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Silicon edges as one-dimensional waveguides for dispersion-free and supersonic leaky wedge waves

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Acoustic waves guided by the cleaved edge of a Si(111) crystal were studied using a laser-based angle-tunable transducer for selectively launching isolated wedge or surface modes. A supersonic leaky wedge wave and the fundamental wedge wave were observed experimentally and confirmed theoretically. Coupling of the supersonic wave to shear waves is discussed, and its leakage into the surface acoustic wave was observed directly. The velocity and penetration depth of the wedge waves were determined by contact-free optical probing. Thus, a detailed experimental and theoretical study of linear one-dimensional guided modes in silicon is presented. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4737011]

The investigation of guided waves plays an increasing role in science and technology. This includes ultrasonic waves confined by the boundaries of the particular structure such as tubes or rails. The tendency is to replace bulk waves by guided waves in nondestructive evaluation (NDE) and structural health monitoring (SHM) by permanently installed sensors.1 The two-dimensional (2D) surface acoustic waves (SAWs) and one-dimensional (1D) wedge waves (WWs), which are guided by the surface or wedge tip, respectively, are more confined, since the penetration depth into the bulk material is on the order of one wavelength. While SAWs are well understood and have found widespread use in industrial applications, e.g., in information technology and as sensors, WWs still need to be studied in more detail.

The 1D acoustic edge or wedge waves were discovered by numerical calculations in 1972 independently by Lagasse2 and Maradudin et al.3 The apex of a wedge functions as waveguide for these dispersion-free elastic waves in both isotropic and anisotropic solids.3 Despite their outstanding physical properties and their potential for unique applications, progress was hampered by the lack of non-dispersive high-quality wedges and versatile experimental methods. Main applications are expected in NDE, sensors, structural edge studies, e.g., of defects in cutting tools or turbine blades, and signal processing.

Methods well known from SAWs have been applied to WWs such as interdigital transducers,5 shear piezoelectric transducers,6 and laser-based excitation and detection.7 In most studies, dispersive WWs have been studied, which develop, for example, when the wedge tip contains defects,8,9 is truncated,10,11 or curved,12 or when at least one of the faces is loaded with an adsorbate13 or coated with a film.14

For selective excitation of the waveguide modes, a laser-operated angle-tunable wedge transducer was developed that allowed the separate excitation of WWs, SAWs, and of a pseudo-wedge wave (p-WW). The supersonic p-WW has neither been observed nor predicted before and demonstrates the importance of anisotropy. With a continuous-wave probe-laser-deflection setup, the main features of the launched 1D guided waves, such as their velocity and depth penetration, were analyzed. The coupling of the p-WW mode with surface and shear modes was studied experimentally and theoretically.

A silicon wedge consisting of two (111) planes, forming an angle of 70.5°, was obtained by cleavage of a 3 mm thick single-crystalline Si(111) plate along another (111) plane. The edge was parallel to the (110) direction and the wedge was mirror-symmetric about the Si(110) plane. Wave propagation along this cleaved wedge did not exhibit any attenuation or dispersion within a length of 1 cm.

In the angle-tunable wedge transducer introduced here, which was coupled to the sample edge, first longitudinal acoustic waves with plane fronts were excited in liquid gallium. The upper surface of gallium was constrained by a sapphire plate (see Fig. 1(a)). This plate was irradiated by 1 ns laser pulses with 1.064 μm wavelength and 1 mJ energy. The laser spot was shaped into a narrow 6 mm long line stretched parallel to the edge. The plate could be precisely turned around the axis perpendicular to the sample edge by a stepper motor, with an angular step of 0.225°. Thus, the angle of incidence of the longitudinal wave ϕ could be adjusted to generate the mode with the phase velocity υ = υL/υL = θ/θ of (ϕ) and wave vector parallel to the surface. The longitudinal velocity υL in liquid gallium is about 2.85 km/s, so it can be used for efficient excitation of modes with faster velocities. The advantage of this transducer is that the desired wedge or surface mode can be selectively excited with maximal efficiency, while all other modes are suppressed. The amplitude of the excited mode exhibits a resonant dependence on ϕ with a width of about 2°, as shown in Fig. 1(b).

Detection of the waves was realized by the probe-laser-deflection method, which is sensitive to the slope or inclination parallel to the edge un = ∂υn/∂x1, where υn is the displacement component normal to one of the wedge faces and x1 is parallel to the edge and wave propagation direction. The overall bandwidth of the generation—detection setup was 400 MHz, giving access to acoustic pulses of 3 ns.

The source—probe distance could be scanned by a translation stage driven by a stepper motor. Waveforms recorded
at different propagation distances compose the life lines in the time–space domain for each mode and deliver their velocity. Using multiple spatial steps increases the accuracy substantially, since the error decreases approximately with the number of steps $N$ as $1/\sqrt{N}$. For each distance, the correlation function was calculated, and the time position of the pulse was identified by its maximum. The step length was $100 \mu m$ with $N \sim 25$.

The probe position with respect to the wedge tip could also be scanned perpendicular to the edge to measure the dependence of pulse shape and magnitude on the distance from the edge tip for different modes. Note that WWs provide a unique way to make this dependence visible, which is hardly possible in such a direct way for SAWs, especially in opaque solids. The spatial steps in this direction were $5 \mu m$, which was on the same order as the size of the probe spot. For $3$ ns WW pulses, the spatial extension is about $10 \mu m$.

Efficient methods for calculating the velocities and displacements of WWs are a semi-analytical version of the finite element method [1,16,17] and an approach based on a representation of the displacement field as a linear combination of products of two Laguerre functions [2,18,19]. These properties are obtained from the smallest eigenvalues and corresponding eigenvectors of a Hermitian matrix. In both approaches, the complex displacement field $\tilde{u} (x_1, x_2, x_3, t)$, associated with the $j$th eigenvector, is represented in the form $\tilde{u} (x_1, x_2, x_3, t) = \exp(ik(x_1 - vt)) \hat{U}_j (x_2, kx_3)$. The $x_1$ axis is chosen along the tip of the wedge, the positive part of the $x_2$ axis is in one of the two surfaces, and $v_j$ is the velocity along the edge (frequency divided by $k$) corresponding to the $j$th eigenvector.

Both methods deliver one velocity $v_1$ corresponding to a totally localized wave. This is the fundamental wedge mode with the lowest velocity and displacements of flexural character decreasing monotonically away from the wedge apex. Eigenmodes of higher eigenvalues degenerate into SAWs.

For the material parameters of silicon $\rho = 2328$ kg/m$^3$, $C_{11} = 165.6$ GPa, $C_{12} = 63.94$ GPa, and $C_{44} = 79.51$ GPa (Ref. 20), the speed of the fundamental wedge mode is $v_1 = 4.21$ km/s (FEM) and $v_1 = 4.22$ km/s (with Laguerre functions), whereas for the SAW with wavevector parallel to the edge $v_{SAW} = 4.54$ km/s.

By continuously decreasing the transducer angle $\varphi$, separate modes with ascending velocities could be seen. The slowest mode was the fundamental WW propagating at $4.21$ km/s for $\varphi \approx 43^\circ$ and then the SAW with $4.54$ km/s was generated at $\varphi \approx 39^\circ$. Further angle scanning revealed a wave at $\varphi \approx 32^\circ$, propagating with a velocity of $5.4$ km/s, which is substantially above the SAW velocity of Si(111) in this direction. The three waveforms are shown as upper trace in Fig. 2 and were detected at the very edge with the transducer position adjusted to some intermediate angle in order to excite all three waves simultaneously, albeit with reduced efficiency. The lower trace presents the waves detected at distance of $250 \mu m$ from the edge. Here, the slowest wedge mode has completely vanished, while the fast wedge mode exhibits relatively low localization.

Scanning the source–probe distance provides the life lines for the acoustic modes, as shown in Fig. 3. The inclination of each life line is determined by the speed of the particular mode. Here the retarded time corresponds to the time frame traveling with a speed of $4.22$ km/s; hence the fundamental wedge mode is depicted by an almost horizontal line at $320$ ns. A few tens of nanoseconds ahead of the WW lies the SAW, which is faster and therefore represented by a tilted life line, whereas the p-WW is the fastest excited mode. All three lines originate from the same point, the source. The velocities of both wedge modes were extracted from these measurements using the correlation function method, yielding $v_{WW} = 4.21$ km/s for the fundamental wedge mode, which is exactly the theoretical speed and $v_{p}$, $wW = 5.4$ km/s for the fast p-WW. The latter theoretically not yet described mode cannot be considered as an entirely
localized mode with a real wavevector in the $x_1$ direction, propagating in a wedge with traction-free sides.

In calculations of wave properties based on the Laguerre-function method, higher eigenvalues have been disregarded so far. For the related and simpler case of SAWs, it can be shown that a representation of the displacement field in terms of Laguerre functions describes leaky waves in the continuum of the frequency spectrum of acoustic waves with fixed wavevector along the surface quite well. The same can be expected for wedge geometries. In Fig. 4, we have plotted the normalized squared displacement at the wedge tip $|U_j(0,0)|^2$ as a function of the velocity $v_j$ along the edge. The normalization is such that the integral $\int |U_j(\eta,\zeta)|^2 \, d\eta \, d\zeta$ over the cross section of the wedge is a constant, independent of mode index $j$. This function has been slightly smoothed with a Lorentzian. A clearly visible peak is found at 5.4 km/s indicating an acoustic mode in the continuum of the velocity spectrum with associated large displacements at the wedge tip. The inset in Fig. 4 shows the same quantity for an isotropic wedge with Lamé constants approximating the elastic moduli of silicon. Note that here the pseudo-mode peak of p-WW is absent.

The p-WW couples with other waves of lower velocity, so that their superposition satisfies the free boundary conditions. Specifically, coupling can occur with two acoustical modes, the bulk shear waves, and the SAW that exists on both wedge faces. To understand these processes, Fig. 5 displays the phase velocities for the slow and fast shear waves (s-SW and f-SW) and the true and fast pseudo surface waves (SAW and p-SAW) of the silicon wedge, as a function of the angle $\alpha$ between the edge and the corresponding propagation direction on the Si(111) face. The shear wave velocities $v_{\text{SW}}$ were calculated from the appropriate eigenvalues $\rho v_{\text{SW}}^2$ of the Christoffel matrix $C_{ijkl}$, where $\rho$ is the density, $C$ is the tensor of elastic moduli, and $I$ is a unit vector along the propagation direction.

The condition for wave coupling is $v = v_{p\text{-WW}} \cos(\alpha)$, where $v_{p\text{-WW}} \cos(\alpha)$ is depicted in Fig. 5 for one face of the wedge. The open triangle and circle indicate coupling to the SAW propagating in the direction of $\alpha = 29^\circ$ and to the slow shear wave (s-SW) propagating along the surface with $\alpha = 23^\circ$, respectively. The emission of SWs into the bulk is limited to wavevectors on the surface of a cone, which is deformed because of anisotropy. The cone angle $2\alpha_{s}(0)$ is a function of the angle $\theta$ in the $(x_2, x_3)$ plane, as it is shown in the inset of Fig. 5. Here $\theta$ is counted from the plane of symmetry of the wedge, which is the (1-10) plane. Because of symmetry, the quadrant $0 < \theta < 90^\circ$ is shown only. In the plane of symmetry, the cone angle has its maximum ($\alpha_{s}(0) = 25^\circ$), and in the normal plane (001), it has its minimum ($\alpha_{s}(90^\circ) = 19^\circ$).

These processes are analogous to the well-known coupling of p-SAW with a bulk wave; the fast surface wave emits a shear wave, which has a lower velocity. In contrast to p-SAWs, p-WWs provide the unique possibility of detecting the emitted SAWs, which propagate across the wedge faces.

FIG. 3. Retarded time as a function of propagation distance with respect to the fixed velocity of the WW (4.22 km/s) to determine the phase velocities of SAW and p-WW.

FIG. 4. Normalized squared modulus of edge mode displacements as a function of velocity along the surface, using 64 Laguerre functions. Inset: result for an isotropic wedge with Lamé constants $\lambda = 52.48 \text{ GPa}$ and $\mu = 68.05 \text{ GPa}$ and density of silicon.
A scan of the p-WW perpendicular to the edge is shown in Fig. 6. In the region close to the edge localization prevails, and the wavefront remains plane and normal to the edge. At distances beyond 100 μm, the p-WW decays and a surface corrugation is generated by the SAW propagating in the direction of $a = 29^\circ$ with velocity $v_{\text{SAW}} = 4.74 \text{ km/s}$. If wavevector and group velocity point into the same direction, the delay of the SAW arrival at a distance $d$ from the edge with respect to the p-WW is given by $\Delta t = d \left( \frac{v_{p,-WW}^2 - v_{\text{SAW}}^2}{v_{p,-WW}v_{\text{SAW}}} \right)^{1/2} \approx d/10 \text{ km/s}$, which results in 40 ns delay at $d = 400 \mu m$. This dependence is shown in Fig. 6 by the white dashed line. The inclination angle of the experimental wavefront coincides with the calculated one quite well, confirming the coupling between p-WW and SAW. Note that the depth dependence of the properly localized wedge mode appears as horizontal trace in the time–depth plot, because its wavefront remains plane and normal to the edge.

In summary, guided waves were launched with an optical transducer that allows the effective selective excitation of isolated wedge or surface modes. Besides the dispersion-free fundamental WW, a supersonic p-WW was selectively excited in a silicon wedge. With the theoretical approach using Laguerre functions the fast wedge mode could be described. Mode coupling between this wedge mode and other acoustic modes is discussed.

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