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Abstract: We analyze the optimal dynamic scale and structure of a two-sector-economy, where each sector produces one consumption good and one specific pollutant. Both pollutants accumulate at different rates to stocks that damage the natural environment, which acts as a dynamic driving force for the economy. Our analysis shows that along the optimal time-path (i) the overall scale of economic activity may be less than maximal, (ii) the time-scale of structural change is mainly determined by the longer-lived pollutant, (iii) the optimal control of emissions may be non-monotonic. In particular the last result raises important questions about the design of optimal environmental policies.

Keywords: dynamic economy-environment interaction, multi-pollutant emissions, non-monotonic control, optimal scale, stock pollution, structural change, time scales

JEL-classification: Q20, O10, O41

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1 Introduction

The natural environment is being damaged by the stocks of various pollutants, which are produced in different sectors of the economy, accumulate according to different dynamic relationships and damage different environmental goods. As an example, think of the two economic sectors "agriculture" and "industry". Nitrate and pesticide run-off from agricultural cultivation accumulates in the groundwater and decreases its quality as drinking water (UNEP 2002), and carbon dioxide emissions from fossil fuel combustion in the industrial sector accumulate in the atmosphere and contribute to the problem of global climate change (IPCC 2001). In general, the different pollutants exhibit different internal dynamics, because their respective natural degradation processes differ. This has dynamic repercussions on the evolution of the economy.

In this paper, we study the mutual interaction over time between the structure and scale of economic activity on the one hand, and the dynamics of a complex natural environment on the other hand. For that sake we analyze a model which comprises two economic sectors, each of which produces one consumption good and, at the same time, gives rise to one specific pollutant. Both pollutants accumulate to stocks which display different internal dynamics, in the sense that the respective natural deterioration rates differ, and damage different environmental goods. This amounts to a *total* analysis of economy-environment interactions, which goes beyond the traditional *partial* equilibrium approach of many contributions to environmental economics.

To some extent, our interest in this issue was inspired by Daly's (1992, 1996, 1999) discussion of the concept of an optimal biophysical scale of the aggregate economy relative to the surrounding natural environment, and of the relation between sustainable scale, efficient allocation and fair distribution of an economy. We will address these issues here for one particular instance of economy-environment-interaction, namely emissions and stocks of pollution. In particular, we are interested in the following questions: Under which conditions does a stationary scale and structure of the economy exist? More generally, how does the scale and structure of economic activity change over time? Is this dynamic process monotonic? What is the time-scale of structural change and how is it influenced by the different time-scales and constraints of the economic and environmental systems?

These questions are only sparsely examined in the economic literature. Many studies assume that it is the *flow* of emissions which causes environmental problems. This neglects stock accumulation and, thus, an essential dynamic environmental constraint on economic action. *Stock pollution* has been taken into account by some authors, e.g. by Falk and Mendelsohn (1993), Forster (1973), Luptacik and Schubert (1982) or Van der Ploeg and Withagen (1991). This is usually done on a highly aggregated level, such that only one pollutant is taken into account. The case of *several stock pollutants*, which all contribute to climate change, has been studied by Michaelis (1992, 1997, 1999). He is interested in finding cost-effective climate policy measures in the multi-pollution case for a given structure of the economy and does not explicitly consider the dynamics of the production side of the economy. Moslener and Requate (2001) challenge the global warming potential as a useful indicator when there are many greenhouse gases with different dynamic characteristics. Faber and Proops (1998: chap. 11) or Keeler, Spence and Zeckhauser (1971) explicitly study the dynamics of *different production sectors with pollution*, assuming one single pollutant. Winkler (2002) analyzes optimal structural change of a two-sector economy characterized by two stock quantities: the capital stock and the stock of a pollutant which is emitted from the more capital-intense sector. Baumgärtner and Jöst (2000) study the optimal (static) structure of a two-sector economy where each sector produces a specific emission. Aaheim (1999) studies a model similar to ours. He analyzes optimal growth of a two-sector economy which gives rise to three different stock pollutants and which is constrained by a policy target concerning the aggregate level of pollution.

In an optimal control framework we explicitly determine the optimal scale and structure over time of a multi-pollution economy, using a linear approximation around the steady-state. Our analysis shows that along the optimal time-path (i) the overall scale of economic activity may be less than maximal, (ii) the time-scale of structural change is mainly determined by the longer-lived pollutant, (iii) the control of emissions may be non-monotonic. In particular the last result raises important questions about the design of optimal environmental policies.

Although our modelling approach is inspired by Ramsey-type optimal growth models, which have previously been used to study steady state growth with environmental pollution (e.g. by Keeler, Spence and Zeckhauser 1971, Plourde 1972, Smith 1977, Van der Ploeg and Withagen 1991, Gradus and Smulders 1993, 1996, Siebert 1998: chaps. 15, 16) we are essentially concerned with the issue of structural change. Therefore, in this paper we do not focus on steady states but analyze the explicit time-dependence of the solution. Furthermore, we study an economy without any potential for growth, as this allows us to focus on the structural-change-effect, which may be obscured by growth effects otherwise. The sole genuine generator of dynamics in our model is the accumulation of pollutant stocks in the natural environment.

The paper is organized as follows. In section 2 we present the model. Section 3 is devoted to a formal analysis of the optimal dynamic scale and structure of the economy. Section 4 illustrates the formal results by numerical simulations. Section 5 concludes.

2 The model

We study a two sector economy with one scarce non-accumable factor of production, say labor, two consumption goods, and two pollutants that accumulate to stocks. Welfare is determined by the amounts consumed of both consumption goods as well as by the environmental damage caused by the two stocks of pollutants.

The production of consumption goods in sectors 1 and 2 of the economy is described by two production functions, $y_i = P^i(l_i)$ for $i = 1, 2$, where l_i denotes the amount of the primary factor allocated in sector i. With index l denoting derivatives with respect to the sole argument l_i, $P_l^i \equiv dP^i/dl_i$ and $P_{ll}^i \equiv d^2P^i/dl_i^2$, the production functions are assumed to exhibit the following standard properties:

$$P^i(0) = 0 \ , \quad P_l^i > 0 \ , \quad \lim_{l_i \to 0} P_l^i = +\infty \ , \quad P_{ll}^i < 0 \qquad (i = 1, 2) \ . \tag{1}$$

That is, the production functions are assumed to be characterized by a positive and decreasing marginal product of the sole factor of production, labor. Labor is essential for production, i.e. with vanishing labor input output vanishes also, and with input going to infinity output goes to infinity as well. In addition, the marginal product approaches infinity as the factor input goes to zero. The latter three assumptions are purely technical which will later guarantee the existence of a unique solution.

Since we want to analyze a non-growing economy, we consider the case of a fixed supply of labor. With $\lambda > 0$ as the total available amount of labor in the economy, consumption possibilities are described by

$$y_1 = P^1(l_1) \ , \tag{2}$$

$$y_2 = P^2(l_2) \ , \tag{3}$$

$$l_1 + l_2 \leq \lambda \ . \tag{4}$$

In addition to the consumption good, each sector yields a pollutant which comes as a joint output in fixed proportion to the desired output. That is, there is no abatement technology:

$$e_i = \alpha_i y_i \qquad \text{with } \alpha_i > 0 \qquad (i = 1, 2) \ . \tag{5}$$

Without loss of generality we set $\alpha_i = 1$ $(i = 1, 2)$. This is just a choice of scale for the emissions and the stocks of pollutants and does not alter any results. Both flows of pollutants, e_1 and e_2, add to the respective stock of the pollutant, which deteriorates at the constant rate δ_i:

$$\dot{s}_i = e_i - \delta_i s_i \qquad \text{with } 0 < \delta_i < 1 \qquad (i = 1, 2) \ . \tag{6}$$

Social instantaneous welfare V depends on consumption of both goods, y_1 and y_2, and on the damage to environmental quality which hinges upon the stocks of pollutants s_1 and s_2. We assume that the welfare function be quasi-linear:

$$V(y_1, y_2, s_1, s_2) = U(y_1, y_1) - S(s_1, s_2) \ , \tag{7}$$

where $U = U(y_1, y_2)$ represents welfare gains due to consumption. The quasi-linearity of the welfare function amounts to the assumption that welfare losses due to environmental damage $S = S(s_1, s_2)$ are not too important compared with welfare gains due to increased consumption.

The function U is assumed to exhibit the usual property of positive and decreasing marginal welfare in both consumption goods. For reasons of tractability we assume that neither consumption good influences marginal welfare of the other. With index i denoting partial derivative with respect to argument y_i, i.e. $U_i \equiv \partial U / \partial y_i$ and $U_{ij} \equiv \partial U / \partial y_i \partial y_j$ with $i, j = 1, 2$, the assumptions are:

$$U_i > 0 \ , \quad \lim_{y_i \to 0} U_i = +\infty \ , \quad U_{ii} < 0 \ , \quad U_{ij} = 0 \qquad (i, j = 1, 2 \text{ and } i \neq j) \ . \quad (8)$$

The stocks of pollutants are assumed to exert an increasing marginal damage in such a manner that the total welfare effect S is the weighted sum of the effects of the stocks of both pollutants:

$$S = \frac{\sigma_1}{2} s_1^2 + \frac{\sigma_2}{2} s_2^2 \qquad \text{with } \sigma_1, \sigma_2 > 0 \ . \tag{9}$$

Hence, the damage function S exhibits the following properties (index i denotes partial derivative with respect to argument s_i, i.e. $S_i \equiv \partial S / \partial s_i$ and $S_{ij} \equiv \partial S / \partial s_i \partial s_j$ with $i, j = 1, 2$):

$$S_i > 0 \ , \quad S_{ii} > 0 \ , \quad S_{ij} = 0 \qquad (i, j = 1, 2 \text{ and } i \neq j) \ . \tag{10}$$

The assumption $S_{ij} = 0$ means that the different pollutant stocks decrease welfare independently. This is plausible if they damage different environmental goods. For instance, nitrates and pesticides from the agricultural sector contaminate the groundwater and decrease its quality as drinking water (UNEP 2002), while carbon dioxide emissions from fossil fuel combustion in the industrial sector contribute to global climate change (IPCC 2001). As a consequence, the welfare effect of one additional unit of one of these pollutants does not depend on the amount of the other. Note that the overall welfare function V is strictly concave and additively separable (because of $U_{ij} = S_{ij} = 0$) in all four variables (y_1, y_2, s_1, s_2).

Since we are interested in studying questions related to the scale as well as the structure of activity in this economy, we introduce new dimensionless variables in the following way:

$$c = \frac{l_1 + l_2}{\lambda} \qquad \text{and} \qquad x = \frac{l_1}{l_1 + l_2} \ . \tag{11}$$

The variable c stands for the *scale* of economic activity. It may take values between 0 and 1 and indicates what fraction of the total available amount of labor is devoted to economic activity. The remaining fraction $1 - c$ is left idle. The variable x stands for the *structure* of economic activity. It indicates what fraction of the total labor employed in production, $l_1 + l_2$, is allocated to sector 1. The remaining fraction $1 - x$ of labor employed in production is allocated to sector 2. x may take values between 0 (all labor in production allocated to sector 2) and 1 (all labor in production allocated to sector 1). The variables l_1 and l_2 can then be expressed in terms of c and x:

$$l_1 = l_1(c, x) = cx\lambda \qquad \text{and} \qquad l_2 = l_2(c, x) = c(1 - x)\lambda \ .$$

This allows us to replace l_1 and l_2 from the problem. For notational convenience, we introduce new production functions F^i which depend directly on c and x, and which are defined in the following way:

$$F^i(c, x) \equiv P^i(l_i(c, x)) \qquad \text{for all } c, \ x \ . \tag{12}$$

From (1) and (12) one obtains that the F^i have the following properties:

$$F_c^1 = xP_l^1\lambda > 0 \ , \qquad \lim_{c\to 0} F_c^1(x\neq 0) = +\infty \ , \tag{13}$$

$$F_x^1 = cP_l^1\lambda > 0 \ , \tag{14}$$

$$F_c^2 = (1-x)P_l^2\lambda > 0 \ , \qquad \lim_{c\to 0} F_x^1(x\neq 1) = +\infty \ , \tag{15}$$

$$F_x^2 = -cP_l^2\lambda < 0 \ . \tag{16}$$

3 Optimal scale and structure of the economy

3.1 Intertemporal optimization

We maximize the discounted intertemporal welfare over c and x,

$$\int_0^\infty [U(y_1,y_2) - S(s_1,s_2)]e^{-\rho t}dt \ , \tag{17}$$

where ρ denotes the discount rate and $y_i = F^i(c,x)$ $(i,j=1,2)$, subject to the dynamic constraints for the state variables s_1 and s_2 that are given by equations (6):

$$\dot{s}_i = F^i(c,x) - \delta_i s_i \qquad \text{with } \ 0<\delta_i<1 \qquad (i=1,2) \ . \tag{18}$$

In addition, the following restrictions for the control variables c and x hold:

$$0 \le c \le 1 \qquad 0 \le x \le 1 \ . \tag{19}$$

Corner solutions with $x=0$ or $x=1$ cannot be an optimal outcome since either case would imply, due to assumptions (1) and (8), that marginal utility of one consumption good would go to infinity while marginal utility of the other good would remain finite. Similarly, a corner solution with $c=0$ cannot be an optimal outcome since in that case marginal utility of both consumption goods would go to infinity while marginal damage from environmental pollution would remain finite. Hence, the only remaining potential corner solution of the restrictions (19), which we have to control for explicitly, is:

$$c \le 1 \ . \tag{20}$$

We introduce two costate variables, p_1 and p_2, and a Kuhn-Tucker parameter, p_c. The current value Hamiltonian of the problem then reads

$$\begin{aligned} \mathcal{H}(c,x,s_1,s_2;p_1,p_2,p_c) &= U(F^1(c,x),F^2(c,x)) - \left(\frac{\sigma_1}{2}s_1^2 + \frac{\sigma_2}{2}s_2^2\right) \\ &\quad + p_1\left[F^1(c,x) - \delta_1 s_1\right] \\ &\quad + p_2\left[F^2(c,x) - \delta_2 s_2\right] \\ &\quad + p_c\left[1-c\right] \ . \end{aligned} \tag{21}$$

Since both control variables, c and x, are always strictly positive, the two state variables, s_1 and s_2, are always nonnegative and the Hamiltonian $\mathcal{H}$ is continuously differentiable with respect to c and x, the first order conditions of the control problem are:

$$
\begin{aligned}
U_1 F_c^1 \; + \; U_2 F_c^2 \; + \; p_1 F_c^1 \; + \; p_2 F_c^2 \; - \; p_c \; &= \; 0 \; , && (22)\\
U_1 F_x^1 \; + \; U_2 F_x^2 \; + \; p_1 F_x^1 \; + \; p_2 F_x^2 \; &= \; 0 \; , && (23)\\
\sigma_1 s_1 \; + \; (\delta_1 + \rho) p_1 \; &= \; \dot{p}_1 \; , && (24)\\
\sigma_2 s_2 \; + \; (\delta_2 + \rho) p_2 \; &= \; \dot{p}_2 \; , && (25)\\
p_c \geq 0 \; , \; p_c (1 - c) \; &= \; 0 \; , && (26)
\end{aligned}
$$

plus the dynamic constraints (18) and the restriction (20). Because of the concavity of the optimized Hamiltonian $\mathcal{H}$ (see appendix A.1) these necessary conditions are also sufficient if, in addition, the transversality conditions

$$\lim_{t \to \infty} p_i(t)\, e^{-\rho t} \cdot s_i(t) = 0 \qquad (i = 1, 2) \; , \tag{27}$$

hold. Note that the optimal path is also unique due to the strict concavity of the optimized Hamiltonian.

3.2 Stationary state

Setting $\dot{p}_1 = 0$, $\dot{p}_2 = 0$, $\dot{s}_1 = 0$ and $\dot{s}_2 = 0$ in the system of first order conditions (18), (20) and (22)–(25) yields the necessary and sufficient conditions for an optimal stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$, in which neither the scale nor the structure of economic activity nor the stocks of pollution accumulated in the environment change over time. From conditions (24) and (25) we derive for the costate variables p_i $(i = 1, 2)$:

$$p_i = -\frac{\sigma_i s_i}{\delta_i + \rho} \qquad (i = 1, 2) \; . \tag{28}$$

Inserting (28) in (22) and (23) yields the necessary and sufficient conditions for an optimal stationary state:

$$
\begin{aligned}
\left(U_1 - \frac{\sigma_1 s_1}{\delta_1 + \rho} \right) F_c^1 \; + \; \left(U_2 - \frac{\sigma_2 s_2}{\delta_2 + \rho} \right) F_c^2 \; - p_c \; &= \; 0 \; , && (29)\\
\left(U_1 - \frac{\sigma_1 s_1}{\delta_1 + \rho} \right) F_x^1 \; + \; \left(U_2 - \frac{\sigma_2 s_2}{\delta_2 + \rho} \right) F_x^2 \; &= \; 0 \; . && (30)
\end{aligned}
$$

From the signs of the F_j^i $(i = 1, 2$ and $j = c, x)$ and p_c stated in (13)–(16) and (26), it follows that:

$$U_i(F^1(c^\star, x^\star), F^2(c^\star, x^\star)) \geq \frac{\sigma_i s_i^\star}{\delta_i + \rho} \qquad (i = 1, 2) \; , \tag{31}$$

where the ">" sign indicates a corner solution ($c^\star = 1$), and

$$s_i^\star = \frac{F^i(c^\star, x^\star)}{\delta_i} = const. \qquad (i = 1, 2) \; . \tag{32}$$

The interpretation of the two conditions (31) is that in an interior (corner) optimal stationary state the scale and structure of economic activity are such that for each sector the marginal welfare gain due to consumption of that sector's output equals (is greater than) the aggregate future marginal damage from that sector's current emission which comes as an inevitable by-product with the consumption good.[2]

An optimal stationary state exists if the system (31)–(32) of four equations for the four unknowns $(c^\star, x^\star, s_1^\star, s_2^\star)$ has a solution with $0 < c^\star \leq 1$ and $0 < x^\star < 1$. In the following, we shall concentrate on the case of an interior stationary state with $c^\star < 1$. Hence, we assume that the total labor amount λ exceeds $\bar{\lambda}$ as given in the following proposition.

Proposition 1:
An interior optimal stationary state of the economy exists if the total available amount of labor λ in the economy is greater than some threshold value $\bar{\lambda} = \bar{l}_1 + \bar{l}_2$, where the $\bar{l}_i$ are specified by the following implicit equations:

$$U_i(P^1(\bar{l}_1), P^2(\bar{l}_2)) = \frac{\sigma_i P^i(\bar{l}_i)}{\delta_i^2 + \delta_i \rho} \qquad (i = 1, 2) \ .$$

Proof: *In the appendix.*

Note that the interior optimal stationary state is also unique due to the strict concavity of the maximized Hamiltonian (appendix A.1).

In order to study the properties of the interior optimal stationary state $(c^\star, x^\star)$ some comparative statics can be done with the conditions (31). The results are stated in the following proposition.

Proposition 2:
The interior optimal stationary state has the following properties:

$$\begin{aligned}
&\frac{dc^\star}{d\delta_1} > 0 \ , \quad && \frac{dx^\star}{d\delta_1} > 0 \ , \\
&\frac{dc^\star}{d\delta_2} > 0 \ , \quad && \frac{dx^\star}{d\delta_2} < 0 \ , \\
&\frac{dc^\star}{d\lambda} < 0 \ , \quad && \frac{dx^\star}{d\lambda} = 0 \ , \\
&\frac{dc^\star}{d\rho} > 0 \ , \quad && \frac{dx^\star}{d\rho} \gtreqless 0 \quad \textit{for} \quad \frac{U_{22} F_x^2}{U_{11} F_x^1} \gtrless \frac{\sigma_2/(\rho+\delta_2)^2}{\sigma_1/(\rho+\delta_2)^2} \ .
\end{aligned}$$

Proof: *In the appendix.*

These results can be interpreted as follows. The higher the natural deterioration rate δ_i of pollutant i $(i = 1, 2)$, the higher – ceteris paribus – the optimal stationary scale of economic activity, $c^\star$, and the more labor is employed in sector i relative

[2] Note that taking account of discounting and natural degradation of the respective pollution stock the net present value of the accumulated damage of one marginal unit of pollution sums up to the right hand side of (31), as $\int_0^\infty \sigma_i s_i^\star e^{-(\rho+\delta_i)t} dt = \frac{\sigma_i s_i^\star}{\rho+\delta_i}$ $(i = 1, 2)$.

to the other sector. The latter is expressed by a higher value of $x^\star$ for sector 1 and a lower value of $x^\star$ for sector 2. An increase in the labor supply λ of the economy does not affect the optimal stationary structure of economic activity, $x^\star$. Yet, it decreases the optimal stationary value of $c^\star$. Note that this result is due to the circumstance that the *absolute* optimal stationary scale of economic activity, $l_1^\star + l_2^\star$, remains unaffected by an increase of λ and, therefore, the *relative* value $c^\star = (l_1^\star + l_2^\star)/\lambda$ decreases. Finally, an increase in the discount rate ρ increases the optimal stationary scale of economic activity, $c^\star$, while its effect on the optimal stationary structure of economic activity, $x^\star$, is ambiguous. It depends on the specification of the welfare and production functions as well as on the environmental parameter values.

3.3 Optimal dynamic path and local stability analysis

In the following we solve the optimization problem by linearizing the resulting system of differential equations around the stationary state. As we have assumed an interior stationary state, the optimal path will also be an interior optimal path at least in a neighborhood of the interior stationary state. Hence, we restrict the analysis to the case of an interior solution, i.e. $c^\star < 1$. As shown in the appendix, the optimal dynamics of the two control variables c, x and the two state variables s_1, s_2 can be described by a system of four coupled first order autonomous differential equations:

$$\begin{aligned} \dot{c} &= \Gamma(c, x, s_1, s_2) \ , && (33) \\ \dot{x} &= \Xi(c, x, s_1, s_2) \ , && (34) \\ \dot{s}_1 &= \Sigma^1(c, x, s_1) \ , && (35) \\ \dot{s}_2 &= \Sigma^2(c, x, s_2) \ . && (36) \end{aligned}$$

Linearizing around the stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$ yields the following approximated dynamic system:

$$\begin{pmatrix} \dot{c} \\ \dot{x} \\ \dot{s}_1 \\ \dot{s}_2 \end{pmatrix} \approx J^\star \begin{pmatrix} c - c^\star \\ x - x^\star \\ s_1 - s_1^\star \\ s_2 - s_2^\star \end{pmatrix} \quad \text{with} \quad J^\star = \begin{pmatrix} \Gamma_c & \Gamma_x & \Gamma_{s_1} & \Gamma_{s_2} \\ \Xi_c & \Xi_x & \Xi_{s_1} & \Xi_{s_2} \\ \Sigma_c^1 & \Sigma_x^1 & \Sigma_{s_1}^1 & \Sigma_{s_2}^1 \\ \Sigma_c^2 & \Sigma_x^2 & \Sigma_{s_1}^2 & \Sigma_{s_2}^2 \end{pmatrix} \qquad (37)$$

where $J^\star$ is the Jacobian evaluated at the stationary state. In general, $J^\star$ has two negative eigenvalues (ν_1, ν_2) and two positive eigenvalues (ν_3, ν_4). Hence, the system dynamics exhibits saddlepoint stability, i.e. for all initial stocks of pollutants, s_1^0 and s_2^0, there exists a unique optimal path converging asymptotically towards the stationary state. Because of the transversality conditions (27) the optimal path is restricted to the stable hyperplane, which is spanned by the eigenvectors associated with the negative eigenvalues. Given the eigenvalues and the eigenvectors, which are calculated in the appendix, the explicit system dynamics in a neighborhood around the stationary state is given by:

$$c = c^\star + (s_1^0 - s_1^\star) \frac{\bar{F}_x^2(\nu_1 + \delta_1)}{\bar{F}_c^1 \bar{F}_x^2 - \bar{F}_x^1 \bar{F}_c^2} e^{\nu_1 t} - (s_2^0 - s_2^\star) \frac{\bar{F}_x^1(\nu_2 + \delta_2)}{\bar{F}_c^1 \bar{F}_x^2 - \bar{F}_x^1 \bar{F}_c^2} e^{\nu_2 t} \ , \qquad (38)$$

$$x = x^\star - (s_1^0 - s_1^\star)\frac{\bar{F}_c^2(\nu_1+\delta_1)}{\bar{F}_c^1\bar{F}_x^2 - \bar{F}_x^1\bar{F}_c^2}\, e^{\nu_1 t} + (s_2^0 - s_2^\star)\frac{\bar{F}_c^1(\nu_2+\delta_2)}{\bar{F}_c^1\bar{F}_x^2 - \bar{F}_x^1\bar{F}_c^2}\, e^{\nu_2 t} , \quad (39)$$

$$s_1 = s_1^\star + (s_1^0 - s_1^\star)\, e^{\nu_1 t} , \quad (40)$$

$$s_2 = s_2^\star + (s_2^0 - s_2^\star)\, e^{\nu_2 t} , \quad (41)$$

where $s_1^0 = s_1(0)$ and $s_2^0 = s_2(0)$ denote the initial stocks of pollutants and $\bar{F}_j^i = F_j^i(c^\star, x^\star)$ the functions F_j^i ($i = 1,2$ and $j = c, x$) evaluated at the stationary state.

The equations (38)–(41) imply that both the control and the state variables converge exponentially towards their stationary state values with the rates ν_1 and ν_2 in a neighborhood around the stationary state. Hence, the time-scale of convergence, as a measure for the overall speed of convergence, is given by the eigenvalue with the smaller absolute value, $\min[|\nu_1|, |\nu_2|]$. If $4\sigma_i/U_{ii}(\bar{F}^1, \bar{F}^2)$ is small compared with $(\rho + 2\delta_i)^2$ for $(i = 1,2)$ – i.e. if harmfulness is small, marginal welfare from consumption is strongly increasing at the stationary state consumption level or the discount and deterioration rates are large – the eigenvalues ν_1 (A.25) and ν_2 (A.26) can be approximated by a Taylor series. The eigenvalues then read:

$$\nu_1 \approx -\delta_1 + \frac{\sigma_1}{U_{11}(\bar{F}^1, \bar{F}^2)(\rho + 2\delta_1)} , \quad (42)$$

$$\nu_2 \approx -\delta_2 + \frac{\sigma_2}{U_{22}(\bar{F}^1, \bar{F}^2)(\rho + 2\delta_2)} . \quad (43)$$

Hence, the time-scale of convergence is mainly determined by the deterioration rate of the longer-lived pollutant. Proposition 3 summarizes these results.

Proposition 3:
For the linear approximation (37) around the stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$ the following statements hold:

(i) The explicit system dynamics is given by equations (38)–(41).

(ii) The stationary state is saddlepoint-stable.

(iii) The time-scale of convergence towards the stationary state is given by the eigenvalue with the smaller absolute value, $\min[|\nu_1|, |\nu_2|]$. *If $4\sigma_i/U_{ii}(c^\star, x^\star)$ is small compared with $(\rho + 2\delta_i)^2$ $(i = 1, 2)$, it is mainly the natural deterioration rate of the longer-lived pollutant which determines the time-scale of convergence.*

We now turn to the qualitative behavior of the optimal path in a neighborhood around the stationary state. According to equations (40) and (41) the stocks of the two pollutants converge monotonically towards their stationary state values $s_1^\star$ and $s_2^\star$. In order to show that the optimal paths for the control variables c and x may be non-monotonic we differentiate equations (38) and (39) with respect to t:

$$\dot{c} = \nu_1(s_1^0 - s_1^\star)\frac{\bar{F}_x^2(\nu_1+\delta_1)}{\bar{F}_c^1\bar{F}_x^2 - \bar{F}_x^1\bar{F}_c^2}\, e^{\nu_1 t} - \nu_2(s_2^0 - s_2^\star)\frac{\bar{F}_x^1(\nu_2+\delta_2)}{\bar{F}_c^1\bar{F}_x^2 - \bar{F}_x^1\bar{F}_c^2}\, e^{\nu_2 t} , \quad (44)$$

$$\dot{x} = -\nu_1(s_1^0 - s_1^\star)\frac{\bar{F}_c^2(\nu_1+\delta_1)}{\bar{F}_c^1\bar{F}_x^2 - \bar{F}_x^1\bar{F}_c^2}\, e^{\nu_1 t} + \nu_2(s_2^0 - s_2^\star)\frac{\bar{F}_c^1(\nu_2+\delta_2)}{\bar{F}_c^1\bar{F}_x^2 - \bar{F}_x^1\bar{F}_c^2}\, e^{\nu_2 t} . \quad (45)$$

The optimal path is non-monotonic if $\dot{c}$ or $\dot{x}$ change their sign, i.e. if the paths $c(t)$ or $x(t)$ exhibit a local extremum for positive times t. According to the signs of the ν_i and F_j^i ($i = 1, 2$ and $j = c, x$) and given that $\nu_1 \neq \nu_2$, $c(t)$ exhibits a unique local extremum if $\text{sgn}(s_1^0 - s_1^\star) \neq \text{sgn}(s_2^0 - s_2^\star)$, and $x(t)$ exhibits a unique local extremum if $\text{sgn}(s_1^0 - s_1^\star) = \text{sgn}(s_2^0 - s_2^\star)$.[3] Note that the local extremum of either $c(t)$ or $x(t)$ can occur at negative times t which is meaningless in the context of this analysis. In this case we would observe monotonic optimal paths for both control variables c and x for times $t > 0$.

Solving $\dot{c}(t) = 0$ and $\dot{x}(t) = 0$ for t, using expressions (44) and (45) for $\dot{c}$ and $\dot{x}$, yields:

$$\hat{t} = \begin{cases} \ln\left[\frac{\nu_2(s_2^0 - s_2^\star)\bar{F}_x^1(\nu_2+\delta_2)}{\nu_1(s_1^0 - s_1^\star)\bar{F}_x^2(\nu_1+\delta_1)}\right](\nu_1 - \nu_2)^{-1}\ , & \text{if} \quad \text{sgn}(s_1^0 - s_1^\star) \neq \text{sgn}(s_2^0 - s_2^\star) \\ \ln\left[\frac{\nu_2(s_2^0 - s_2^\star)\bar{F}_c^1(\nu_2+\delta_2)}{\nu_1(s_1^0 - s_1^\star)\bar{F}_c^2(\nu_1+\delta_1)}\right](\nu_1 - \nu_2)^{-1}\ , & \text{if} \quad \text{sgn}(s_1^0 - s_1^\star) = \text{sgn}(s_2^0 - s_2^\star) \end{cases} . \quad (46)$$

The following properties summarizes the qualitative behavior of the optimal control path.

Proposition 4:
For the qualitative behavior of the optimal path for the linear approximation (37) around the stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$ the following statements hold:

(i) The stocks of pollutants $s_1(t)$ and $s_2(t)$ converge exponentially, i.e. monotonically, towards their stationary state values $s_1^\star$ and $s_2^\star$.

(ii) If $\hat{t}$ as given by equation (46) is positive, then the optimal path is non-monotonic, where $\hat{t}$ denotes the time at which the optimal path has a local extremum. In particular, if $sgn(s_1^0 - s_1^\star) \neq sgn(s_2^0 - s_2^\star)$, the optimal path for $c(t)$ is non-monotonic and the optimal path for $x(t)$ is monotonic. If $sgn(s_1^0 - s_1^\star) = sgn(s_2^0 - s_2^\star)$, the optimal path for $x(t)$ is non-monotonic and the optimal path for $c(t)$ is monotonic.

4 Numerical simulation

In this section we illustrate the results derived in section 3 by numerical simulations of the original, non linearized optimization problem (17)–(19). All numerical optimizations were derived with the advanced optimal control software package MUSCOD-II (Diehl et al. 2001), which exploits the multiple shooting state discretization (Leineweber et al. 2003).

There are four different qualitative scenarios which have to be examined. First, both stocks of pollutants exhibit the same harmfulness but differ in their deterioration rates, i.e. $\sigma_1 = \sigma_2$, $\delta_1 < \delta_2$. Second, the two pollutants differ in their harmfulness but have equal deterioration rates, i.e. $\sigma_1 < \sigma_2$, $\delta_1 = \delta_2$. Third, the pollutants differ in both harmfulness and deterioration rates and the more harmful pollutant has the higher deterioration rate, i.e. $\sigma_1 < \sigma_2$, $\delta_1 < \delta_2$. Fourth, both

[3] Note that $\nu_i + \delta_i < 0$, which can easily be verified from equations (A.25) and (A.26).

harmfulness and deterioration rates are different but the more harmful pollutant has a lower deterioration rate, i.e. $\sigma_1 < \sigma_2$, $\delta_1 > \delta_2$. Furthermore, each of the four scenarios splits in four subcases depending on the initial stocks of pollutants (both initial stocks below, only first stock above, only second stock above and both stocks above the stationary state values).

In the following we discuss these four different scenarios. The parameter values used for the numerical optimization have primarily been chosen in such a way as to clearly illustrate the different effects, and do not necessarily reflect the characteristics of real environmental pollution problems. For all numerical examples, the total labor supply λ has been chosen such as to guarantee an interior stationary state scale $c^\star < 1$. As it is not possible to optimize numerically over an infinite time horizon, the time horizon has been set to 250 years and all parameters have been chosen in such a way that the system at time $t = 250$ is very close to the stationary state (for a more convenient exposition, the figures show just times up to $t = 125$). The parameter values for the numerical optimization are listed in the appendix.

In the first scenario ($\sigma_1 = \sigma_2$), both stocks of pollutants exhibit the same harmfulness but the deterioration rate is smaller for the first pollutant than for the second. Figure 1 shows the result of a numerical optimization of this case. In this example the initial stocks for both pollutants are above their stationary state values ($s_1^0 = 30, s_2^0 = 30$). The optimal path for the structure exhibits a non monotonic behavior as expected from proposition 4. Further, we expect that the optimal stationary state structure $x^\star$ is clearly below 0.5, indicating that relatively more labor is employed in the second sector, because as the second stock of pollutant deteriorates at a higher rate the aggregate intertemporal damage of one unit of emissions is smaller for the second pollutant.[4] This expectation is confirmed by the simulation.

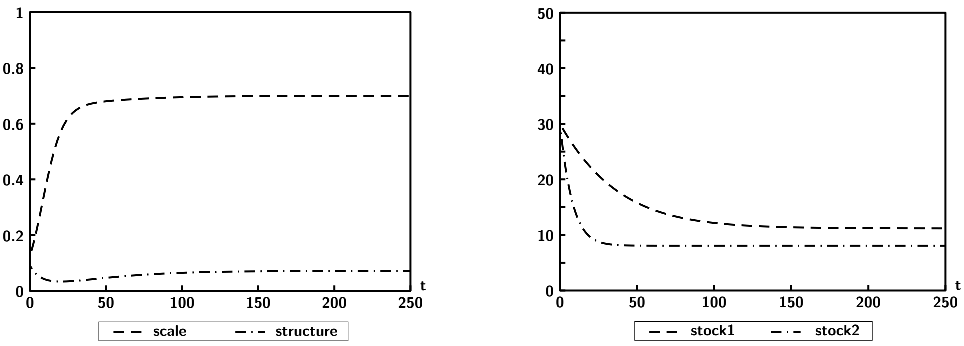

Figure 1: Optimal paths for scale and structure (left) and the two stocks of pollutants (right) for the case $\sigma_1 = \sigma_2$, $\delta_1 < \delta_2$. Numerical optimization parameters are given in the appendix.

In the second scenario ($\sigma_1 < \sigma_2, \delta_1 = \delta_2$), the two stocks of pollutants are of

[4]Note that both consumption goods are equally valued by the representative consumer, i.e. $\mu_1 = \mu_2$ (see appendix).

different harmfulness but the deterioration rate for the two pollutants are equal. The result of a numerical optimization of this case is presented in figure 2. In this example the initial stock for the first (second) pollutant is above (below) their stationary state values ($s_1 = 40, s_2 = 0$). Now, the optimal path for the scale exhibits a non monotonic behavior as expected from proposition 4. Further, we expect that the optimal stationary state structure $x^\star$ is clearly above 0.5, indicating that relatively more labor is employed by the second sector, because as the second stock of pollutant is less harmful the aggregate intertemporal damage of one unit of emissions is smaller for the second pollutant. This expectation is confirmed by the simulation.

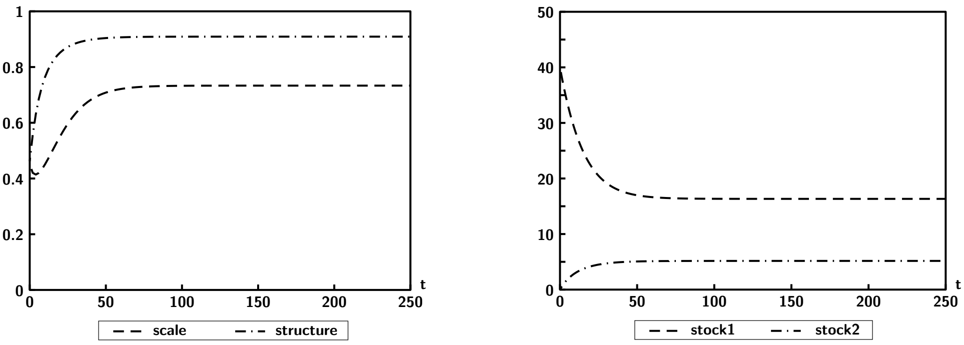

Figure 2: Optimal paths for scale and structure (left) and the two stocks of pollutants (right) for the case $\sigma_1 < \sigma_2$, $\delta_1 = \delta_2$. Numerical optimization parameters are given in the appendix.

The third scenario ($\sigma_1 < \sigma_2, \delta_1 < \delta_2$) – both harmfulness and deterioration rates are different and the more harmful pollutant has the higher deterioration rate – is the most interesting as neither of the two pollutants exhibits a priori more favorable dynamic characteristics for the economy. Hence, we are not able to predict which production sector will be used to a greater extent in the stationary state. Furthermore, non monotonic paths – if they occur – are likely to be more pronounced than in the other cases. Figure 3 shows the optimal paths for a numerical example for all four subcases (initial pollutant stocks above or below stationray state levels for one and both pollutants). Of course, the long run stationary state to which the economy converges, is the same in all four subscenarios, as all parameters are identical except for the initial stocks of the two pollutants. Nevertheless, the optimal paths and especially their convergence towards the stationary state is quite different for the four subcases. As expected from proposition 4, we observe that – if at all – the optimal path for the structure is non-monotonic if both stocks start above or below their stationary state values (subcases a and d) and the optimal path for the scale is non-monotonic if one initial stock is higher and one is lower than their stationary state values (subcase b). We also see that both, structure and scale, may exhibit monotonic optimal paths (subcase c).

In the fourth scenario ($\sigma_1 < \sigma_2, \delta_1 > \delta_2$), where both pollutants exhibit different harmfulness and deterioration rates but the second pollutant is more harmful and

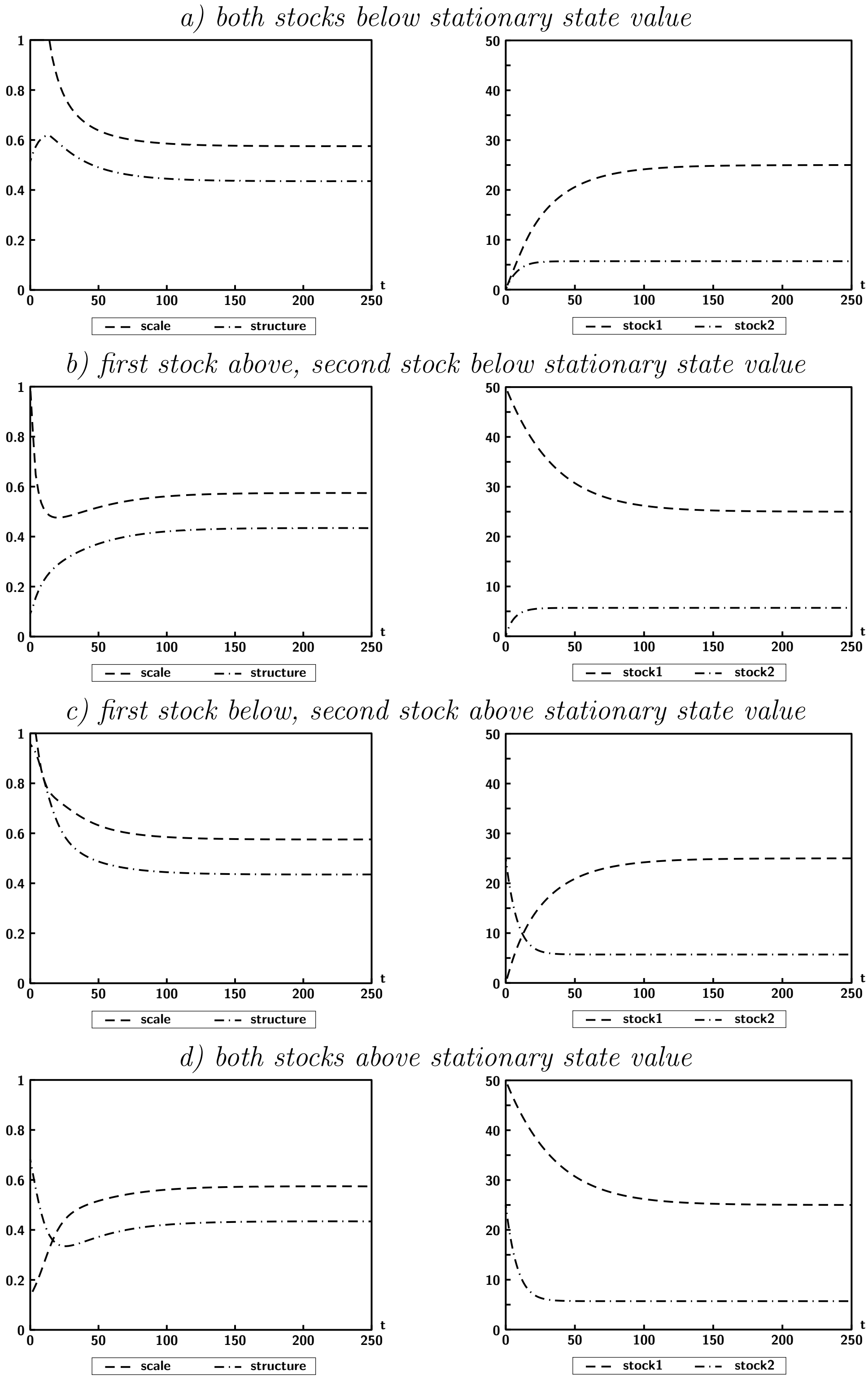

Figure 3: Optimal paths for scale and structure (left) and the two stocks of pollutants (right) for the case $\sigma_1 < \sigma_2$, $\delta_1 < \delta_2$ and all four subscenarios. Numerical optimization parameters are given in the appendix.

has the lower deterioration rate, the first pollutant exhibits clearly more favorable dynamic properties than the second pollutant. In this case the economy will nearly exclusively use the first production sector. Although non-monotonicities in the optimal paths for scale and structure can occur according to proposition 4, they are unincisive. As nothing new can be learned from this case, we do not show a numerical optimization example.

5 Conclusion

In this paper, we have studied the mutual interaction over time between the structure and scale of economic activity on the one hand, and the dynamics of a complex natural environment on the other hand in a total analysis. We have studied a two-sector-economy, in which each sector produces one consumption good and one specific pollutant. The pollutants of both sectors were assumed to differ in their environmental impact in two ways: (i) with respect to their harmfulness and (ii) with respect to their time-scales of accumulation in the environment. We have shown that a stationary state for such a multi-pollution economy exists, and that the optimal dynamics of structural change depends on the intertemporal characteristics of the different pollutants as well as their environmental harmfulness.

Most of the results are intuitive. First of all, if emissions differ either in their environmental harmfulness or in their deterioration rates, we should have structural change towards the sector emitting the less harmful or the shorter-lived pollutant. However, if harmfulness and deterioration rates differ and if the environmentally less harmful emission is also the longer-lived pollutant, no general conclusion concerning the direction of structural change can be drawn. Most importantly, our formal analysis as well as the numerical simulations show that it is likely that the optimal control paths, i.e. the change in structure or scale of the economy, are non-monotonic over time. If a non-monotonic control is optimal, our simulations suggest that the local extremum of the control path may be strongly pronounced and that it occurs at the beginning of the control path. These results are similar to the conclusions of Moslener and Requate (2001), who show that for interacting pollutants ($S_{ij} \neq 0$) which accumulate on different time scales the optimal emission abatement strategies are non-monotonic. We have obtained the non-monotonicity result in a model framework which is, in principle, much simpler than that of Moslener and Requate (2001), as we have assumed that there is neither an interaction between the pollutants ($S_{ij} = 0$) nor between the consumption goods ($U_{ij} = 0$) of both sectors.

This non-monotonicity-result challenges common policy advise which recommends – as a rule of thumb – to reduce (increase) emissions, if the corresponding stocks of pollutants are too high (low) compared to their optimal steady state level. In contrast to this simple rule of thumb, our analysis suggests that if pollutants accumulate on different time-scales, if they differ in environmental harmfulness, and if environmental damage is taken into account by decision-makers, then such a simple policy advice may be inefficient. The degree of inefficiency of the traditional monotonic policy recommendations in more complex settings, such as the one discussed here, could be evaluated by comparing the changes in welfare of the optimal non-monotonic path with the best monotonic control of the system. Even if we have not evaluated the welfare implications of different types of control, we are convinced that the difference between monotonic and non-monotonic control paths may be significant. Our simulations suggest that the local extremum of a non-monotonic control path may be strongly pronounced (up to 10–15 % "over-shooting") and that it occurs at the beginning of the time horizon. Hence, the welfare effect of the "hump" by which the optimal non-monotonic path deviates from the best monotonic control is not diminished too much by discounting.

As for the generality of this result, our analysis as well as recent work by Moslener and Requate (2001) and Jöst, Quaas and Schiller (2003) suggest that it is likely that with more than two interacting control variables only a non-monotonic control is optimal. Because real world environmental problems are normally characterized by the fact that stocks of pollutants interact and accumulate on different time-scales, this result may be relevant for a large number of environmental problems, such as climate change, depletion of the ozone layer, groundwater contamination, acidification of soil and surface water, biodiversity loss, etc. In all these cases, when formulating policy advice it is important to take account of the history, the empirical parameter values and the complex dynamic relationships of the problem – even when answering the most simple of all policy questions: should we emit more or less of a particular pollutant?

Appendix

A.1 Concavity of the optimized Hamiltonian

For a more convenient presentation we show the strict concavity of the optimized Hamiltonian $\mathcal{H}^o$ without taking into account the restriction $c \leq 1$, i.e. $p_c = 0$. Obviously, if the unrestricted optimization problem is strictly concave then the restricted optimization problem is also strictly concave.

A sufficient condition for strict concavity of the optimized Hamiltonian is that the Hessian $H = \frac{\partial^2 \mathcal{H}^o}{\partial i \partial j}$ $(i, j = c, x, s_1, s_2)$ is negative definite. The Hessian H reads:

$$H = \begin{pmatrix} \mathcal{H}^o_{cc} & \mathcal{H}^o_{cx} & 0 & 0 \\ \mathcal{H}^o_{xc} & \mathcal{H}^o_{xx} & 0 & 0 \\ 0 & 0 & -\sigma_1 & 0 \\ 0 & 0 & 0 & -\sigma_2 \end{pmatrix} \tag{A.1}$$

Hence, H is negative definite if the reduced Hessian $H' = \frac{\partial^2 \mathcal{H}^o}{\partial i \partial j}$ $(i, j = c, x)$ is negative definite, i.e. $\mathcal{H}^o_{cc}, \mathcal{H}^o_{xx} < 0$ and $\det H' > 0$. Using that $p_i = -U_i$ on the optimal path (A.15), one obtains:

$$\begin{aligned} \mathcal{H}^o_{cc} &= U_{11}(F^1_c)^2 + U_1 F^1_{cc} + U_{22}(F^2_c)^2 + U_2 F^2_{cc} p_1 F^1_{cc} + p_2 F^2_{cc} \\ &= U_{11}(F^1_c)^2 + U_{22}(F^2_c)^2 \ < \ 0. \end{aligned} \tag{A.2}$$

$$
\begin{aligned}
\mathcal{H}^o_{xx} &= U_{11}(F^1_x)^2 + U_1 F^1_{xx} + U_{22}(F^2_x)^2 + U_2 F^2_{xx} p_1 F^1_{xx} + p_2 F^2_{xx} \\
&= U_{11}(F^1_x)^2 + U_{22}(F^2_x)^2 \ < \ 0. \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}^o_{xc} = \mathcal{H}^o_{cx} &= U_{11} F^1_c F^1_x + U_1 F^1_{cx} + U_{22} F^2_c F^2_x + U_2 F^2_{cx} + p_1 F^1_{cx} + p_2 F^2_{cx} \\
&= U_{11} F^1_c F^1_x + U_{22} F^2_c F^2_x \gtreqless 0. \qquad \text{(A.4)}
\end{aligned}
$$

$$
\begin{aligned}
\det H' &= \left[U_{11}(F^1_c)^2 + U_{22}(F^2_c)^2\right]\left[U_{11}(F^1_x)^2 + U_{22}(F^2_x)^2\right] \\
&\quad - \left[U_{11} F^1_c F^1_x + U_{22} F^2_c F^2_c\right]^2 \\
&= U_{11} U_{22}\left[(F^1_c)^2 (F^2_x)^2 + (F^1_x)^2 (F^2_c)^2 - 2 F^1_c F^1_x F^2_c F^2_x\right] \ > \ 0. \qquad \text{(A.5)}
\end{aligned}
$$

A.2 Proof of Proposition 1

Inserting equation (32) in equation (31) and employing definition (12) yields as a condition for an optimal stationary path:

$$
U_i(P^1(l^\star_1), P^2(l^\star_2)) \geq \frac{\sigma_i P^i(l^\star_i)}{\delta_i^2 + \delta_i \rho} \qquad (i = 1, 2) \ . \qquad \text{(A.6)}
$$

We derive $\bar{l}_i$ by solving (A.6) for $l^\star_i$ assuming equality. $\bar{l}_i$ is the maximal amount of labor which will be assigned to production process P^1 in an optimal stationary state. Hence, if the total labor supply λ exceeds the sum $\bar{l}_1 + \bar{l}_2$, then not all labor will be used for economic activity and the optimal stationary state will be an interior solution.

A.3 Proof of Proposition 2

The proposition for $dc^\star/d\lambda$ and $dx^\star/d\lambda$ can easily be verified by differentiation of equation (11). According to equation (31) for an interior stationary path

$$
U_i(c^\star, x^\star) = \frac{\sigma_i s^\star_i}{\delta_i + \rho} \qquad (i = 1, 2) \qquad \text{(A.7)}
$$

holds. Furthermore, differentiation of U_i with respect to c and x yields:

$$
\begin{aligned}
\frac{\partial U_1}{\partial c} &= U_{11} F^1_c < 0 \ , \qquad \frac{\partial U_1}{\partial x} = U_{11} F^1_x < 0 \ , \\
\frac{\partial U_2}{\partial c} &= U_{22} F^2_c < 0 \ , \qquad \frac{\partial U_2}{\partial x} = U_{22} F^2_x > 0 \ .
\end{aligned} \qquad \text{(A.8)}
$$

Hence, we derive:

$$
\frac{dc^\star}{d\delta_1} = \underbrace{\frac{\partial c^\star}{\partial U_1}}_{<0} \underbrace{\frac{\partial U_1}{\partial \delta_1}}_{<0} > 0 \ , \qquad \frac{dx^\star}{d\delta_1} = \underbrace{\frac{\partial x^\star}{\partial U_1}}_{<0} \underbrace{\frac{\partial U_1}{\partial \delta_1}}_{<0} > 0 \ . \qquad \text{(A.9)}
$$

$$\frac{dc^\star}{d\delta_2} = \underbrace{\frac{\partial c^\star}{\partial U_2}}_{<0}\underbrace{\frac{\partial U_2}{\partial \delta_2}}_{<0} > 0 \ , \qquad \frac{dx^\star}{d\delta_2} = \underbrace{\frac{\partial x^\star}{\partial U_2}}_{>0}\underbrace{\frac{\partial U_2}{\partial \delta_2}}_{<0} < 0 \ . \tag{A.10}$$

$$\frac{dc^\star}{d\rho} = \underbrace{\frac{\partial c^\star}{\partial U_1}}_{<0}\underbrace{\frac{\partial U_1}{\partial \rho}}_{<0} + \underbrace{\frac{\partial c^\star}{\partial U_2}}_{<0}\underbrace{\frac{\partial U_2}{\partial \rho}}_{<0} < 0 \ . \tag{A.11}$$

$$\begin{aligned} \frac{dx^\star}{d\rho} &= \underbrace{\frac{\partial x^\star}{\partial U_1}}_{<0}\underbrace{\frac{\partial U_1}{\partial \rho}}_{<0} + \underbrace{\frac{\partial x^\star}{\partial U_2}}_{>0}\underbrace{\frac{\partial U_2}{\partial \rho}}_{<0} = \frac{\sigma_1/(\delta_1+\rho)^2}{U_{11}F_x^1} - \frac{\sigma_2/(\delta_2+\rho)^2}{U_{22}F_x^2} \ , \\ \Rightarrow \frac{dx^\star}{d\rho} &\gtrless \Leftrightarrow \frac{U_{22}F_x^2}{U_{11}F_x^1} \gtrless \frac{\sigma_2/(\rho+\delta_2)^2}{\sigma_1/(\rho+\delta_2)^2} \ . \end{aligned} \tag{A.12}$$

A.4 Derivation of the differential equation system

Assuming an interior solution, i.e. $c^\star < 1$, the necessary and sufficient conditions (22) and (23) become:

$$(U_1+p_1)F_c^1 + (U_2+p_2)F_c^2 = 0 \ , \tag{A.13}$$
$$(U_1+p_1)F_x^1 + (U_2+p_2)F_x^2 = 0 \ . \tag{A.14}$$

Thus, for an interior optimal path the following equations hold:

$$p_i = -U_i \qquad (i=1,2) \ . \tag{A.15}$$

Differentiation of (A.15) with respect to time and inserting into equations (24) and (25) yields, together with the equations of motion (18), a system of four differential equations in the four unknowns c, x, s_1 and s_2:

$$\sigma_1 s_1 - U_1(\delta_1+\rho) + U_{11}(F_c^1\dot{c} + F_x^1\dot{x}) = 0 \ , \tag{A.16}$$
$$\sigma_2 s_2 - U_2(\delta_2+\rho) + U_{22}(F_c^2\dot{c} + F_x^2\dot{x}) = 0 \ , \tag{A.17}$$
$$\dot{s_1} - F^1 + \delta_1 s_1 = 0 \ , \tag{A.18}$$
$$\dot{s_2} - F^2 + \delta_2 s_2 = 0 \ . \tag{A.19}$$

The conditions (A.16)–(A.19) for an interior optimal solution can be rearranged to yield the system (33)–(36) of four coupled autonomous differential equations, where:

$$\Gamma = \frac{[U_1(\delta_1+\rho)-\sigma_1 s_1]U_{22}F_x^2 - [U_2(\delta_2+\rho)-\sigma_2 s_2]U_{11}F_x^1}{U_{11}U_{22}df} \ , \tag{A.20}$$
$$\Xi = \frac{[U_2(\delta_2+\rho)-\sigma_2 s_2]U_{11}F_c^1 - [U_1(\delta_1+\rho)-\sigma_1 s_1]U_{22}F_c^2}{U_{11}U_{22}df} \ , \tag{A.21}$$
$$\Sigma^1 = F^1 - \delta_1 s_1 \ , \tag{A.22}$$
$$\Sigma^2 = F^2 - \delta_2 s_2 \ , \tag{A.23}$$

and $df \equiv F_c^1F_x^2 - F_x^1F_c^2 < 0$.

A.5 Eigenvalues and eigenvectors of the Jacobian

From (33)–(36) and (A.20)–(A.23) it follows that

$$J^* = \begin{pmatrix} \rho + \frac{\delta_1 F_c^1 F_x^2 - \delta_2 F_x^1 F_c^2}{df} & \frac{(\delta_1-\delta_2) F_x^1 F_x^2}{df} & -\frac{\sigma_1 F_x^2}{U_{11} df} & \frac{\sigma_2 F_x^1}{U_{22} df} \\ \frac{(\delta_2-\delta_1) F_c^1 F_c^2}{df} & \rho + \frac{\delta_2 F_c^1 F_x^2 - \delta_1 F_x^1 F_c^2}{df} & \frac{\sigma_1 F_c^2}{U_{11} df} & -\frac{\sigma_2 F_c^1}{U_{22} df} \\ F_c^1 & F_x^1 & -\delta_1 & 0 \\ F_c^2 & F_x^2 & 0 & -\delta_2 \end{pmatrix} . \tag{A.24}$$

where all functions are evaluated at the stationary state. The eigenvalues ν_i and eigenvectors ξ_i are the solutions of the the equation $J^\star \cdot \xi = \nu \cdot \xi$. The four eigenvalue are:

$$\nu_1 = \frac{1}{2}\left[\rho - \sqrt{(\rho + 2\delta_1)^2 - \frac{4\sigma_1}{U_{11}}}\right] < 0 , \tag{A.25}$$

$$\nu_2 = \frac{1}{2}\left[\rho - \sqrt{(\rho + 2\delta_2)^2 - \frac{4\sigma_2}{U_{22}}}\right] < 0 , \tag{A.26}$$

$$\nu_3 = \frac{1}{2}\left[\rho + \sqrt{(\rho + 2\delta_1)^2 - \frac{4\sigma_1}{U_{11}}}\right] > 0 , \tag{A.27}$$

$$\nu_4 = \frac{1}{2}\left[\rho + \sqrt{(\rho + 2\delta_2)^2 - \frac{4\sigma_2}{U_{22}}}\right] > 0 . \tag{A.28}$$

The eigenvectors associated with the negative eigenvalues ν_1 and ν_2 are:

$$\xi_1 = \left(\frac{F_x^2(\nu_1 + \delta_1)}{df}, -\frac{F_c^2(\nu_1 + \delta_1)}{df}, 1, 0\right) , \tag{A.29}$$

$$\xi_2 = \left(-\frac{F_x^1(\nu_2 + \delta_2)}{df}, \frac{F_c^1(\nu_2 + \delta_2)}{df}, 0, 1\right) . \tag{A.30}$$

A.6 Parameter values for the numerical optimization

We used a Cobb-Douglas welfare function for the numerical optimizations,

$$U(y_1, y_2) = \mu_1 \ln(y_1) + \mu_2 \ln(y_2) , \tag{A.31}$$

and the following production functions:

$$P^1(l_1) = \sqrt{l_1} , \qquad P^2(l_2) = \sqrt{l_2} . \tag{A.32}$$

As common parameters for all numerical optimizations we set $\mu_1 = \mu_2 = 0.5$, $\lambda = 1$ and $\rho = 0.03$. In addition, we used the following parameters for the different scenarios:

Figure	σ_1	σ_2	δ_1	δ_2	s_1	s_2
1	0.01	0.01	0.02	0.1	30	30
2	0.003	0.03	0.05	0.05	40	0
3a	0.002	0.02	0.02	0.1	0	0
3b	0.002	0.02	0.02	0.1	50	0
3c	0.002	0.02	0.02	0.1	0	25
3d	0.002	0.02	0.02	0.1	50	25

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