)}\right)\\
=\;&-q_i^{(1)}\left(0-p_i^{(1)}\right)^2-\left(1-q_i^{(1)}\right)\left(0-p_j^{(1)}\right)^2\\
&\qquad+\delta\left[-q_i^{(2)}\left(0-p_i^{(2)}\right)^2-\left(1-q_i^{(2)}\right)\left(0-p_j^{(2)}\right)^2\right]\\
=\;&-q_i^{(1)}\left(p_i^{(1)}\right)^2-\left(1-q_i^{(1)}\right)\left(p_j^{(1)}\right)^2-\delta\left[q_i^{(2)}\left(p_i^{(2)}\right)^2+\left(1-q_i^{(2)}\right)\left(p_j^{(2)}\right)^2\right]
\end{aligned}
$$

The equitable payoffs which serve as a reference point when the median voter $\tau=0$ decides how kindly party i is treating her are given by:

$$
\begin{aligned}
&\pi_0^{e_i}\left(c_{0i0}^{(1)},c_{0i0}^{(2)}\right)\\
=\;&\frac{1}{2}\left[\max_{p_i^{(1)},p_i^{(2)}}\pi_0\left(c_{0i0}^{(1)},c_{0i0}^{(2)},p_i^{(1)},p_j^{(1)},p_i^{(2)},p_j^{(2)}\right)\right.\\
&\qquad\left.+\min_{p_i^{(1)},p_i^{(2)}}\pi_0\left(c_{0i0}^{(1)},c_{0i0}^{(2)},p_i^{(1)},p_j^{(1)},p_i^{(2)},p_j^{(2)}\right)\right]\\
=\;&-\frac{1}{2}c_{0i0}^{(1)}-\left(1-c_{0i0}^{(1)}\right)\left(p_j^{(1)}\right)^2-\frac{1}{2}\delta c_{0i0}^{(2)}-\delta\left(1-c_{0i0}^{(2)}\right)\left(p_j^{(2)}\right)^2
\end{aligned}
$$

Note that we have used that the best policy for voter $\tau=0$ is $p=0$ whereas the worst policy policy is given by $p=-1$ or $p=+1$.

The median voter believes the kindness of party i to be:

$$\begin{aligned}
& \lambda_{0i0}\left(b_{0i}^{(1)}, b_{0i}^{(2)}, c_{0i0}^{(1)}, c_{0i0}^{(2)}\right) \\
= \; & \pi_0\left(c_{0i0}^{(1)}, c_{0i0}^{(2)}, b_{0L}^{(1)}, b_{0R}^{(1)}, -P, +P\right) - \pi_0^{e_i}\left(c_{0i0}^{(1)}, c_{0i0}^{(2)}\right) \\
= \; & c_{0i0}^{(1)}\left[\frac{1}{2} - \left(b_{0i}^{(1)}\right)^2\right] + \delta c_{0i0}^{(2)}\left[\frac{1}{2} - \left(b_{0i}^{(2)}\right)^2\right] \\
= \; & c_{0i0}^{(1)}\left[\frac{1}{2} - \left(b_{0i}^{(1)}\right)^2\right] + \delta c_{0i0}^{(2)}\left[\frac{1}{2} - P^2\right]
\end{aligned}$$

where we have used that $\left(b_{0i}^{(2)}\right)^2 = P^2$ in equilibrium.

Let us now assume without loss of generality that the right-wing party has been elected in the first period. Since identical material payoffs accrue to the median voter from both parties in the second period, it suffices to compare psychological payoffs from voting for the left-wing party and voting for the right-wing party.

The kindness of the median voter to the left-wing party after the first period for an equilibrium where a left-wing government chooses $-x$ and a right-wing government chooses $+x$ in the first period is given by:

$$\begin{aligned}
\kappa_{0L} &= \frac{1}{2}\left[(-P+x)^2 - (-P-x)^2\right] + 4\delta\left(\frac{1}{2} - a_{0R}^{(2)}\right)P^2 \\
&= -2Px + 4\delta\left(\frac{1}{2} - a_{0R}^{(2)}\right)P^2
\end{aligned}$$

where we have updated beliefs properly such that $a_{0L}^{(1)} = 0$ and $a_{0R}^{(1)} = 1$ after the first period. Similarly, the kindness of the median voter to the right-wing party amounts to:

$$\begin{aligned}
\kappa_{0R} &= \frac{1}{2}\left[(P+x)^2 - (P-x)^2\right] + 4\delta\left(a_{0R}^{(2)} - \frac{1}{2}\right)P^2 \\
&= 2Px + 4\delta\left(a_{0R}^{(2)} - \frac{1}{2}\right)P^2
\end{aligned}$$

The median voter maximizes $\kappa_{0L}\lambda_{0L0} + \kappa_{0R}\lambda_{0R0}$, since she is indifferent between both parties with respect to material payoffs. Note that $\kappa_{0L} = -\kappa_{0R}$ and that $\frac{\partial \kappa_{0R}}{\partial a_{0R}^{(2)}} > 0$. Thus the right-wing party will be re-elected if $\lambda_{0R0} \geq \lambda_{0L0}$, i.e., if the median voter thinks that the right-wing party is at least as kind as the left-wing party.

The friendliness the median voter attributes to the right-wing party is given by:

$$\begin{aligned}\lambda_{0R0} &= c^{(1)}_{0R0}\left(-x^2+\frac{1}{2}\right)+\delta c^{(2)}_{0R0}\left[\frac{1}{2}-P^2\right]\\ &= -x^2+\frac{1}{2}+\delta c^{(2)}_{0R0}\left[\frac{1}{2}-P^2\right]\end{aligned}$$

Note that we have applied $c^{(1)}_{0R0}=1$, since we have to update beliefs after the first period and we consider the case where the right-wing party has been elected in the first period. x denotes the equilibrium choice of the right-wing party. Note that due to the symmetry of the problem, this implies that in equilibrium the left-wing party would choose $-x$.

Similarly, λ_{0L0} can be computed as:

$$\lambda_{0L0}=\delta c^{(2)}_{0L0}\left[\frac{1}{2}-P^2\right]$$

Let us now assume that the right-wing party is elected also in the second period. Then in equilibrium we have $c^{(2)}_{0R0}=1$ and $c^{(2)}_{0L0}=0$ at the beginning of the second period. We obtain:

$$\lambda_{0L0}=0$$

The right-wing party will be re-elected if:

$$\begin{aligned}\lambda_{0R0} &\geq \lambda_{0L0}\\ -x^2+\frac{1}{2}+\delta\left[\frac{1}{2}-P^2\right] &\geq 0\\ x^2 &\leq \frac{1}{2}+\delta\left(\frac{1}{2}-P^2\right)\end{aligned}$$

Note that the right wing-party can always choose $x=P$ and be re-elected if $P\leq\frac{\sqrt{2}}{2}$. However, if $P>\frac{\sqrt{2}}{2}$, then the party will only be re-elected if

$$x\leq\sqrt{\frac{1}{2}+\delta\left(\frac{1}{2}-P^2\right)}$$

Hence, the optimal policy by the right-wing government in the first period is given by:

$$p^* = \begin{cases} P & \text{for } P \leq \frac{\sqrt{2}}{2} \\ \sqrt{\frac{1}{2} + \delta\left(\frac{1}{2} - P^2\right)} & \text{for } P > \frac{\sqrt{2}}{2} \end{cases}$$

Equivalently, the left-wing party would choose $-p^*$ in the first period.

It remains to be shown that it is in fact optimal for a right-wing party to choose p^* and get re-elected instead of choosing P and not being re-elected for $P > \sqrt{2}/2$. The gain in utility in the first period by choosing P amounts to $(P - p^*)^2$. The loss in utility as a consequence of the policy conducted by the left-wing party in the second period is given by $\delta(2P)^2$. A sufficient condition for choosing p^* to be optimal is $P^2 < \delta(2P)^2$ (as $(P - p^*)^2 \leq P^2$). This is equivalent to $\delta \geq \frac{1}{4}$.

We summarize our observations by the following proposition:

Proposition 2

The following sequential reciprocity equilibrium exists for $\delta \geq \frac{1}{4}$. All voters with $\tau < 0$ vote for the left-wing party in each period, all voters with $\tau > 0$ always vote for the right-wing party. The median voter chooses each party with equal probability in the first period. The median voter does not replace the incumbent party at the beginning of the second period if its first-period policy lies in the interval $[-p^*, +p^*]$. The government is replaced otherwise. If the left-wing party has been elected, it adopts the policy $-p^*$ in the first period. A right-wing government adopts p^*. Thus, in equilibrium, the incumbent government is always re-elected at the beginning of the second period. In the second period, each government chooses its favorite policy, i.e., a left-wing government chooses $-P$, whilst a right-wing government adopts $+P$.

Let us now reconsider the assumption that all voters with $\tau < 0$ vote for the left-wing party while all voters with $\tau > 0$ vote for the right-wing party at the beginning of the second period. This assumption does not lead to contradictions for the following reasons. Consider an equilibrium where the median voter votes for the right-wing party, then all voters with $\tau > 0$ also profit more from voting for the right-wing party as P is closer to their favorite position and the right-wing party is even more kind from their perspective than from the median voter's perspective. While the later claim is intuitively appealing, we show in section A of the appendix that it is in fact true. All voters with $\tau < 0$ can be assumed to vote for the left-wing party because it would adopt a policy with higher material payoffs. In addition, these voters cannot be kind to any party, since their votes are not pivotal. An analogous argument holds for equilibria where the median voter votes for the left-wing party.

Interestingly, our results do not depend on the importance of reciprocity for voter behavior which is measured by parameters $Y_{\tau i}$ (with $\tau \in \mathcal{N}$ and $i \in \{L, R\}$). As long as parameters $Y_{\tau i}$ are strictly positive (albeit very small) and $Y_{\tau L} = Y_{\tau R}\ \forall \tau \in [-1; +1]$, our results continue to hold.

If $\delta < \frac{1}{4}$, then an equilibrium exists where both political parties will choose their favorite policies $-P$ or $+P$ respectively in every period. Intuitively, for low values of δ parties are too myopic to be willing to incur losses in the first period in order to be re-elected and adopt their favorite policies in the second period.

Let us now examine whether there is a chance that the left-wing party is elected in the second period if the right-wing government was chosen in the first period. Thus we have to check whether an equilibrium exists with $c^{(2)}_{0L0} = 1$, $c^{(1)}_{0R0} = 0$ and

$$\begin{aligned} \lambda_{0L0} &\geq \lambda_{0R0} \\ \delta\left[\frac{1}{2} - P^2\right] &\geq \frac{1}{2} - x^2 \end{aligned}$$

This time, x represents the choice by the right-wing party in the candidate equilibrium. We have shown already that our assumption $\delta > 1/4$ guarantees that the incumbent government would always prefer a policy of $x = 0$ in the first period to $-P$ or $+P$ if this ensures re-election. But for $x = 0$ the above inequality simplifies to:

$$\begin{aligned} \delta\left[\frac{1}{2} - P^2\right] &\geq \frac{1}{2} \\ \frac{1}{2}\frac{\delta - 1}{\delta} &\geq P^2 \end{aligned}$$

Hence, we get a contradiction since a profitable deviation exists for the right-wing government as it could adopt $x = 0$ and be re-elected. We obtain:

Proposition 3

No sequential reciprocity equilibrium exists in which the incumbent government is not re-elected after the first period.

This is an interesting observation, since it implies that the existence of reciprocal motives may give a strong advantage to the incumbent party. In the next section, we illustrate that the incumbent party will usually be re-elected even if it yields material payoffs that are lower for the median voter compared to the material payoffs the oppositional party would deliver.

6 Introducing A Handicap for the Incumbent Party

Assume that the incumbent party will create losses Δ ($\Delta > 0$) for all voters if it is re-elected. At the same time, the party generates some extra benefits B for itself ($B \geq 0$). One explanation may be that parties become better at extracting rents the longer they hold office. We introduce this assumption to illustrate the severity of the incumbent's advantage.

Let us note first that in any subgame-perfect Nash equilibrium (without reciprocity), the incumbent party will never be re-elected, since the median voter and possibly also some more voters will become swing voters in the second period. Voting for the oppositional party yields higher payoffs for them.

However, with reciprocity the case may be different. Let us consider an equilibrium where the median voter will always re-elect the incumbent party. Without loss of generality, we assume again that the right-wing party has been elected in the first period. Let us first note that the terms λ_{0i0} are not affected by our modification since the introduction of Δ lowers $\pi_0\left(c_{0i0}^{(1)}, c_{0i0}^{(2)}, b_{0L}^{(1)}, b_{0R}^{(1)}, -P, +P\right)$ and $\pi_0^{e_i}\left(c_{0i0}^{(1)}, c_{0i0}^{(2)}\right)$ by the same amount. However, the kindness of the median voter to both parties κ_{0i} needs to be modified because the existence of additional benefits B increases the size of the gift the median voter can give.

$$\kappa_{0R}\left(a_R^{(2)}=1\right) = 2Px+2\delta P^2-\frac{1}{2}\delta B+\delta B \tag{1}$$

$$\kappa_{0R}\left(a_R^{(2)}=0\right) = 2Px-2\delta P^2-\frac{1}{2}\delta B \tag{2}$$

Voting for the right-wing party in the second period will be optimal if:

$$-\Delta+\lambda_{0R0}\kappa_{0R}\left(a_R^{(2)}=1\right)\geq\lambda_{0R0}\kappa_{0R}\left(a_R^{(2)}=0\right) \tag{3}$$

where we have taken into account the fact that $\lambda_{0L0}=0$. Note that we have assumed that $Y_{0L}=Y_{0R}=1$. This assumption is not crucial to our results but simplifies the exposition. Inserting (1) and (2) into (3), we obtain:

$$\left(-x^2+\frac{1}{2}+\delta\left(\frac{1}{2}-P^2\right)\right)(4\delta P^2+\delta B)\geq\Delta \tag{4}$$

Thus, the right-wing party is re-elected if its choice of x is sufficiently low:

$$x\leq\sqrt{\frac{1}{2}+\delta\left(\frac{1}{2}-P^2\right)-\frac{\Delta}{\delta(4P^2+B)}} \tag{5}$$

If Δ is rather large, then there may be no solution for x. This is intuitively clear, because it seems implausible that the incumbent party would ever be re-elected if Δ were very large. As it is never optimal for the right-wing party to choose a policy larger than P in the first period, the optimal choice in the first period is given by:

$$p^*:=\min\left\{\sqrt{\frac{1}{2}+\delta\left(\frac{1}{2}-P^2\right)-\frac{\Delta}{\delta(4P^2+B)}},P\right\}$$

Hence, we obtain the following proposition:

Proposition 4

Assume $\delta\geq\frac{1}{4}$, $\frac{1}{2}+\delta\left(\frac{1}{2}-P^2\right)\geq\frac{\Delta}{\delta(4P^2+B)}$ *and* $Y_{\tau L}=Y_{\tau R}=1\ \forall\tau\in[-1;+1]$. *Then the following sequential reciprocity equilibrium exists for the game where the incumbent party is able to extract rents. All voters with* $\tau<0$ *vote for the left-wing party in each period, all voters with* $\tau>0$ *always vote for the right-wing party. The median voter chooses each party with equal probability in the first period. The median voter does not replace the incumbent party at the beginning of the second period if its first-period policy lies in the interval* $[-p^*,+p^*]$. *If the left-wing party has been elected, it* *adopts the policy $-p^*$ in the first period. A right-wing government adopts p^*. Thus, in equilibrium, the incumbent government is always re-elected at the beginning of the second period. The government is replaced otherwise. In the second period, each government chooses its favorite policy, i.e., a left-wing government chooses $-P$, whilst a right-wing government adopts $+P$.*

It is straightforward to show that the behavior of voters $\tau \neq 0$ is also optimal. The proof is very similar to the respective proof for proposition 2.

This proposition highlights the fact that the incumbent's advantage is strong enough to ensure re-election of the incumbent party although this may reduce material payoffs for a majority of voters. The effect is that the incumbent party may choose a policy that is closer to the median voter's position is strengthened further. The larger Δ, the more moderate the policy implemented by the incumbent party. Intuitively, if Δ is very large, then the incentive to vote for the incumbent party is very low. The incumbent party then has to make up for this disadvantage by adopting a very friendly policy.

7 Discussion and Conclusion

One might wonder how our results would be affected if there where positive benefits to parties from holding office. Interestingly, this modification would not change our findings. It would not change the friendliness of parties towards the median voter which yields the crucial inequality determining the equilibrium policy in the first period. However, it would affect the size of the gift the median voter is able to give to parties. But if positive benefits accrue to parties from holding office, then our results would be even more robust in models comprising more than two periods. The reason is that these benefits would strengthen the incentives for parties to please the median voter because holding office would be more attractive. Without personal benefits, holding office is attractive to parties just because they can pursue their favorite policies in the last period of the game.

One might also be interested in the case where the main problem is that politicians may not be be sufficiently motivated to perform socially advantageous tasks. We consider this case in the appendix. There we show that reciprocal motives of voters induce politicians to exert substantially more effort on socially desirable tasks as this makes voters feel gratitude which in turn induces voters to re-elect the incumbent government. Again the existence of reciprocal motives of voters puts the incumbent party at a substantial advantage.

There is one disadvantage when applying the concept of Dufwenberg and Kirchsteiger (2004) to our model, namely that we use overall discounted material payoffs when calculating the expressions for friendliness. This may seem implausible, because it implies that nice or unkind behavior in the first period is considered more important than kind behavior in the second period, even at the beginning of the second period. Instead one might introduce discounting of past payoffs when forming updated expressions for kindness, because it seems plausible that the memory of past kind (or unkind) behavior fades away over time. However, introducing a modification along these lines would not affect the basic effects in our model, but would merely strengthen the incentives to implement policies which are perceived as kind by the median voter. In particular, such a modification would seem necessary for extensions of our analysis to more than two periods. It seems likely that our findings would carry over to a multi-period environment.

The definition of the equitable payoff is necessarily arbitrary to some extent.[6] One simple generalization would be to define the equitable payoffs as:

$$\pi_j^{e_i}\left((b_{ik})_{k\neq i}\right) = (1-\alpha)\max_{a_i\in A_i}\pi_j\left(a_i,(b_{ik})_{k\neq i}\right) + \alpha\min_{a_i\in A_i}\pi_j\left(a_i,(b_{ik})_{k\neq i}\right)$$

where $0 < \alpha < 1$. Then α would be a parameter describing how easily players are satisfied with the behavior of others. If α were very small, then players could not be satisfied easily. In the appendix we show that this more general specification often leads to similar results. However, if α is very large, then voters may always feel that the incumbent party is mean which implies that they never re-elect the government.

[6]Cf. Rabin (1993).

Our paper has granted three insights. Firstly, reciprocity may have important consequences in elections as it may deliver an important channel how past behavior of politicians may affect future voter behavior. In standard models with rational utility-maximizing voters, past behavior of the government can only affect voter behavior insofar as it reveals hidden information about characteristics of the government (e.g. about competence or preferences). In a way, we provide a microeconomic foundation for retrospective voting which is often assumed to solve the dilemma that past misbehavior of politicians cannot be punished by rational utility-maximizing voters.[7] Secondly, reciprocity may make it more likely that moderate positions (e.g. the position of the median voter) are adopted even if parties cannot commit to future policies. Thirdly, we have shown that the very existence of reciprocal motives puts the incumbent at a substantial advantage. Interestingly, a party may be re-elected although this causes material payoffs to be lower for a majority of voters.

[7] See e.g. the models presented in Gersbach (2004).

A Proof that no Profitable Deviation Exists for Voters with $\tau \neq 0$

We show that $\lambda_{\tau R\tau} > \lambda_{\tau L\tau}$ for $\tau > 0$ in an equilibrium where the right-wing party is elected in the first period and in the second period. This implies that no profitable deviation exists for any voter with $\tau > 0$. It is easy to show that $\lambda_{\tau L\tau} = 0$. The reason for this is that the left-wing party is not elected in equilibrium. Thus, it is sufficient to show that $\lambda_{\tau R\tau} > 0$. The friendliness of the right-wing party to voter τ is given by:

$$\begin{aligned}
\lambda_{\tau R\tau} &= \pi_\tau - \pi_\tau^{e_R} \\
&= -c^{(1)}_{\tau R\tau}(\tau - p^*)^2 - \delta c^{(2)}_{\tau R\tau}(\tau - P)^2 + \frac{1}{2}c^{(1)}_{\tau R\tau}(1+\tau)^2 + \frac{1}{2}c^{(2)}_{\tau R\tau}\delta(1+\tau)^2 \\
&= -(\tau - p^*)^2 - \delta(\tau - P)^2 + \frac{1}{2}(1+\delta)(1+\tau)^2
\end{aligned}$$

We have assumed that the right-wing party chooses p^* and P in the first and second period respectively. We have also applied $c^{(1)}_{\tau R\tau} = c^{(2)}_{\tau R\tau} = 1$. Note also that the worst policy that the right-wing party can choose from the perspective of voter τ ($\tau > 0$) is -1. Hence, maximal losses in each period are given by $(1+\tau)^2$.

Now we compute the derivative of $\lambda_{\tau R\tau}$ with respect to τ:

$$\begin{aligned}
\frac{\partial \lambda_{\tau R\tau}}{\partial \tau} &= -2(\tau - p^*) - 2\delta(\tau - P) + (1+\delta)(1+\tau) \\
&= (2p^* - \tau) + \delta(2P - \tau)
\end{aligned}$$

For small positive values of τ $\lambda_{\tau R\tau}$ is increasing, i.e., voters with small positive values of τ think that the right-wing party is kind. However, $\lambda_{\tau R\tau}$ is quadratic in τ and is decreasing for large values of τ. Hence, $\lambda_{\tau R\tau}$ is positive for any $\tau > 0$ if it is positive for $\tau = 1$. Thus, we compute:

$$\begin{aligned}
\lambda_{1R1} &= -(1 - p^*)^2 - \delta(1 - P)^2 + \frac{1}{2}(1+\delta)(1+1)^2 \\
&= (2 - (1-p^*)^2) + \delta\left(2 - (1-P)^2\right)
\end{aligned}$$

This expression is clearly positive. Thus, we have shown that any voter with $\tau > 0$ thinks that the right-wing government is more friendly than the left-wing government in an equilibrium where the right-wing party is elected in the first period and in the second period. Hence, no profitable deviation exists for voters with $\tau > 0$.

B Reciprocity and the Effort of Politicians

Here we show that reciprocity may also be important if there is the problem that politicians may not be sufficiently motivated to pursue socially optimal tasks, e.g., politicians may derive more utility from pleasant tasks such as holding speeches at summits or visiting receptions than from tedious work on structural reforms. Then it is well-known that politicians cannot be motivated enough by elections alone to pursue socially beneficial tasks.[8]

We start from the two-period model presented in this paper, but introduce the following modifications. We consider two identical politicians or political parties A and B which derive the following per-period utility if they hold office:

$$u_i = W - c(e)$$

where $i \in \{A, B\}$. W denotes the wage of the politicians which can also be interpreted as the intrinsic benefit from holding office due to the prestige involved with holding such a position. We assume that it is large enough for holding office to be always desirable. Politicians suffer some costs $c(e)$ from exerting effort e on socially beneficial tasks ($c(0) = 0$, $c'(e) > 0$, $c''(e) < 0$).

We only consider one representative voter with per-period utility:

$$v = b(e)$$

We assume $b(0) = 0$, $b'(e) > 0$ and $\lim_{e \to \infty} b(e) = B < \infty$.[9] For simplicity we assume that the discount factor for politicians and for voters is 1.

As a benchmark case, let us again examine the game without reciprocal motives. Then the following result is obvious:

Proposition 5

The following subgame-perfect Nash equilibrium exists. At the beginning of period 1, *the representative voter randomizes between both politicians with equal probability.*

[8] Cf. e.g. Gersbach (2004).

[9] The assumption $\lim_{e \to \infty} b(e) = B < \infty$ is necessary, because we need a maximal benefit that politician can create for the voter to compute the equitable payoffs. Note that one would have to replace the maximum in the definition of equitable payoffs with the supremum.

Then the elected politicians chooses $e = 0$. At the beginning of period 2, the representative voter again randomizes between both politicians with equal probability. In the last period, the politician will always choose the lowest possible effort level $e = 0$.

Hence, the politician cannot be motivated to exert any effort on socially desirable tasks.

We now turn to the case with reciprocity. Analogously to our basic model, the voter will re-elect the incumbent politician at the beginning of the second period, if she believes his friendliness to be larger than 0, which is the friendliness of the politician who does not hold office. This is optimal because the voter is indifferent between both politicians with respect to material payoffs.

Consider the case where politician A has held office in the first period. Then the friendliness of politician A for the case where he is re-elected amounts to:

$$\begin{aligned}\lambda_A &= \left(b\left(e_A^1\right) - \frac{1}{2}(0+B)\right) + \left(0 - \frac{1}{2}(0+B)\right) \\ &= b\left(e_A^1\right) - B\end{aligned}$$

where we have used that the equitable payoff is $\frac{1}{2}(b(0)+B)$. This is the average of the best and the worst the politician can do from the representative voter's view. We have also taken into account the fact that the politician will always choose $e = 0$ in the second period. In addition, we have used that the politician is not creating any benefits when exerting no effort, i.e. $b(0) = 0$, and that the maximal level of benefits the politician can create for the voter amounts to B. e_A^1 denotes the effort chosen by politician A in the first period.

It is optimal for politician A to choose e_A^1 such that $\lambda_A = 0$. This behavior guarantees re-election. Any choice of e larger than this level would imply wasteful overwork from the politician's perspective. Therefore in equilibrium we have:

$$e_A^1 = b^{-1}(B)$$

We summarize our results by the following proposition:

Proposition 6

The following sequential reciprocity equilibrium exists. At the beginning of period 1, *the representative voter randomizes between both politicians with equal probability. Then the elected politicians chooses $e = b^{-1}(B)$. The politician will be re-elected; he would not be re-elected if he chose a lower level of effort. In the last period, the politician will always choose the lowest possible effort level $e = 0$.*

Thus, we have an effect similar to the effect in our basic model, namely that the reciprocity of voters may induce politicians to choose policies which are less favorable to them but more favorable to the electorate.

C A more General Specification of Equitable Payoffs

We now discuss how the more general specification of equitable payoffs would affect our results. Let us assume that equitable payoffs are given by:

$$\pi_j^{e_i}\left((b_{ik})_{k\neq i}\right) = (1-\alpha)\max_{a_i\in A_i}\pi_j\left(a_i,(b_{ik})_{k\neq i}\right) + \alpha\min_{a_i\in A_i}\pi_j\left(a_i,(b_{ik})_{k\neq i}\right)$$

where $0 < \alpha < 1$. Then the friendliness of party i $i \in \{L, R\}$ amounts to:

$$\lambda_{\tau i\tau}\left(b_{\tau i}^{(1)}, b_{\tau i}^{(2)}, c_{\tau i\tau}^{(1)}, c_{\tau i\tau}^{(2)}\right) = c_{\tau i\tau}^{(1)}\left[\alpha(1+\tau)^2 - \left(\tau - b_{\tau i}^{(1)}\right)^2\right] + \delta c_{\tau i\tau}^{(2)}\left[\alpha(1+\tau)^2 - (P-\tau)^2\right]$$

Let us now consider a candidate equilibrium where the incumbent is re-elected in the second period. W.l.o.g. we assume again that the right-wing party has been elected in the first period. Let us also assume that all voters with $\tau > 0$ vote for the right-wing candidate and all voters with $\tau < 0$ vote for the left-wing candidate at the beginning of the second period.[10]

Then we obtain:

$$\lambda_{0i0} = \left[\alpha - x^2\right] + \delta\left[\alpha - P^2\right]$$

where x represents the choice of policy by the right-wing party in the first period. The above expression is equal to zero if:

$$x = \sqrt{\alpha(1+\delta) - \delta P^2}$$

[10] This assumed behavior of voters can be shown to be optimal unless α is rather small and voters care very much about reciprocity, i.e., for very large values of $Y_{\tau i}$.

If α is rather small, i.e, $\alpha(1+\delta) < \delta P^2$, then the root does not exist. Similarly to the basic case, we obtain that the optimal choice of policy in the first period is given by:

$$p^* = \begin{cases} P & \text{for } P \leq \sqrt{\alpha} \\ \sqrt{\alpha(1+\delta) - \delta P^2} & \text{for } P > \sqrt{\alpha} \end{cases}$$

For $\alpha = \frac{1}{2}$ we obtain the same solution as in our basic model. Note that p^* is increasing in α. Thus, for rather small values of α which means that voters are quite demanding, politicians choose a policy that is very close to the median voter's position.

Hence, we have shown that our results also hold for this more general specification of equitable payoffs.

Proposition 7

The following sequential reciprocity equilibrium exists for $\delta \geq \frac{1}{4}$ and $\alpha(1+\delta) < \delta P^2$.[11] *All voters with $\tau < 0$ vote for the left-wing party in each period, all voters with $\tau > 0$ always vote for the right-wing party. The median voter chooses each party with equal probability in the first period. The median voter does not replace the incumbent party at the beginning of the second period if its first-period policy lies in the interval $[-p^*, +p^*]$. The government is replaced otherwise. If the left-wing party has been elected, it adopts the policy $-p^*$ in the first period. A right-wing government adopts p^*. Thus, in equilibrium, the incumbent government is always re-elected at the beginning of the second period. In the second period, each government chooses its favorite policy, i.e., a left-wing government chooses $-P$, whilst a right-wing government adopts $+P$.*

[11] The equilibrium may not exist if α is very small and parameters $Y_{\tau i}$ are sufficiently large, because some voters with $\tau > 0$ may find it optimal to vote for the left-wing party in the second period if the right-wing party has been elected at the beginning of the first period. Intuitively, a small value of α means that voters are very demanding. If, in addition, reciprocity is very important, voters with $\tau > 0$ derive utility from punishing the right-wing party which makes voting for the left-wing party a profitable deviation. We do not give the exact conditions here, because they are tedious to derive.

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