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Frank Jöst,
Martin Quaas
and Johannes Schiller

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by Frank Jöst†, Martin Quaas‡, Johannes Schiller§

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Abstract

Population growth is often viewed as a most oppressive global problem with respect to environmental deterioration. In this paper, we investigate the optimal development of a coupled system comprising population, economy, and the natural environment as subsystems. In our formal dynamic model these are interrelated by the society's economic decisions on consumption, birthrate, and emissions. Considering Hicks neutral technical progress, we find a steady state with growing population and declining per capita emissions, all other variables remaining constant over time. We investigate the comparative static properties of the steady state, and the dynamic behavior of the system. In numerical simulations we show that simple variations in the dynamics of the subsystems lead to complex and sometimes qualitatively different behavior of the coupled system. This is a challenge for policy advice based on such intertemporal optimization models.

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†Corresponding author. Alfred-Weber-Institute of Economics, University of Heidelberg, Grabengasse 14, D-69117 Heidelberg, Germany. email: frank.joest@uni-hd.de

‡Interdisciplinary Institute for Environmental Economics, University of Heidelberg, Bergheimerstr. 20, D-69115 Heidelberg, Germany. email: quaas@eco.uni-heidelberg.de

§UFZ Centre for Environmental Research Leipzig-Halle, Germany email: schiller@alok.ufz.de

1 Introduction

In the 2001 report "Footprints and Milestones: Population and Environmental Change", the United Nations Population Fund states that changes in demographic variables such as size, growth rates or distribution of population have an important impact on the environment. However, even if population and environmental change are closely linked to each other, the exact relationship between environmental quality and population is complex. The amount and type of emissions are not only determined by the number of people living on earth, but also depend on production technologies and consumption patterns. Hence, even a growing population does not necessarily lead to an increasing deterioration of environmental quality. If e.g. highly polluting consumption is substituted by goods of less polluting character, or technical progress occurs, overall environmental quality may improve even with an increasing population.

The complex interdependencies between demographic change, economic development and the use of the environment might be one reason why one can only find few articles in environmental economics dealing with this issues.[1] The existing economic literature on the relationship between population and the environment may be divided into two categories.

Most of the contributions describe a situation typical for rural areas in less developed countries. These areas are characterized by small agricultural production units. In such a rural production system usually even young children contribute to the output of a family, e.g. by collecting firewood or looking after cows. Therefore households feel the incentive to have more children in order to achieve a higher output. However, additional children have an impact on the output of other families which is not taken into account in the individual decision making of families. Dasgupta (1993, 2000) and Shah (1998) analyze these population externalities similar to the common pool problem. They show that without coordination in the decision about the number of children such a society may find itself in an inferior situation, i.e. the number of children is too high and profits are too low compared to a social optimum. A similar, but dynamic model structure is explored by Nerlove (1991) and Nerlove, Mayer (1997). However, they do not primarily analyze population externalities, but they discuss conditions for a stationary state in which environmental quality and population are constant.

Models which are more appropriate for the situation of a country with industrial production have been developed by Cronshaw and Requate (1997) and Harford (1998). Using a static model with exogenous fertility, Cronshaw and Requate (1997) analyze the impact of population growth on the environment and production applying comparative-static methods. Harford (1998) investigates the relationship between population growth and environmental problems within a dynamic model with endogenous fertility, but he neglects the production side of the economy.

Our brief review of the existing literature shows, that all these contributions analyze

[1]The complexity of population problems might also be the reason for opposite positions about the impact of population growth on the environment in the literature. Examples for such different positions are Simon, Steinmann (1991) and Ehrlich and Ehrlich (1990). An extensive discussion of the complex interdependencies between demographic, economic, and environmental systems as well as the difficulties to analyze them in economic modelling is provided by Jöst (2003).

only some aspects of the relationship between population growth, economic development, and environmental problems. They mainly deal with rural environmental problems, and neglect that the main source of global environmental deterioration is industrial production. Taking these shortcomings into account, we aim to extend these contributions by integrating the following four characteristics of the relationship between demographic change, economic development and environmental deterioration:

- People decide on the number of their children, i.e. fertility is endogenous.
- Output is mainly produced by industrial production systems with emissions as unwanted by-products which may accumulate in the surrounding natural environment.
- Industrial production processes are characterized by the possibility to mitigate emissions, and during the course of time technical progress may occur, which allows for an increase in output with the same amount of inputs and emissions.
- Environmental deterioration is caused by the stock of pollutants, which results from industrial production.

We integrate these aspects into an economic growth model in the following manner: We assume that population growth and environmental deterioration are the results of the decisions of households with respect to consumption, pollution and the number of children. Concerning population dynamics, we abstract from age structure and assume mortality to be exogenous. The environment is modelled as a stock of pollutants degrading environmental quality. The economic system is modelled using an approach based on a physical capital stock. The model is formulated as an optimal control model and presented in the form of six coupled differential equations. We investigate the existence and properties of a steady state because in our framework of intertemporal optimization such a state may be interpreted as some kind of 'sustainable development' of the coupled system. In this context, exploring the steady state enlightens some characteristics of a sustainable development in the sense of a long-term optimal development of the economic-environmental-demographic system, taking into account that production of goods causes environmental damage and that this damage negatively influences the utility of present and future generations. For the steady state, we derive comparative static results which yield insights into the change of the optimal size of steady-state population, per-capita consumption, and environmental damage if the valuation of children, or the environmental absorption capacity and the preference for environmental quality change.

Concerning the dynamics of the system, we investigate the effect of the different time scales of the economic, the environmental, and the demographic subsystems on the development of the coupled system. In particular, we discuss consequences of variations in subsystem dynamics for the convergence of the coupled system towards the steady state.

Using numerical techniques, we analyze characteristics of the optimal evolution of the model economy over time. Such a discussion may be helpful in discussing the consequences of policy measures to control population growth and degradation properties of the natural environment.

The paper is organized as follows. In section 2, we develop the intertemporal optimization model and present the first- and second-order conditions. Section 3 discusses the characteristics of the steady state and presents some comparative static results. In section 4, we analyze the dynamic behavior of the system in the neighborhood of the steady state, and we simulate the optimal development of the economy over time. Finally, section 5 summarizes our results and gives an outlook on further work.

2 The Model

2.1 The intertemporal welfare function

Extending the approach of Yip and Zhang (1997:100) and Barro and Sala-I-Martin (1995, chapter 9) we use the following intertemporal welfare function U for the model in continuous time.

$$U = \int_0^\infty u(c(t), n(t), S(t)) \cdot \exp(-\rho t) dt. \tag{1}$$

$c(t)$ denotes the consumption per capita, $n(t)$ is the per-capita birth rate, and $S(t)$ is the stock of pollutant in the environment at time t. $u(c, n, S)$ denotes instantaneous utility stemming from own consumption $c(t)$, from having $n(t)$ newly-born children, and from environmental quality, with $u_c > 0$ and $u_n > 0$ reflecting non-satiation in consumption and children. In addition, we assume that welfare of the society depends on environmental quality. The state of the environment depends on the stock of pollutants $S(t)$ determined by the accumulation of emissions $e(t)$ which are unwanted by-products of the production of goods. This stock of pollutants enters the instantaneous utility function $u(\cdot)$ in negative form, which implies that $u_S < 0$.

In order to keep the derivation of the optimal development of our system as simple as possible, we use the following log-linear utility function.[2]

$$u(c, n, S) = \ln c + \nu \frac{n^{1-\epsilon} - 1}{1 - \epsilon} + \sigma \ln \left(\bar{S} - S\right) \tag{2}$$

In equation (2), ν, ϵ, σ and $\bar{S}$ are strictly positive constants, furthermore we assume that $\epsilon < 1$. $\bar{S}$ represents an upper bound for the stock of pollutants, which shall never be reached. By modelling the household's preferences for environmental quality in this way, we acknowledge the fact that human life is not possible without the life-supporting functions of the natural environment. Hence, a minimal level of environmental quality is necessary for the existence of the economy. This is translated into our model by the requirement that the level of pollutants in the environment must not exceed a maximum level, denoted by $\bar{S}$, which may be interpreted as the carrying capacity of

[2]This is an extension of the utility function used by Yip and Zhang (1997:100).

the system.[3]
The parameter $\rho > 0$ is constant in time. The term $\exp(-\rho t)$ in equation (1) accounts for the general discounting of the future and can be interpreted as a measure for the society's altruism towards its successor generations: the smaller ρ is, the more it cares for them. Independently of the population size, this accounts for the 'intergenerational' time preference of the society, i.e. for the fact that utility of future generations is usually the more discounted, the further in the future they live.

2.2 The development of the demographic, environmental and economic subsystems

The dynamics of the three subsystems population, environment, and economy are described by three stock variables and the corresponding control variables. As outlined above, $N(t)$ denotes the absolute population size with $n(t)$, the per-capita birth rate, being the corresponding control variable. The stock of pollutant in the environment, $S(t)$, is controlled by the per-capita emissions $e(t)$, and per-capita physical capital, $k(t)$, is controlled by per-capita consumption $c(t)$. Omitting time dependence, we formulate the three differential equations determining the dynamics of the subsystems. Neglecting the age structure and assuming a constant death rate d equation (3) expresses the dynamics of population growth.[4]

$$\dot{N} = (n - d)N. \tag{3}$$

Per-capita capital accumulation $\dot{k}$ is given by

$$\dot{k} = f(k, e, t) - c - (n - d)k - bnk - b_0 n. \tag{4}$$

The per-capita output $f(\cdot)$ in equation (4) is divided into consumption, investment, and costs to raise children. Following Barro and Sala-I-Martin (1995:312), we interprete $(n - d)$ as net per-capita birth rate. The term $-(n - d)k$ expresses the fact that each new population member has to be provided with the per-capita amount of capital for k to remain constant.
The term $-bnk - b_0 n$, with b and b_0 being positive constants, denotes the costs of raising children. Here, average per-capita capital is used as a proxy for the relative size of the opportunity costs of women raising children. This is an easy way to model the fact that these costs are higher in more developed countries (Barro, Sala-I-Martin

[3] At this point, it is important to mention that for the results derived in the remainder of the paper the *absolute size* of $\bar{S}$ is of no importance. Rather, it is the *existence* of this upper bound to the pollution stock which turns out to be crucial to many outcomes of the model. Secondly, we want to note that the integral sum of emissions over time is not bounded as we model the use of the natural environment as a sink for pollution like a renewable resource and as time runs to infinity.

[4] In the model, we treat N and n as continuous variables. This approximation is valid because we exclusively regard large numbers for population size N. Hence, n denotes an average birth rate. With the same rational, instead of regarding an individual's probability to die, we employ an average death rate d for the whole population.

1995:313). The second part, $b_0 n$, represents a share of opportunity costs for raising children that is independent of the women's working opportunities.
Production is described by a production function $F(N, Nk, Ne, t)$ with constant returns to scale. The production inputs are labor, capital and emissions.[5] We assume that every person supplies one unit of labor. Hence, the total labor input is equal to the population size N. We further assume that there exists an abatement technology to reduce emissions from the production process. This is integrated into the production function by assuming that emissions are inputs into the production process and can be substituted by other input factors. Furthermore we assume that the production function is time-dependent, i.e. that technical progress may occur. We specify the following functional form, expressed in per capita terms:

$$f(k, e, t) = \frac{F(N, Nk, Ne, t)}{N} = k^{\alpha} e^{\beta} \exp(xt). \tag{5}$$

where $\alpha, \beta > 0$ and $\alpha + \beta < 1$. The parameter x denotes the exogenous and constant rate of Hicks-neutral technical progress.
Finally, equation (6) describes the dynamics of the stock of pollutant in the natural environment in a simplified way.

$$\dot{S} = Ne - \delta S. \tag{6}$$

Here, we assume that the pollutant is equally distributed throughout the environment. Pollution degradation is proportional to the concentration of the pollutant in the environmental system with δ being the natural degradation rate of the pollutant.[6]

2.3 Conditions for the optimal development

The optimal development of the coupled demographic-economic-environmental system is derived from the maximization of the intertemporal welfare function (1) with respect to the three restrictions (3), (4), and (6). In order to solve the maximization problem we define the following present-value Hamiltonian (omitting time arguments)

$$\begin{aligned} H \;=\; & u(c, n, S) \exp(-\rho t) \\ & + \lambda^k \left[f(k, e, t) - c - (n - d)k - bnk - b_0 n \right] \\ & + \lambda^N (n - d) N \\ & + \lambda^S \left[Ne - \delta S \right] \end{aligned} \tag{7}$$

We get the first order conditions (FOC) for a maximum by taking the derivatives with respect to control (i. e. c, n, e) and state (i. e. k, N, S) variables. Denoting a derivative

[5] In physical terms, emissions are unwanted outputs of production. However, for purposes of analysis they are formally treated as production inputs (Siebert 1998: chapter 3).

[6] For CO_2 this assumption is reasonable if one exclusively considers the anthropogenic CO_2 excess above the natural level. Furthermore, this excess has to be comparatively small and timescales regarded must not be too long. For a critical comment on the use of a single differential equation for the description of the accumulation of greenhouse gases in the environment see Joos, Müller-Fürstenberger, Stephan (1999) and Moslener, Requate (2001).

with respect to one of the control or state variables by the corresponding subscript, we get the following equations:

$$
\begin{aligned}
H_c &= 0 & u_c \exp(-\rho t) - \lambda^k &= 0 && (8)\\
H_n &= 0 & u_n \exp(-\rho t) - \lambda^k \left[k + bk + b_0\right] + \lambda^N N &= 0 && (9)\\
H_e &= 0 & \lambda^k f_e + \lambda^S N &= 0 && (10)\\
H_k &= -\dot{\lambda}^k & \lambda^k \left[f_k - (n-d) - bn\right] &= -\dot{\lambda}^k && (11)\\
H_N &= -\dot{\lambda}^N & \lambda^N (n-d) + \lambda^S e &= -\dot{\lambda}^N && (12)\\
H_S &= -\dot{\lambda}^S & u_S - \lambda^S \delta &= -\dot{\lambda}^S && (13)
\end{aligned}
$$

As we are interested in the optimal time paths of the three control variables, n, c, and e, we have to eliminate the conjugate variables λ^k, λ^N, and λ^S from the set of equations (8) to (13). We get three differential equations which determine the optimal time paths of the control variables c, n, and e. Together with the differential equations (3), (4), and (6), and the functional form of the utility and production function, this yields a set of six coupled ordinary differential equations determining the dynamics of the economy (For the calculations see appendix A.1).

$$
\begin{aligned}
\dot{N} &= (n-d)N && (14)\\
\dot{k} &= k^\alpha e^\beta \exp(xt) - c - (n-d)k - bnk - b_0 n && (15)\\
\dot{S} &= Ne - \delta S. && (16)\\
\frac{\dot{n}}{n} &= -\frac{\rho}{\epsilon} + \left[\beta k^\alpha e^\beta \exp(xt) + \left[(1+b)k + b_0\right]\left(\frac{\dot{c}}{c} + \rho\right) - (1+b)\dot{k}\right] \frac{n^\epsilon}{\nu \epsilon\, c} && (17)\\
\frac{\dot{c}}{c} &= \alpha k^{\alpha-1} e^\beta \exp(xt) - bn - \rho - (n-d) && (18)\\
\frac{\dot{e}}{e} &= -\frac{1}{1-\beta}\left[\rho + \delta + (n-d) + \frac{\dot{c}}{c} - \alpha\frac{\dot{k}}{k} - \frac{\sigma N c}{\left(\bar{S} - S\right) \beta k^\alpha e^{\beta-1} \exp(xt)}\right] && (19)
\end{aligned}
$$

For every set of initial conditions $N(t=0)$, $k(t=0)$ and $S(t=0)$, the solution of this set of equations consists of the optimal time paths of the six endogenous variables. The transversality condition (Michel 1982)

$$\lim_{t\to\infty} H^0 = 0,[7] \tag{20}$$

allows us to derive values for the control variables for $t \to \infty$. The first-order conditions (8)-(13), together with the transversality condition (20) are in general not sufficient for a maximum. They are, however, sufficient if the *maximized Hamiltonian* H^0 is concave in the state variables (Arrow, Kurz 1970). We show in appendix A.2 that the following inequalities must hold for every solution of the equations (14)-(19) to ensure

[7] H^0 is the *maximized* Hamiltonian. It is the function H after we have substituted the control variables by (8)-(10).

that (8)-(13) are sufficient.

$$\begin{aligned} \frac{\partial^2 H^0}{\partial N^2} &\leq 0 \quad \text{and} & (21) \\ \frac{\partial^2 H^0}{\partial N^2}\frac{\partial^2 H^0}{\partial k^2} - \left(\frac{\partial^2 H^0}{\partial N \partial k}\right)^2 &\geq 0. & (22) \end{aligned}$$

These two conditions impose restrictions on the parameter values of the utility function. In particular, it is necessary that $\nu \leq 1 - \epsilon$. That means, we have to assume that the relative weight of the birth rate $\frac{\nu}{1-\epsilon}$ in the utility function is smaller than the relative weight of the per-capita consumption, which is normalized to one.

3 Steady state analysis

In a first step we analyze the steady state of the model and present some results of a comparative static analysis of the steady state with respect to important parameters of the model. The steady state could be interpreted as a 'sustainable development', because it is the result of an intertemporal optimization taking into account the interests of future generations with respect to consumption and environmental quality. And the model takes into account, that the size of the future generation is itself the result of a decision of the present generation. The calculations in the appendix A.3 show that in the steady state of the model per capita emissions are declining and population is increasing; all other variables stay constant. The following proposition summarizes these results.

Proposition 1
Given the assumptions of our model, all steady state growth rates are determined uniquely by the following relation:

$$\dot{k} = \dot{S} = \dot{n} = \dot{c} = 0 \text{ and } \frac{\dot{N}}{N} = -\frac{\dot{e}}{e} = \frac{x}{\beta}. \tag{23}$$

Proof:
See appendix A.3.

□

It may be surprising that even if we start with Hicks-neutral technical progress which is augmenting all inputs, in the steady state, it is only used in order to improve the productivity of the per-capita emissions. This improvement allows for a growing population without reducing per capita output and environmental quality. Environmental quality is constant, because total emissions Ne are constant in the steady state. It holds that $d/dt(Ne) = \dot{N}e + N\dot{e} = ne(\dot{N}/N + \dot{e}/e = 0)$ given the result of proposition 1. Because all entries in our utility function are constant in the steady state, we have a constant current utility level per capita.
It is further important to mention that the steady state population growth rate depends only on technical parameters, i.e. the rate of technical progress x and the exponent β

of the emissions of the Cobb-Douglas production function. This means, that the population growth rate of our demographic-economic-environmental system is independent from the preferences of the social planer concerning the number of children. In particular, the steady state growth rates are independent from the rate of time preference ρ.
The following corollary follows directly from proposition 1.

Corollary 1
Without technical progress the steady state is a stationary state, in which all variables stay constant.

After having derived the growth rates of variables in the steady state, we transform the population size N and per-capita emissions e into $\hat{N} = N \exp(-\frac{x}{\beta}t)$ and $\hat{e} = e \exp(\frac{x}{\beta}t)$. These variables are constant in the steady state. We can use these new variables, in order to transform the system of equations (14)-(19) into to the new system of equations (56) - (61) in appendix A.4. If we substitute the steady state growth rates of the endogenous variables, which are zero for the transformed variables, we can calculate the steady state values explicitly. These are given in appendix A.5 by equations (63) - (68). The following proposition states under which conditions a unique steady state with the growth rates given by proposition 1 exists:

Proposition 2
If the positive parameters ν and ϵ of the utility function and the parameters α, β and x of the production function as well as d, b and ρ fulfill the following inequality with $b_0 > 0$, a unique steady state exists.

$$\nu\rho \frac{(1-\alpha)(1+b)\frac{x}{\beta} + (1-\alpha)bd + \rho}{\beta bd + (1+b)x + (\beta + (1+b)\alpha)\rho} > \left(d + \frac{x}{\beta}\right)^{\epsilon}. \tag{24}$$

The steady state is characterized by the values $\hat{N}^*, k^*, S^*, n^*, c^*$ and $\hat{e}^*$ given in the appendix A.5.

Proof:
See appendix A.5.

□

The set of parameters which fulfills condition (24) is non-empty. In particular, (24) holds for sufficient high values of ν and ϵ and sufficiently small values of α.
In order to learn more about the steady state values of important variables it is useful to do some comparative static analysis of the steady state. In particular, we are interested to learn how the steady state population size ($\hat{N}^*$), per-capita consumption (c^*) and the state of the environment (S^*) change if the parameters of the utility function change. These are the weight of children (ν) in the utility function, the inverse intertemporal elasticity of substitution of birth rate (ϵ),[8], the weight of environmental damage (σ) and the carrying capacity of the environment ($\bar{S}$).

[8] For this interpretation of ϵ see Barro and Sala-I-Martin (1995:64).

	$\hat{N}^*$	c^*	S^*
weight of children in the utility function ν	$+$	$-$	$+$
inverse intertemporal elasticity of substitution of birth rate ϵ	$+$	$-$	$+$
weight of environmental damage in the utility function σ	$-$	0	$-$
'carrying capacity' of the environment $\bar{S}$	$+$	0	$+$

Table 1 *The results of the comparative statics of the steady state-values of population size N, per capita consumption c and pollution stock S with respect to the parameters ν and σ of the utility function, the direct costs b_0 of having children and the 'carrying capacity' of the environment, $\bar{S}$. A $+$ $(-)$ indicates a rise (fall) of the respective quantity with an increase of the parameter. The* 0 *indicates that the steady state value of the respective quantity is independent of the parameter under consideration.*

Of course one can do a lot of more comparative static analysis of the steady state. However, it is in our view reasonable to restrict the analysis on the parameters representing valuation of the demographic and the environmental system. The results are summarized in table 1. The calculations are done in A.6.
Concerning the three parameters of the utility function, ν, ϵ, and σ, we get the following results: If the weight ν of children increases compared to consumption and environmental damage, per-capita consumption decreases and population size and the stock of pollutants increase. The same result holds for a decrease in the intertemporal elasticity of substitution ϵ of newly born children increases. Row three of table 1 summarizes the impact of a change in the weight of environmental damage σ: A social planner with a higher weight of environmental damage prefers lower steady state population size and lower steady state environmental deterioration. Per capita consumption is not influenced by a change σ.
Finally, we summarize the impacts of an increasing carrying capacity of the environment, which is given by the maximum tolerable level of immissions $\bar{S}$. In the case of a higher $\bar{S}$ the population size and the stock of pollutants are higher, while per capita consumption c is constant. As the additional calculations in the appendix A.6 show, the number of children n and the environmental quality $\bar{S} - S$ is higher. Hence, per capita utility is increasing (c.f. appendix A.6). Thus, in a situation with a higher carrying capacity it is optimal to have a higher population and environmental quality.

4 Dynamic analysis

In the preceding section we have derived conditions for the existence of a steady state and analyzed its properties. In the following we investigate the behavior of our model economy outside the stedy state. We proceed in two steps: In the first step, we analyze how a change in the dynamics of one subsystem alters the dynamics of the coupled system in the neighborhood of the steady state. In particular, we are interested to learn how a change in the exogenously given internal dynamics of one subsystem, alters the dynamic behavior of the whole system. The internal dynamics of the subsystems are

given by the parameters d, x, ρ and δ, which may be interpreted as time scales of the subsystems Population, Economy and Environment, respectively.
This interpretation may be illustrated as follows: If the birth rate is zero, the Population stock declines exponentially with the constant rate d. The same is true for the stock of pollutants, if there are no emissions. Similarly, x and ρ are such rates, which govern the technical progress and the intertemporal decision making process.
In second step, we investigate the transition dynamics of the system into the steady state. In particular, we analyze the characteristics of typical optimal control paths, and discuss how they depend on different initial conditions.

4.1 Dynamic behavior in the neighborhood of the steady state

Our first step in analyzing the dynamic behavior of our model is to linearize the system of equations (56) – (61) in the neighborhood of the steady state. To simplify notation, we define

$$\mathbf{z} := \left(\hat{N} - \hat{N}^*, k - k^*, S - S^*, n - n^*, c - c^*, \hat{e} - \hat{e}^*\right)^T . \tag{25}$$

The vector $\mathbf{z}$ measures the distance of each endogenous variable from its steady state value. Taking into account that $\dot{\mathbf{z}} = \left(\dot{\hat{N}}, \dot{k}, \dot{S}, \dot{n}, \dot{c}, \dot{\hat{e}}\right)^T$, the linearized system in the neighborhood of the steady state is given by the following vector-equation (Feichtinger and Hartl 1986:133):

$$\dot{\mathbf{z}} = \mathcal{J}^*\mathbf{z} + O(\mathbf{z}^2). \tag{26}$$

$\mathcal{J}^*$ is the Jacobian matrix of the system of equations (56) – (61) evaluated at the steady state.[9] In the following, we neglect the error term $O(\mathbf{z}^2)$. Thus, the general solution of the linearized system (26) is determined by:

$$\mathbf{z}(t) = \mathbf{z}(0)\,\exp\left(\mathcal{J}^* t\right). \tag{27}$$

Denoting the Eigenvalues of $\mathcal{J}^*$ with μ_i, $i = 1, \ldots, 6$ and the six corresponding Eigenvectors with $\mathbf{v_i}$, $i = 1, \ldots, 6$, we may rewrite the general solution as follows:

$$\mathbf{z}(t) = \sum_{i=1}^{6} a_i\,\exp(\mu_i t)\,\mathbf{v_i}, \tag{28}$$

where the scalars a_i are determined by the initial conditions $\mathbf{z}(0) = \mathbf{z_0} = \sum_{i=1}^{6} a_i\,\mathbf{v_i}$.
The vector space, which contains the solutions of (26), may be divided in two subspaces. One of them is spanned by the Eigenvectors $\mathbf{v_i}$, which correspond to the negative Eigenvalues (respectively, the complex Eigenvalues with a negative real part). This is the stable subspace, because solutions in this subspace run into the steady state in the course of time. The other one is the instable subspace, spanned by the Eigenvectors, which correspond to the positive Eigenvalues.

[9] The full Jacobian $\mathcal{J}^*$ is described in Appendix A.7.

In our numerical calculations, we find three negative and three positive, real Eigenvalues of $\mathcal{J}^*$ for a wide range of parameters.[10] Figure 1 illustrates this result, which plots the negative Eigenvalues against parameters representing the timescales of the subsystems. Except for very small rates of technical progress $x < x_k$ and very small discount rates $\rho < \rho_k$, we always find three negative Eigenvalues,[11] even if we vary each of the parameters δ, x, d and ρ over a very wide range.
Therefore, we restrict the following argumentation on the case of three negative Eigenvalues, i.e. the case where a stable saddle-path exists. In this case, the optimal path in the neighborhood of the steady state is located in the stable subspace. Thus, the solution (28) of the linearized system reduces to

$$\mathbf{z}(t) = a_1\,\mathbf{v_1}\,\exp(\mu_1\,t) + a_2\,\mathbf{v_2}\,\exp(\mu_2\,t) + a_3\,\mathbf{v_3}\,\exp(\mu_3\,t). \tag{29}$$

In general, we could expect that the absolute values of the negative eigenvalues are different, and without loss of generality we assume $|\mu_1| < |\mu_2| < |\mu_3|$. These three negative Eigenvalues can be interpreted as time scales of the coupled dynamic system in the neighborhood of the steady state: After a time $t_i = 1/|\mu_i|$, the variable i has declined on o fraction $1/\mathbf{e}$ – where $\mathbf{e}$ is Euler's number – of its initial value a_i.
Thus from (29) it follows that the share of the eigenvalues of the greatest absolute value vanishes quickly and the system runs at least along the direction of the eigenvector $\mathbf{v}_1$ into the steady state. For large t, the dynamics in the neighborhood of the steady state is determined by the related eigenvalue $|\mu_1|$, and (29) reduces to $\mathbf{z}(t) = a_1\,\exp(\mu_1\,t)\,\mathbf{v_1}$.
With the interpretation of the negative Eigenvalues μ_i as time scales of the optimal dynamics of the coupled system, we may now analyze how the behavior of the coupled system changes, if the dynamic behavior of the subsystems change. The latter is determined by the following parameters: The internal time scale of the demographic subsystem is described by the death rate d, the dynamics of the economic system is described by the rate of technical progress x, the time structure of the decision making of the society is given by the time preference rate ρ, and the dynamic behavior of the environmental subsystem is characterized by the natural deterioration rate δ.
In order to analyze the change in the dynamic behavior of the coupled system with respect to a change in the exogenously given dynamics of the subsystems, we calculate numerically the change of the eigenvalues of our linearized system near the steady state with respect to a change of the above mentioned parameters. The results of these calculations are summarized in the following observations.

[10] In our calculations, we use the following parameter set. We choose the unit of time being a decade. The following rates are to be interpreted as units per decade: $d = 0.1$, $\delta = 0.1$, $x = 0.3$ (and $x = 0.1$ in the transition dynamics), and $\rho = 0.1$ per decade. The technology-parameters are as follows: $\alpha = 0.2$ (relating to Yip and Zhang 1997) and $\beta = 0.04$ (relating to Kümmel et al. 2002:417). The costs of raising children are given by $b = 0.1$ and $b_0 = 0.035$, respectively. These parameters relate to the parameters chosen by Barro and Sala-I-Martin (1995:319-320). Parameters of the utility function are: $\nu = 0.9$, $\epsilon = 0.01$, $\sigma = 0.33$ and $\bar{S} = 1$. These parameters satisfy the sufficient conditions (21) and (22). The time horizon for our simulation is ten decades.

[11] In figure 1 (a) and (c), only the two negative Eigenvalues of smallest absolute value are shown. The third negative Eigenvalue has a very high absolute value. Therefore, it has been omitted in figure 1 (a) and (c) for illustrative reasons.

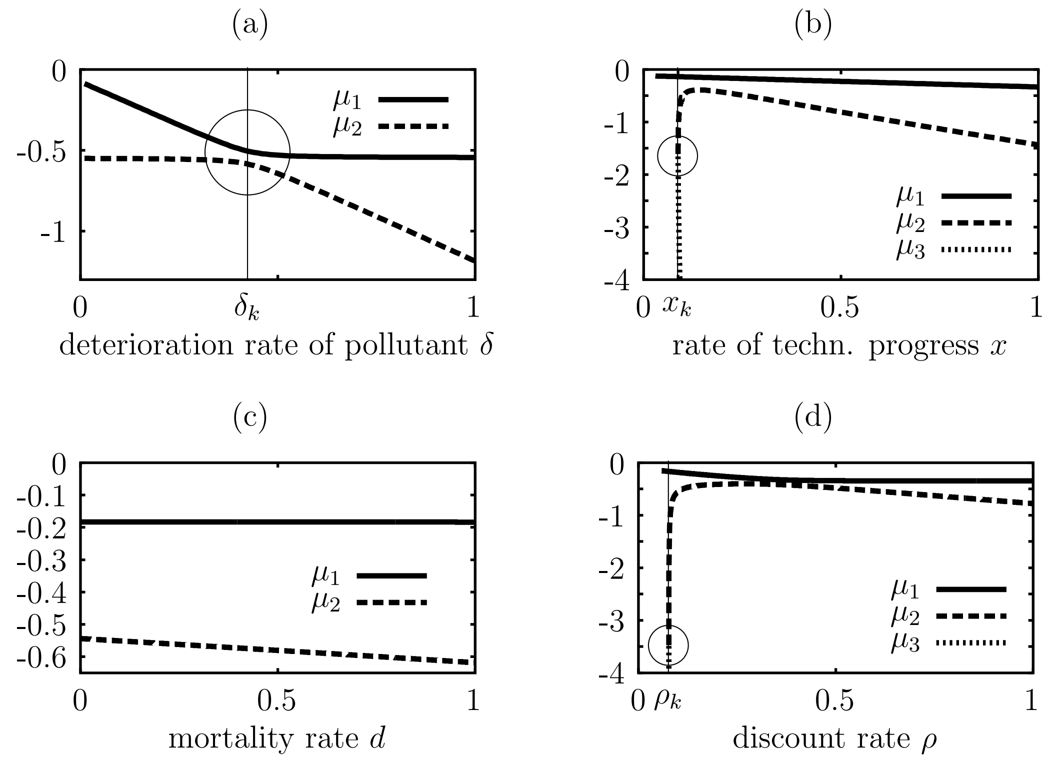

Figure 1: Comparative statics of the negative eigenvalues $\mu_1 \geq \mu_2 \geq \mu_3$. The emphasized values are $\delta_k = 0.43$, $x_k = 0.087$ and $\rho_k = 0.077$. To keep the illustration clear, we have omitted the smallest Eigenvalue μ_3 in figures (a) and (c). For the same reason, in (b) and (c), only a small part of μ_3 is shown.

Observation 1 *A change in the internal dynamics of a subsystem may yield a qualitative change in the evolution of the coupled system.*

Observation 1 may be shown by figure 1 (a). If the deterioration rate for emissions δ is high or low, the dynamic behavior of the whole system in the neighborhood of the steady state is mainly determined by the time scale μ_1. For low or high δ, the eigenvalue μ_1 is substantially lower than the other eigenvalues. Hence, in the neighborhood of the steady state, every optimal path will ultimately follow the direction of $\mathbf{v_1}$ (see page 12).

Between low and high deterioration rates, there is a range in which μ_1 and μ_2 are very close to each other (circle in figure 1 (a)): the components of the optimal path related to both eigenvectors $\mathbf{v_1}$ and $\mathbf{v_2}$ vanish only slowly. For $\delta \approx \delta_k$, the optimal path for large t is given by $\mathbf{z}(t) \approx a_1 \exp(\mu_1 t)\, \mathbf{v_1} + a_2 \exp(\mu_2 t)\, \mathbf{v_2} \approx (a_1 \mathbf{v_1} + a_2 \mathbf{v_2}) \exp(\mu_1 t)$. Thus, optimal paths could lie within the whole plane spanned by the eigenvectors $\mathbf{v_1}$ and $\mathbf{v_2}$. This is a qualitatively different dynamic behavior in comparison to a situation with low or high natural deterioration rates.

Observation 2 *A change of the internal time scale of one subsystem may cause a non-monotonic change of time-scales of the whole system.*

Observation 2 is illustrated by figures 1 (b) and (d). Figure 1 (b) shows that a change in the rate of technical progress x could lead to a non-monotonic change in the eigenvalue μ_2: If technical progress is small, the velocity at which the whole system moves towards the steady state, is declining. This is reflected by a decline of the absolute value of μ_2 if x is small and increasing. However, if the rate of technical progress is high, the opposite behavior occurs.

A similar non-monotonic behavior occurs with respect to a change of the discount rate ρ. For small values of ρ, μ_2 is increasing and it achieves a maximum, where μ_2 is very close to μ_1. For further increasing values of ρ, μ_2 is declining.

Observation 3 *The characteristic time-scales of the coupled system depend on the internal time-scales of all three subsystems.*

Figure 1 shows that all time-scales of the coupled system given by the three negative eigenvectors μ_1, μ_2, μ_3 change if the internal dynamics of one subsystem given by the respective parameters δ, x, d, and ρ changes. Hence, the dynamic behavior of the coupled system depends on the internal time scales of all subsystems, as stated in the observation. This yields the following conclusion: If we want to analyze the behavior of the total system, we have to take into account the internal dynamics of all subsystems simultaneously.

4.2 Transition dynamics

In order to investigate the dynamic characteristics of the system outside the 'neighborhood of the steady state', we solve conditions (56)-(61) numerically for the set of

parameters given on page 12 and initial conditions, which guarantee that the necessary conditions are also sufficient.[12]
Our numerical procedure is to start with a state of the system in the neighborhood of the steady state and integrate the optimality conditions backwards in time. [13] Considering the example of figure 3, which shows a typical optimal path, we may illustrate the essential characteristics of the optimal dynamics of the model economy. Every optimal path is determined by six time-dependent variables: $\hat{N}$, k, S, n, c and $\hat{e}$. Thus, the phase space has six dimensions. Figure 2 shows three projections of this phase space into the planes spanned by the stock variables N, k and S and their corresponding control variables n, c and e. The dotted lines give the direction of the eigenvectors $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$ corresponding to the three negative eigenvalues μ_1, μ_2 and μ_3. The cross section of the dotted lines determines the steady state values of the respective variables. As mentioned above, the optimal path runs at least along the direction of the eigenvector $\mathbf{v_1}$ into the steady state.[14]
Because in general, the eigenvectors have non-vanishing components in all directions, and because the optimal path runs at least into the steady state along the direction of $\mathbf{v_1}$, we generally need non-monotonic controls in order to achieve the steady state.[15] This is e.g. illustrated by the right-hand side of figure 2. Additionally, these figures show that the change in the variables is relatively high in the beginning and becomes lower the more we approach the steady state. This follows from the characteristics of the linearized system (29): the dynamic behavior of our model close to the steady state is determined by the slowest time scale μ_1. The more distant the system is from its steady state, the higher the contribution of the faster time scales μ_2 and μ_3 to its development is.
Finally, we analyze the influence of a variation in the initial values of the state variables on the development of the system. In particular, we illustrate how the optimal control paths change if we have small changes in the initial values. This is done by numerical simulations summarized in figure 3. In a first simulation we start with the initial values $\hat{N}_0, \hat{k}_0, \hat{S}_0$. The resulting optimal control path is depicted with a solid line. In a next step we have changed the initial conditions, such that the initial values are smaller and greater compared to the first simulation. The values are given by $N_0', N_0'', k_0', k_0'', S_0', S_0''$. The results are illustrated by the dotted lines in figure 3. The figures suggest that the behavior of our dynamic system is regular, i.e. a small variation of initial conditions leads to changes in the control path's, but the direction and the general behavior of the control path are very close to the first simulation.
Nevertheless, the initial conditions play a crucial role for the optimal development

[12] The first order conditions for our optimal path in figure 3 for a particular set of initial conditions are also sufficient, if the left hand side of equation (21) is always non-positive and the left hand side of equation (22) is always non-negative. Both conditions are fulfilled, as is readily checked.

[13] We use a Mathematica-script, which employs a Runge-Kutta procedure (e.g. Bronstein and Semendjajew (1991:770).

[14] The exact direction of this eigenvector however, depends crucially on the particular parameter values. As the parameters for this example were chosen in order to generate an illustrative example, the interpretation of the particular direction of $\mathbf{v_1}$ does not seem to be very meaningful.

[15] Further examples of the necessity of non-monotonic optimal control path are found in Moslener, Requate 2001 and Baumgärtner, Jöst, Winkler 2003

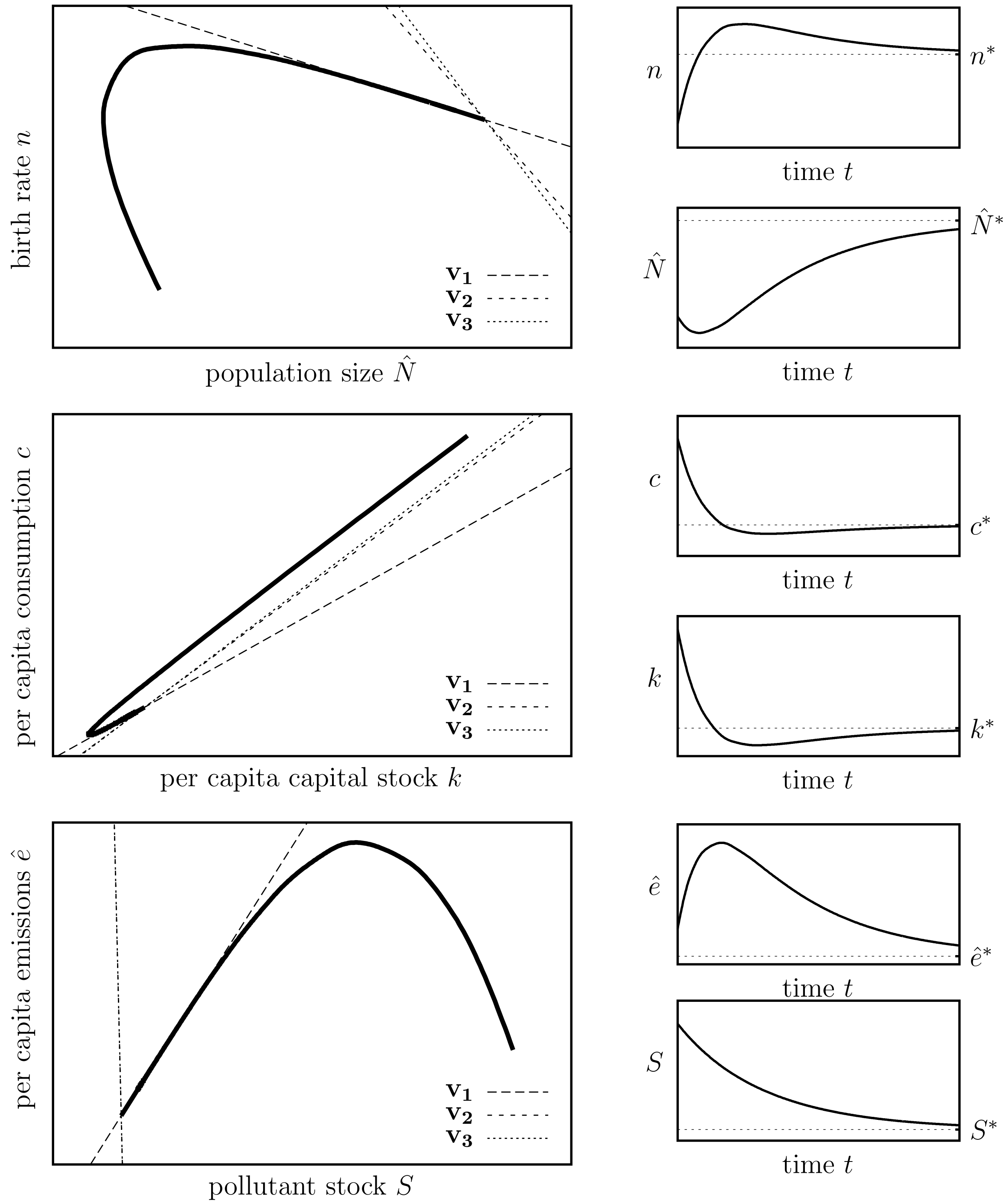

Figure 2: The figure shows a typical optimal path. On left hand side in three projections of the phase space (with the directions of the Eigenvalues $\mathbf{v_1}$,$\mathbf{v_2}$ and $\mathbf{v_3}$); on the Right, the time path of every variable is depicted. The parameters for the calculations are given in footnote 10 on page 12.

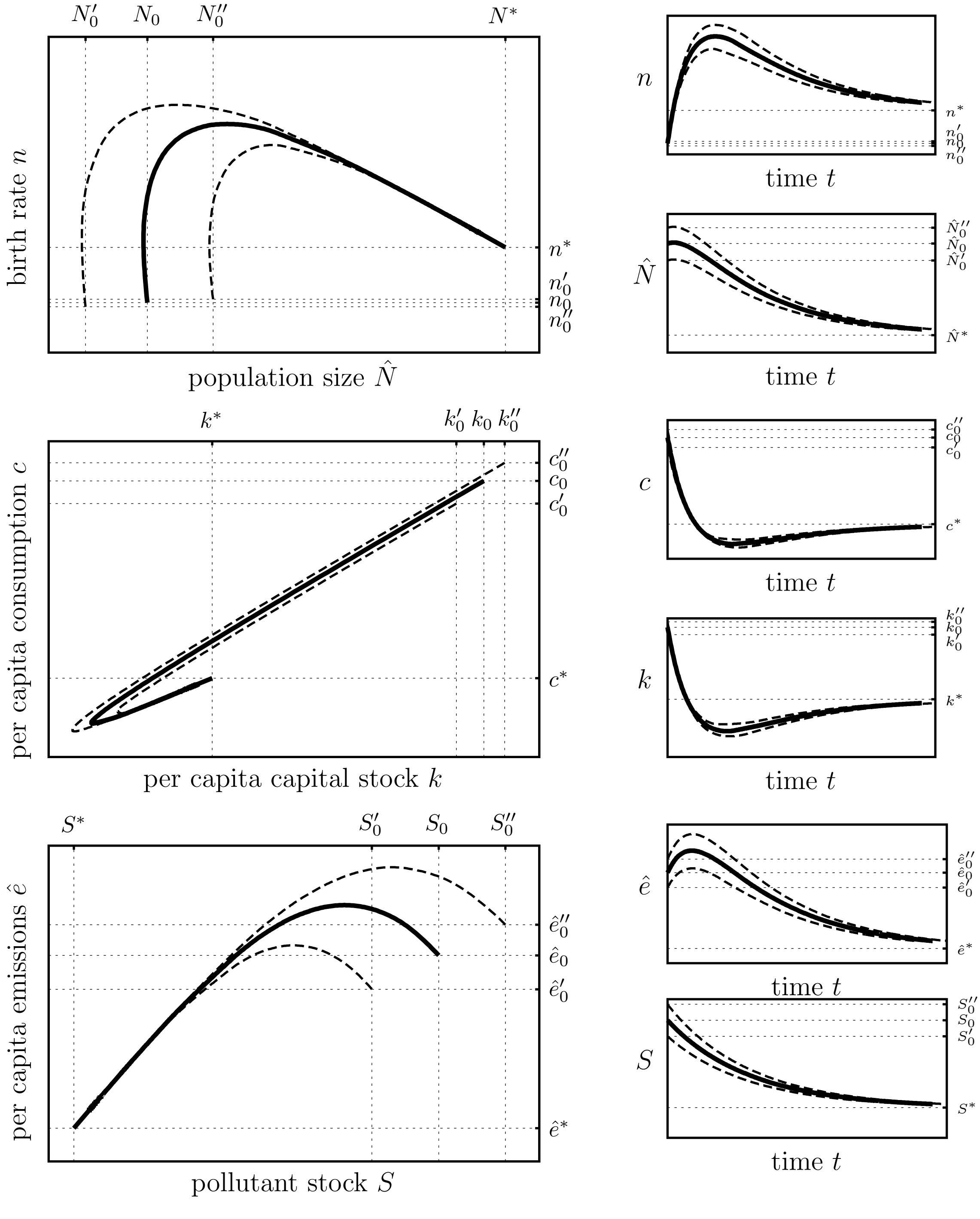

Figure 3: The figure shows the optimal path of figure 2 as well as two other optimal paths for different sets of initial conditions.

of the economy: A big change in initial conditions will result in a big change in the optimal path in general.

5 Conclusions

It was the aim of our analysis to contribute to a better understanding of the dynamic interaction between population growth, economic development and the use of the natural environment as a sink for pollutants.
For this purpose, we have formulated a stylized dynamic model of the complex interdependencies between real demographic, economic and environmental systems.
At this point, we want to note that even if our approach takes into account some of the essential characteristics of the interdependencies between population, economy, and nature, we had to make some rather 'heroic' assumptions. We regard the approach of an intertemporal optimization model with an infinite time horizon as a benchmark case. Nevertheless, it can easily be criticized as highly unrealistic with respect to, e.g., informational requirements and computing capacity of the decision maker.
Further significant simplifications are the assumptions that technical progress is exogenously given, that pollutants are homogenous, and that there is only one natural degradation rate for pollutants. Finally, we have neglected the age structure of the population. We make these assumptions for reasons of simplicity and tractability.
Despite this simplifications, many of our results could only be derived with the assistance of numerical methods, and are valid only for a restricted parameter space. It is an open question whether the extension of the model, e. g. the introduction of an age structure, would yield further insights.
Turning to the results of our paper, we can state that inspite of the simplifying assumptions, the model yields some interesting and important insights into the characteristics of an optimal development path of an economy in which fertility is endogenous and environmental quality is a variable of the decision-making process.
First of all, we have shown that in the long-run optimum the population N is growing and per capita emissions e are declining at the rate x/β. The whole increase in the productivity of inputs is used to reduce the per-capita emissions, while population is increasing. This allows for a constant per-capita utility level in our economy.
We have seen that an increase in the preference for children in comparison to consumption leads to a higher population and lower consumption and environmental quality in the long run, i.e. the steady state. However, if the preference for environmental quality is increasing in comparison to consumption and children, the environmental quality is improving in the long-run, while population size is declining. A higher carrying capacity $\bar{S}$ allows for a higher population and pollution stock. Because the resulting pollution stock increases slower than the carrying capacity, we achieve a higher environmental quality. Since the number of children and per-capita consumption are constant we have a higher utility level in a situation with a higher carrying capacity of the natural systems.
The analysis of the dynamic behavior of our system, in particular the discussion of the the effect of a change of the internal time scales of the three subsystems population,

economy and environment on the coupled system, leads to the following conclusions:

- If we consider a change in the different internal dynamics of the subsystems, the behavior of the total system may change in a non-monotonic manner. Statements concerning real world systems therefore require a detailed knowledge about the internal dynamics of the subsystems under investigation.

- The dynamic behavior of the whole system could be different if we analyze various pollutants. Pollutants with very high natural deterioration rates, e.g., tropospheric ozone, cause a qualitatively different dynamic behavior of the coupled system than medium living stock pollutants, e.g., methane, and this again is qualitatively different from the impacts of stock pollutants with a long life time, e.g. CO_2; (c.f. figure 1 (a)).

- We have seen that the optimal dynamics of the coupled system depends substantially on the internal time scales of population, economy and the environment (c.f. observation 3). Therefore we may not investigate the subsystems separately: In order to analyze the interrelationship between population development and environmental problems, we need an integrative analysis of the dynamic behavior of the whole system.

- The complex dynamic behavior also governs the optimal time paths of the control variables of the coupled system. The investigation of the system's transition dynamics shows that in general a non-monotonic time path of the control variables is necessary in order to achieve the steady state (cf. section 4).

  Reducing the number of children and emissions in a situation where the values of these variables exceed their steady state values is inefficient in general. This result is a challenge for any policy advice based on the results of intertemporal optimization models: On the one hand, simple policy recommendation such as reduce population growth, if population is too high compared to the optimum may cause considerable inefficiencies. On the other hand in the sphere of politics, the recommendation e.g. to increase numbers of children first and to decrease them afterwards, may be confronted with severe difficulties.

Given the optimistic assumptions of strong substitution possibilities between emissions and non-polluting production inputs and of constant technical progress, we find that long-term population growth does not necessarily lead to a constantly deteriorating environment, as is often stated in the debates about population growth and environmental problems. However, sophisticated policies have to be employed to achieve the optimal development.

A Appendix

A.1 Necessary Conditions

If we differentiate equation (8) implicitly with respect to time we get:

$$\dot{\lambda}^k = (\dot{u}_c - \rho u_c)\exp(-\rho t). \tag{30}$$

If we substitute this result and equation (8) into (11) we get for our utility function (2) after some alterations:

$$-\frac{\dot{u}_c}{u_c} = \frac{\dot{c}}{c} = f_k - (n-d) - b\,n - \rho. \tag{31}$$

Differentiation of (9) with respect to time leads to:

$$(\dot{u}_n - \rho u_n)\exp(-\rho t) - \dot{\lambda}^k\left[(1+b)k + b_0\right] - \lambda^k(1+b)\dot{k} + \lambda^N\dot{N} + \dot{\lambda}^N N = 0. \tag{32}$$

Multiplying (12) by N and inserting (3) gives:

$$\lambda^N\dot{N} + \lambda^S N e = -\dot{\lambda}^N N.$$

Together with equation (10) multiplied by e and (8) we get:

$$\lambda^N\dot{N} + \dot{\lambda}^N N = u_c e f_e \exp(-\rho t);$$

If we insert this expression into equation (32) we get together with (30) and (8):

$$(\dot{u}_n - \rho u_n) - (\dot{u}_c - \rho u_c)\left[(1+b)k + b_0\right] - u_c(1+b)\dot{k} + u_c e f_e = 0.$$

and after some alterations

$$\frac{\dot{u}_n}{u_n} = \rho + \left[-e f_e + \left[(1+b)k + b_0\right]\left(\frac{\dot{u}_c}{u_c} - \rho\right) + (1+b)\dot{k}\right]\frac{u_c}{u_n}.$$

Taking into account the specification of our utility and production function (2) and (5) we get:

$$\frac{\dot{n}}{n} = -\frac{\rho}{\epsilon} + \left[\beta k^\alpha e^\beta \exp(xt) + \left[(1+b)k + b_0\right]\left(\frac{\dot{c}}{c} + \rho\right) - (1+b)\dot{k}\right]\frac{n^\epsilon}{\nu\epsilon\, c}.$$

Differentiation of (10) with respect to time leads to:

$$\left(\frac{d}{dt}\left(u_c\, f_e\right) - \rho\left(u_c\, f_e\right)\right) + \dot{\lambda}^S N + \lambda^S\dot{N} = 0.$$

Together with equation (13) multiplied by N and (8), (10) and (3) we obtain:

$$\left(\frac{d}{dt}\left(u_c\, f_e\right) - \rho\left(u_c\, f_e\right)\right) - N u_S - \delta u_c f_e - (n-d)u_c f_e = 0.$$

After some alterations and the substitution of expression

$$\frac{d}{dt}\left(u_c f_e\right) = -\frac{\dot{c}}{c} - (1-\beta)\frac{\dot{e}}{e} + \alpha\frac{\dot{k}}{k}$$

we get:

$$\frac{\dot{e}}{e} = -\frac{1}{1-\beta}\left[\rho + \delta + (n-d) + \frac{\dot{c}}{c} - \alpha\frac{\dot{k}}{k} - \frac{\sigma N c}{\beta\left(\bar{S} - S\right)k^\alpha e^{\beta-1}\exp(xt)}\right]. \tag{33}$$

A.2 Sufficient conditions

In order to show that the first order conditions are even sufficient, we have to show that the maximized Hamiltonian H^0 is concave in its state variables,[16] for given costate variables, using the first order conditions for the control variables. Given the co-state variables in current values, $\hat{\lambda}^i = \lambda^i \exp(\rho t)$, $i \in \{k, N, S\}$, we obtain for the control variables for an optimal path:

$$\begin{aligned}
c &= \frac{1}{\hat{\lambda}^k} && (34)\\
n &= \nu^{\frac{1}{\epsilon}} \left[\hat{\lambda}^k \left[(1+b)k + b_0\right] - \hat{\lambda}^N N\right]^{-\frac{1}{\epsilon}} && (35)\\
e &= \left(-\frac{N\hat{\lambda}^S}{\hat{\lambda}^k k^\alpha \beta}\right)^{\frac{1}{\beta-1}} && (36)\\
f(k,e) &= -\frac{\hat{\lambda}^S}{\hat{\lambda}^k}\frac{Ne}{\beta}. && (37)
\end{aligned}$$

This leads to the following maximized Hamiltonian:

$$\begin{aligned}
H^0 = & -\ln(\hat{\lambda}^k) + \frac{\nu^{\frac{1}{\epsilon}+1}}{1-\epsilon}\left[\hat{\lambda}^k \left[(1+b)k + b_0\right] - \hat{\lambda}^N N\right]^{-\frac{1-\epsilon}{\epsilon}} - \frac{1}{1-\epsilon} + \sigma \ln\left(\bar{S} - S\right)\\
& -\frac{\hat{\lambda}^S}{\beta} Ne - 1 - \hat{\lambda}^k \left[(1+b)k + b_0\right] \nu^{-\frac{1}{\epsilon}} \left[\hat{\lambda}^k \left[(1+b)k + b_0\right] - \hat{\lambda}^N N\right]^{-\frac{1}{\epsilon}}\\
& +\hat{\lambda}^k k d + \hat{\lambda}^N N \nu^{-\frac{1}{\epsilon}} \left[\hat{\lambda}^k \left[(1+b)k + b_0\right] - \hat{\lambda}^N N\right]^{-\frac{1}{\epsilon}} - \hat{\lambda}^N N d + \hat{\lambda}^S N e - \hat{\lambda}^S \delta S\\
= & \frac{\nu - (1-\epsilon)}{1-\epsilon} \nu^{-\frac{1}{\epsilon}} \left[\hat{\lambda}^k \left[(1+b)k + b_0\right] - \hat{\lambda}^N N\right]^{-\frac{1-\epsilon}{\epsilon}}\\
& + \frac{1-\beta}{\beta}\left(\beta\lambda^k\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \left(-\lambda^S N\right)^{-\frac{\beta}{1-\beta}}\\
& - \ln \lambda^k + \sigma\left(\bar{S} - S\right) - 1 + \lambda^k d\, k - \lambda^N d\, N - \lambda^S \delta\, S.
\end{aligned}$$

H^0 is concave in its state variables if the Hessian matrix

$$\mathcal{H} = \begin{pmatrix} \frac{d^2 H^0}{dN^2} & \frac{d^2 H^0}{dN dk} & 0 \\ \frac{d^2 H^0}{dN dk} & \frac{d^2 H^0}{dk^2} & 0 \\ 0 & 0 & \frac{d^2 H^0}{dS^2} \end{pmatrix} \tag{38}$$

is negative semi-definite. We have already used the fact that in our model the cross-derivatives with respect to S are vanishing.
$\mathcal{H}$ is negative semi-definite if the determinants of its principal minors are of alternating sign. Hence we have to show that $\frac{d^2 H^0}{dN^2} \leq 0$, $\frac{d^2 H^0}{dk^2} \leq 0$, $\frac{d^2 H^0}{dS^2} \leq 0$ and that

[16] We discuss sufficiency conditions by the use of the theorem of Arrow and Kurz (1970, Proposition 6) .

$\frac{d^2H^0}{dN^2}\frac{d^2H^0}{dk^2} - \left(\frac{d^2H^0}{dN^2}\right)^2 \geq 0$. This is sufficient, because

$$\frac{d^2H^0}{dS^2} = -\frac{\sigma}{\left(\bar{S}-S\right)^2} \leq 0. \tag{39}$$

The second order derivatives of H^0 with respect to k and N are as follows:

$$\begin{aligned}
\frac{d^2H^0}{dN^2} &= \left(\frac{\lambda^N}{\epsilon}\right)^2 (\nu-(1-\epsilon))\,\nu^{\frac{1}{\epsilon}} \left[\hat{\lambda}^k\left[(1+b)k+b_0\right]-\hat{\lambda}^N N\right]^{-\frac{1}{\epsilon}-1} \\
&\quad + \frac{\left(\lambda^S\right)^2}{1-\beta}\left(\beta\lambda^k\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \left(-\lambda^S N\right)^{-\frac{1}{1-\beta}-1} \\
&= (\nu-(1-\epsilon))\frac{\left((1+b)k+b_0-\nu c n^{-\epsilon}\right)^2}{\nu\epsilon^2c^2N^2n^{-1-\epsilon}} + \frac{\beta}{1-\beta}\frac{k^\alpha e^\beta}{cN^2} \qquad (40) \\
\frac{d^2H^0}{dkdN} &= \frac{\lambda^N\lambda^k(1+b)}{\epsilon^2}(\nu-(1-\epsilon))\,\nu^{\frac{1}{\epsilon}} \left[\hat{\lambda}^k\left[(1+b)k+b_0\right]-\hat{\lambda}^N N\right]^{-\frac{1}{\epsilon}-1} \\
&\quad + \frac{\alpha\lambda^S}{1-\beta}\left(\beta\lambda^k\right)^{\frac{1}{1-\beta}} k^{-\frac{1-\alpha-\beta}{1-\beta}} \left(-\lambda^S N\right)^{-\frac{1}{1-\beta}} \\
&= (\nu-(1-\epsilon))\frac{\left((1+b)k+b_0-\nu c n^{-\epsilon}\right)(1+b)}{\nu\epsilon^2c^2Nn^{-1-\epsilon}} + \frac{\alpha\beta}{1-\beta}\frac{k^{\alpha-1}e^\beta}{cN} \qquad (41) \\
\frac{d^2H^0}{dk^2} &= \left(\frac{\lambda^k(1+b)}{\epsilon}\right)^2 (\nu-(1-\epsilon))\,\nu^{\frac{1}{\epsilon}} \left[\hat{\lambda}^k\left[(1+b)k+b_0\right]-\hat{\lambda}^N N\right]^{-\frac{1}{\epsilon}-1} \\
&\quad - \frac{\alpha(1-\alpha-\beta)}{\beta(1-\beta)}\left(\beta\lambda^k\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}-2} \left(-\lambda^S N\right)^{-\frac{\beta}{1-\beta}} \\
&= (\nu-(1-\epsilon))\frac{(1+b)^2}{\nu\epsilon^2c^2n^{-1-\epsilon}} - \frac{\alpha(1-\alpha-\beta)}{1-\beta}\frac{k^{\alpha-2}e^\beta}{c} \qquad (42)
\end{aligned}$$

It is obvious that in general the Hamiltonian is concave. In particular $\frac{d^2H^0}{dN^2}$ is for $\nu \geq 1-\epsilon$ not negative. Hence it is necessary that ν is sufficiently small. If $\frac{d^2H^0}{dN^2} \leq 0$ than $\nu \leq 1-\epsilon$ and $\frac{d^2H^0}{dk^2} \leq 0$. Thus we have to prove for every optimal path whether or not the following two conditions hold

$$\frac{d^2H^0}{dN^2} \leq 0 \text{ and} \tag{43}$$

$$\frac{d^2H^0}{dN^2}\frac{d^2H^0}{dk^2} - \left(\frac{d^2H^0}{dkdN}\right)^2 \geq 0. \tag{44}$$

A.3 The steady state growth rates

A steady state of the system is characterized by constant growth rates of all variables. Hence, $\frac{\dot{N}}{N}$, $\frac{\dot{k}}{k}$, $\frac{\dot{S}}{S}$, $\frac{\dot{c}}{c}$ and $\frac{\dot{e}}{e}$ are constant.[17]

[17]Because N is growing at a constant rate, all extensive variables are growing at a constant rate.

From $\frac{\dot{N}}{N}$ = constant follows

$$\frac{\dot{n}}{n} = 0, \tag{45}$$

i.e. the number of children per woman is constant in a steady state.
The derivative of equation (6) with respect to time leads after some alterations to the condition

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} + \frac{\dot{e}}{e}. \tag{46}$$

The derivative of (18) with respect to time together with $\dot{n} = 0$ leads to

$$\frac{d}{dt} f_k = \frac{d}{dt} \alpha k^{\alpha-1} e^{\beta} \exp(xt) = 0 \quad \Rightarrow \quad \frac{\dot{k}}{k} = \alpha\frac{\dot{k}}{k} + \beta\frac{\dot{e}}{e} + x. \tag{47}$$

With this result and the derivative of (4) we get

$$\frac{\dot{c}}{c} = (1 + b_0 n)\frac{\dot{k}}{k}. \tag{48}$$

From equation (19), using the same procedure as above, we get:

$$0 = \frac{\dot{N}}{N} + \frac{\dot{c}}{c} + \frac{\dot{S}}{S}\frac{S}{\bar{S} - S} - \alpha\frac{\dot{k}}{k} + (1-\beta)\frac{\dot{e}}{e} - x = b_0 n\frac{\dot{k}}{k} + \frac{\dot{S}}{S}\frac{\bar{S}}{\bar{S} - S}, . \tag{49}$$

From equation (17) together with $\dot{n} = 0$ we get

$$\xi \equiv -\beta f - [(1+b)k + b_0]\left[\frac{\dot{c}}{c} + \rho\right] + (1+b)k\frac{\dot{k}}{k} = -\frac{\rho}{\epsilon}\frac{u_n}{u_c}. \tag{50}$$

If we differ (17) with respect to time together with (47) and (50) we obtain:

$$\begin{aligned}
0 &= \xi\frac{u_c}{u_n}\underbrace{\left[\frac{\dot{u}_c}{u_c} - \overbrace{\frac{\dot{u}_n}{u_n}}^{=0}\right]}_{=-\frac{\dot{c}}{c}} + \underbrace{\left[-\beta f\underbrace{\left[\alpha\frac{\dot{k}}{k} + \beta\frac{\dot{e}}{e} + x\right]}_{=\frac{\dot{k}}{k}} - (1+b)k\frac{\dot{k}}{k}\left[\frac{\dot{c}}{c} + \rho\right] + (1+b)k\left(\frac{\dot{k}}{k}\right)^2\right]\frac{u_c}{u_n}}_{=\frac{\dot{k}}{k}\left[\xi + b_0\left[\frac{\dot{c}}{c} + \rho\right]\right]} \\
0 &= -\xi\frac{u_c}{u_n}\frac{\dot{c}}{c} + \frac{\dot{k}}{k}\left[\xi + b_0\left[\frac{\dot{c}}{c} + \rho\right]\right]\frac{u_c}{u_n} \\
0 &= \frac{\dot{c}}{c}\frac{\rho}{\epsilon} + \frac{\dot{k}}{k}\left[-\frac{\rho}{\epsilon} + b_0\left[\frac{\dot{c}}{c} + \rho\right]\frac{n^{\epsilon}}{\nu\epsilon\, c}\right].
\end{aligned} \tag{51}$$

Together with (48) we get

$$0 = \left[b_0 n\frac{\rho}{\epsilon} + b_0\left[[1 + b_0 n]\frac{\dot{k}}{k} + \rho\right]\frac{n^{\epsilon}}{\nu\epsilon\, c}\right]\frac{\dot{k}}{k}. \tag{52}$$

This equation is fulfilled if $\dot{k} = 0$ or

$$[1 + b_0 n] \frac{\dot{k}}{k} = -\rho(\nu n^{1-\epsilon} c + 1). \tag{53}$$

If c is growing (or declining) at a constant rate, the right hand side of this equation is not constant. Because we have assumed that in the steady state $\dot{k}/k$ is constant, this condition could not hold in a steady state. Thus in the steady state $\dot{k} = 0$ From equation (48) immediately follows that $\dot{c} = 0$. And from (49) we get that $\dot{S} = 0$.
From equations (47) and (46) we get for the variables N and e

$$\frac{\dot{N}}{N} = -\frac{\dot{e}}{e} = \frac{x}{\beta}. \tag{54}$$

Therefore the following conditions hold in a steady state:

$$\dot{k} = \dot{S} = \dot{n} = \dot{c} = 0 \text{ and } \frac{\dot{N}}{N} = -\frac{\dot{e}}{e} = \frac{x}{\beta}. \tag{55}$$

A.4 Transformed optimality conditions

We have seen that N is growing and e is declining in a steady state. In order to have constant steady state values of all variables, we transform the variables $\hat{e} = e \exp(-x/\beta t)$ and $\hat{N} = N \exp(x/\beta t)$. This leads to the following new first order conditions

$$\begin{aligned}
\dot{\hat{N}} &= \left(n - d - \frac{x}{\beta}\right) \hat{N} && (56)\\
\dot{k} &= k^\alpha \hat{e}^\beta - c - (n-d)k - bnk - b_0 n && (57)\\
\dot{S} &= \hat{N}\hat{e} - \delta S. && (58)\\
\dot{n} &= -\frac{\rho n}{\epsilon} + \left[\beta k^\alpha \hat{e}^\beta + [(1+b)k + b_0]\left[\rho + \frac{\dot{c}}{c}\right] - (1+b)\dot{k}\right] \frac{n^{1+\epsilon}}{\nu\epsilon\, c} && (59)\\
\dot{c} &= \left[\alpha k^{\alpha-1}\hat{e}^\beta - bn - \rho - (n-d)\right] c && (60)\\
\dot{\hat{e}} &= -\frac{\hat{e}}{1-\beta}\left[\rho + \delta + (n-d) + \frac{\dot{c}}{c} - \alpha\frac{\dot{k}}{k} - \frac{\sigma \hat{N}\hat{e}c}{\left(\bar{S} - S\right)\beta k^\alpha \hat{e}^\beta}\right] + \frac{x}{\beta}\hat{e} && (61)
\end{aligned}$$

A.5 Proof of Proposition 2

We show the following: The system of equations that results from inserting the conditions

$$\dot{k} = \dot{S} = \dot{n} = \dot{c} = 0 \text{ und } \frac{d}{dt}\hat{N} = \frac{d}{dt}\hat{e} = 0 \tag{62}$$

into the optimality conditions equations in section A.4 has a unique solution $\hat{N}^*$, k^*, S^*, n^*, c^* und $\hat{e}^*$.

This system of equation reads:

$$
\begin{aligned}
n^* &= d + \frac{x}{\beta} && (63)\\
f(k^*, \hat{e}^*) &= c^* + (n^* - d)k^* + b\, n^*\, k^* + b_0 n^* && (64)\\
\hat{N}^*\, \hat{e}^* &= \delta\, S^* && (65)\\
\alpha\, (k^*)^{-1} f(k^*, \hat{e}^*) &= b\, n^* + (n^* - d) + \rho && (66)\\
(n^*)^{-\epsilon} \nu c^* &= \frac{\beta}{\rho} f(k^*, \hat{e}^*) + (1+b)\, k^* + b_0 && (67)\\
\frac{\hat{N}^*\, \hat{e}^*\, \sigma}{\bar{S} - S^*} &= \beta \frac{f(k^*, \hat{e}^*)}{c^*} (\rho + \delta + x)\,. && (68)
\end{aligned}
$$

Given equation (63), the steady-state-birth rate is uniquely determined. It is positive, as the parameters d, x and β are positive. Equation (66) leads to:

$$
f(k^*, \hat{e}^*) = \frac{b\, n^* + (n^* - d) + \rho}{\alpha} k^* \equiv \zeta\, k^*. \tag{69}
$$

Here, we have $\alpha\zeta = b\, n^* + (n^* - d) + \rho = bd + (1+b)x/\beta + \rho > 0$, as all parameters are positive. Inserting equation (64) we get:

$$
c^* = (\zeta - (n^* - d) - b\, n^*)\, k^* - b_0 n^*. \tag{70}
$$

Using this result in equation (67) yields the steady-state-capital stock. As n^* is uniquely determined, k^* is unique as well:

$$
\begin{aligned}
k^* &= \frac{b_0\, (\nu n^* + (n^*)^\epsilon)}{\nu\, (\zeta - (n^* - d) - b\, n^*) - \frac{\beta}{\rho}(n^*)^\epsilon \zeta - (1+b)(n^*)^\epsilon} && (71)\\
&= \frac{\alpha\rho b_0\, (\nu n^* + (n^*)^\epsilon)}{\nu\rho\Xi - \beta(n^*)^\epsilon\, (b\, n^* + (n^* - d) + \rho) - (1+b)(n^*)^\epsilon \alpha\rho}
\end{aligned}
$$

with the abbreviation $\Xi = (1-\alpha)(n^* - d) + (1-\alpha) b\, n^* + \rho = \alpha\, (\zeta - (n^* - d) - b\, n^*)$. Inserting k^* in equation (70), we get the steady-state value of per capita consumption. As (70) is linear in k^*, c^* ist uniquely determined:

$$
\begin{aligned}
c^* &= \frac{\Xi}{\alpha} k^* - b_0 n^*\\
&= b_0 \frac{\rho\, \Xi\, (\nu n^* + (n^*)^\epsilon) - n^*\, (\nu\rho\, \Xi - \beta(n^*)^\epsilon\, (b\, n^* + (n^* - d) + \rho) - (1+b)(n^*)^\epsilon \alpha\rho)}{\nu\rho\, \Xi - \beta(n^*)^\epsilon\, (b\, n^* + (n^* - d) + \rho) - (1+b)(n^*)^\epsilon \alpha\rho}\\
&= b_0\, (n^*)^\epsilon \frac{\rho\, \Xi + \beta n^*\, (b\, n^* + (n^* - d) + \rho) + (1+b) n^* \alpha\rho}{\nu\rho\, \Xi - \beta(n^*)^\epsilon\, (b\, n^* + (n^* - d) + \rho) - (1+b)(n^*)^\epsilon \alpha\rho}.
\end{aligned}
$$

This expression as well as k^* are positive, if and only if the denominator ist positive, i.e.

$$
\nu\rho\, \Xi - \beta(n^*)^\epsilon\, (b\, n^* + (n^* - d) + \rho) - (1+b)(n^*)^\epsilon \alpha\rho > 0.
$$

Inserting $n^* = d + x/\beta$ and Ξ and rearranging proves that this condition is equivalent to the assumption

$$\nu\rho\,\frac{(1-\alpha)(1+b)\frac{x}{\beta} + (1-\alpha)b\,d + \rho}{\beta bd + (1+b)x + (\beta + (1+b)\alpha)\,\rho} > \left(d + \frac{x}{\beta}\right)^{\epsilon}. \tag{24}$$

If this condition holds, k^* and c^* are positive.
From equation (68) we get exactly one solution for the steady-state pollution stock

$$S^* = \bar{S}\frac{\beta\zeta(\rho+\delta+x)k^*}{\delta\sigma c^* + \beta\zeta(\rho+\delta+x)k^*}. \tag{72}$$

S^* is positive, because all parameters as well as k^* and c^* are positive.
Equation (66) has exactly one positive real-valued solution which gives the steady-state value of per capita emissions:

$$\hat{e}^* = e^*\exp(\frac{x}{\beta}t) = \zeta^{\frac{1}{\beta}}(k^*)^{\frac{1-\alpha}{\beta}}. \tag{73}$$

Therefore, N^* calculated from (65) is positive and uniquely determined:

$$\hat{N}^* = N^*\exp(-\frac{x}{\beta}t) = \frac{\delta S^*}{\hat{e}^*}. \tag{74}$$

In each case, the previously calculated steady-state-values of the endogenous variables have to be inserted in the right hand sides of the equations.
The transversality condition holds, as in the steady-state, we have:

$$\begin{aligned}
\lim_{t\to\infty} H^0 &= \lim_{t\to\infty}\left[u(c,n,S)\exp(-\rho\,t) + \frac{x}{\beta}\,\lambda^N\,N\right] \\
&= \lim_{t\to\infty}\left[u(c,n,S)\exp(-\rho\,t) + \frac{x}{\beta}\,\left[-u_n\exp(-\rho\,t) + \lambda^k\,[k+bk+b_0]\right]\right] \\
&= \lim_{t\to\infty}\left[u(c,n,S)\exp(-\rho\,t) + \frac{x}{\beta}\,\left[-u_n\,\exp(-\rho\,t) + u_c\,\exp(-\rho\,t)\;[k+bk+b_0]\right]\right] \\
&= 0.
\end{aligned}$$

Here, we have inserted $\dot{k} = 0$ and $\dot{S} = 0$ from proposition 1 and used the equations $n^* - d = x/\beta$ as well as (9) and (8).

□

A.6 Comparative static of the steady state

Comparative static with respect to ν

Differentiation of (71) with respect to ν gives:

$$\frac{dk^*}{d\nu} = \frac{b_0 n^* k^*}{b_0\,(\nu n^* + (n^*)^{\epsilon})} - (\zeta - (n^*-d) - bn^*)\,\frac{(k^*)^2}{b_0\,(n^* + \nu(n^*)^{\epsilon})} = -\frac{k^* c^*}{b_0\,(\nu n^* + (n^*)^{\epsilon})} \leq 0. \tag{75}$$

Thus we get from equation (70)

$$\frac{dc^*}{d\nu} = (\zeta - (n^* - d) - bn^*)\frac{dk^*}{d\nu} \leq 0. \tag{76}$$

Together with (72) we obtain:

$$\frac{dS^*}{d\nu} = -\frac{\bar{S}b_0 n^* \beta\delta\zeta\sigma(\rho+\delta+x)}{\left(b_0\delta\sigma n^* - (\beta\zeta(\rho+\delta+x)+\delta(d-(1+b)n^*+\zeta)\sigma)k^*\right)^2}\frac{dk^*}{d\nu} \geq 0. \tag{77}$$

From (73) we see that $\frac{d\hat{e}^*}{d\nu} \leq 0$, and hence,

$$\frac{d\hat{N}^*}{d\nu} = \frac{\delta S^*}{\hat{e}^*}\left(\frac{1}{S^*}\frac{dS^*}{d\nu} - \frac{1}{\hat{e}^*}\frac{d\hat{e}^*}{d\nu}\right) \geq 0. \tag{78}$$

Comparative statics with respect to ϵ

Differentiation of (71) with respect to ϵ leads to:

$$\frac{dk^*}{d\epsilon} = \left(\frac{(n^*)^\epsilon k^*}{\nu n^* + (n^*)^\epsilon} + \left(\frac{\beta}{\rho}\zeta - (1+b)\right)\frac{(n^*)^\epsilon (k^*)^2}{b_0\left(\nu n^* + (n^*)^\epsilon\right)}\right)\ln(n^*) \leq 0 \tag{79}$$

Because in the steady state $n^* \leq 1$ the expression above holds with inequality. Hence, we get from (70)

$$\frac{dc^*}{d\epsilon} = (\zeta - (n^* - d) - bn^*)\frac{dk^*}{d\epsilon} \leq 0. \tag{80}$$

Together with equation (72) we get:

$$\frac{dS^*}{d\epsilon} = -\frac{\bar{S}b_0 n^* \beta\delta\zeta\sigma(\rho+\delta+x)}{\left(b_0\delta\sigma n^* - (\beta\zeta(\rho+\delta+x)+\delta(d-(1+b)n^*+\zeta)\sigma)k^*\right)^2}\frac{dk^*}{d\epsilon} \geq 0. \tag{81}$$

From equation (73) immediately follows that $\frac{d\hat{e}^*}{d\epsilon} \leq 0$, and hence

$$\frac{d\hat{N}^*}{d\epsilon} = \frac{\delta S^*}{\hat{e}^*}\left(\frac{1}{S^*}\frac{dS^*}{d\epsilon} - \frac{1}{\hat{e}^*}\frac{d\hat{e}^*}{d\epsilon}\right) \geq 0. \tag{82}$$

Comparative statics with respect to σ

From equation (71) we get that

$$\frac{dk^*}{d\sigma} = 0, \tag{83}$$

Thus from (70) and (73) follows that

$$\frac{dc^*}{d\sigma} = \frac{d\hat{e}^*}{d\sigma} = 0. \tag{84}$$

From equation (72) we get

$$\frac{dS^*}{d\sigma} = \delta c^* \bar{S}\frac{\beta\zeta(\rho+\delta+x)k^*}{\left(\delta\sigma c + \beta\zeta(\rho+\delta+x)k^*\right)^2} > 0. \tag{85}$$

And hence,

$$\frac{d\hat{N}^*}{d\sigma} = \frac{\delta}{\hat{e}^*}\frac{dS^*}{d\sigma} > 0. \tag{86}$$

Comparative statics with respect to b_0

From (71) it follows

$$\frac{dk^*}{db_0} = \frac{k^*}{b_0} > 0. \tag{87}$$

Thus we get from equation (70)

$$\frac{dc^*}{db_0} = (\zeta - (n^* - d) - bn^*)\frac{dk^*}{db_0} - n^* = \frac{c^*}{b_0} > 0. \tag{88}$$

From (72) we obtain after rearrangement:

$$S^* = \bar{S}\frac{\beta\zeta(\rho+\delta+x)\frac{k^*}{b_0}}{\delta\sigma\frac{c^*}{b_0} + \beta\zeta(\rho+\delta+x)\frac{k^*}{b_0}}. \tag{89}$$

From (71) and (70) it follows that $\frac{k^*}{b_0}$ and $\frac{c^*}{b_0}$ are independent from b_0. Thus

$$\frac{dS^*}{db_0} = 0. \tag{90}$$

Furthermore we get

$$\frac{d\hat{N}^*}{db_0} = -\frac{\delta S^*}{\hat{(}e^*)^2}\frac{d\hat{e}^*}{db_0} < 0, \tag{91}$$

because together with $\frac{dk^*}{db_0} > 0$ we get from equation (73) that $\frac{d\hat{e}^*}{db_0} > 0$.

Comparative statics with respect to $\bar{S}$

From (71) immediately follows that

$$\frac{dk^*}{d\bar{S}} = 0. \tag{92}$$

Thus, we get from (70) and (73) that

$$\frac{dc^*}{d\bar{S}} = \frac{d\hat{e}^*}{d\bar{S}} = 0 \tag{93}$$

From equation (72) we obtain

$$\frac{dS^*}{d\bar{S}} = \frac{S^*}{\bar{S}} > 0, \tag{94}$$

and hence

$$\frac{d\hat{N}^*}{d\bar{S}} = \frac{\delta}{\hat{e}^*}\frac{dS^*}{d\bar{S}} > 0. \tag{95}$$

Finally

$$\frac{d(\bar{S} - S^*)}{d\bar{S}} = 1 - \frac{S^*}{\bar{S}} = \frac{\delta\sigma c^*}{\delta\sigma c^* + \beta\zeta(\rho+\delta+x)k^*}. \tag{96}$$

A.7 The Jacobi-Matrix in the steady state

We obtain the Jacobian matrix if we differentiate equations (3), (4), (6) and (18)-(19) with respect to the endogenous variables of our model, N, k, S, n, c und e. These derivatives are calculated in the steady state, and we get the following matrix,

$$\mathcal{J}^* = \begin{pmatrix} n^* - d - \frac{x}{\beta} & 0 & 0 & \hat{N}^* & 0 & 0 \\ 0 & \rho & 0 & -(1+b)k^* - b_0 & -1 & \beta (k^*)^{\alpha} \hat{(e^*)}^{\beta-1} \\ \hat{e}^* & 0 & -\delta & 0 & 0 & \hat{N}^* \\ 0 & \phi_k^n & 0 & \rho & \phi_c^n & \phi_e^n \\ 0 & \phi_k^c & 0 & -(1+b)c^* & 0 & \alpha\beta (k^*)^{\alpha-1} \hat{(e^*)}^{\beta-1} c^* \\ \frac{\hat{e}^*}{\hat{N}^*} \frac{\rho+\delta+x}{1-\beta} & \phi_k^e & \frac{\hat{e}^*}{\bar{S}-S^*} \frac{\rho+\delta+x}{1-\beta} & \phi_n^e & \phi_c^e & \rho + \delta + n^* - d \end{pmatrix} \tag{97}$$

with:

$$\phi_k^n = \frac{\alpha\zeta (n^*)^{1+\epsilon}}{\nu\epsilon k^* c^*} \left(\beta k^* - ((1+b)k^* + b_0)(1-\alpha)\right)$$
$$\phi_c^n = \frac{(1+b)(n^*)^{1+\epsilon} - \nu\rho n^*}{\nu\epsilon c^*}$$
$$\phi_e^n = \frac{\beta\zeta}{\nu\epsilon} \frac{(n^*)^{1+\epsilon}}{c^* \hat{e}^*} \left(\beta k^* + ((1+b)k^* + b_0)\alpha - (1+b)k^*\right)$$

$$\phi_k^c = -\alpha(1-\alpha)(k^*)^{\alpha-2} \hat{(e^*)}^{\beta} c^*$$

$$\phi_k^e = \frac{\hat{e}^*}{k^*} \frac{\alpha}{(1-\beta)} \left[(1-\alpha)(k^*)^{\alpha-1} \hat{e}^{\beta} - \delta - x\right]$$
$$\phi_n^e = \frac{-\hat{e}^*}{(1-\beta)k^*} \left[\alpha \left[(1+b)k^* + b_0\right] - bk^*\right]$$
$$\phi_c^e = \frac{\hat{e}^*}{1-\beta} \left[\frac{\rho+\delta+n^*-d}{c^*} - \frac{\alpha}{k^*}\right]$$

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