Laser-based linear and nonlinear guided elastic waves at surfaces (2D) and wedges (1D)

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The characteristic features and applications of linear and nonlinear guided elastic waves propagating along surfaces (2D) and wedges (1D) are discussed. Laser-based excitation, detection, or contact-free analysis of these guided waves with pump–probe methods are reviewed. Determination of material parameters by broadband surface acoustic waves (SAWs) and other applications in nondestructive evaluation (NDE) are considered. The realization of nonlinear SAWs in the form of solitary waves and as shock waves, used for the determination of the fracture strength, is described. The unique properties of dispersion-free wedge waves (WWs) propagating along homogeneous wedges and of dispersive wedge waves observed in the presence of wedge modifications such as tip truncation or coatings are outlined. Theoretical and experimental results on nonlinear wedge waves in isotropic and anisotropic solids are presented.

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1. Introduction

The interest in ultrasonic guided waves covers a wide field, from nondestructive evaluation (NDE) and structural health monitoring (SHM) of industrial systems with dispersive three-dimensional (3D) ultrasonic waves to basic research on high-quality low-dimensional waveguides, for example, one-dimensional (1D) ultrasonic waves without dispersion and diffraction. Acoustic waves can be guided by different types of boundaries, for example, free boundaries, solid–liquid, solid–gas interfaces, or interfaces between two different solids. Normally such waves propagate along the boundary over substantially long distances.

Even in well-established conventional applications of ultrasonics to macroscopic objects such as rails there is a tendency to use guided waves for inspection, which is connected with reduced cost, less inspection time, and greater coverage [1]. These ultrasonic waves are constrained by the boundaries of the particular geometric structure and include, for example, torsional waves in a tube or waves that are constrained by the boundaries of the particular geometric nature. In NDE applications, substantial computations are needed for data analysis under these conditions.

Currently, practical applications of guided waves for sensing surface damage and cracks in rails, for example, are dominated by conventional transducers and sensors, while the use of lasers is still under development (for a review see [2]). However, a contact-free fast inspection system has been described, using a pulsed laser for excitation of guided ultrasonic waves and multiple air-coupled sensors for detection of transverse-type small cracks with 1 mm depth in the rail head [3]. Recently for SHM purposes also noncontact laser excitation and sensing, as well as laser-based piezo-ceramic transducer excitation and sensing systems have been developed [4]. These detection systems can be used for laser-based wireless power and data transmission schemes in remote guided waves and impedance measurements. Envisaged aircraft, wind turbine, bridge, high-speed rail, and nuclear power plant applications have been discussed [4].

Laser ultrasonics yields wavefield images with a much higher spatial resolution than conventional transducers. Since for laser diagnostics no baseline data is needed, laser scanning techniques are less sensitive to environmental and operational changes. A similar situation is encountered in current studies of material nonlinearity using nonlinear Lamb waves. While most of the investigations have employed conventional transducers only a few studies use a laser interferometer for detection that provides clear advantages over contact finite-size wedge receivers [5]. The goal of these measurements is to identify plasticity damage, e.g., due to dislocation pile-ups, by monitoring generation of the second harmonic of the sound wave prior to initiation of microcracks. The nonlinear acoustic behavior of cracks has also been studied by all-optical probing, since actuators may mask the effect to be measured owing to nonlinear acoustic transformations at their interface with the surface to be probed [6]. As nonlinear signature for the characterization of imperfect contacts between interfaces or crack faces frequency mixing has been selected in these experiments.

In the following we focus on two particular types of guided waves, propagating along a solid half-space, namely surface acoustic waves (SAWs), or along the edge of a solid wedge bounded by two planes, which are called wedge waves (WWs). These waves are localized at the guiding plane or the apex line, respectively, hence they do not probe the whole sample.

In geometries like planar surfaces (2D) of homogeneous media and ideal wedges (1D), guided acoustic waves exist that are not dispersive. In the 1D case, diffraction is absent, too, whereas acoustic mode conversion may happen under certain conditions.

This review concentrates on the contact-free laser excitation and/or detection of 2D SAWs, propagating along plane surfaces, and 1D WWs, travelling along the apex of a wedge. The unique properties of laser radiation such as directivity, power density, and the possibility of focusing the beam allow unique experiments to be performed that are not possible by conventional ultrasonics. Pulsed lasers give access to linear but also strongly nonlinear ultrasonic pulses. The acoustic bandwidth can be easily controlled, which is of great practical interest, since industrial materials may show strong frequency-dependent attenuation, limiting the observation distance. Therefore, it can be anticipated that laser-based waveguide methods will find their niches, besides the usually much cheaper conventional transducer techniques, for quality control of advanced high-quality materials. Especially, this can be expected when spatial resolution, contact-free performance, limited sample access, or sensitivity are of concern.

Surface acoustic waves were discovered in 1885 by Lord Rayleigh [7]. This wave concept was first applied to seismic waves (“infrasound”). Only much later, with the advent of piezoelectric transducers and interdigital structures on piezoelectric materials, was the field of guided-wave ultrasonics born. Laser ultrasonics developed even later, after the discovery of lasers in 1960, and gave access to narrowband and broadband ultrasonic waves, as well as linear and nonlinear elastic pulses [8]. A characteristic feature of SAWs is that they penetrate only about one wavelength deep into the solid with the displacement decaying exponentially. Note that the penetration depth is wavelength dependent and therefore different depths can be probed simultaneously with a broadband SAW pulse. We can say that the surface acts as a waveguide, keeping the elastic energy at the free surface. In fact, these waves propagate even around corners and edges. In SAWs particle motion is elliptically polarized with in-plane and out-of-plane displacement components that can be measured optically. Furthermore, plane waves with defined propagation direction and minimized diffraction losses can be launched by focusing the exciting laser pulse to a narrow line at the surface.

When a length scale is introduced into the system, for example by a layer, the dispersion-free SAWs become dispersive. The resulting dispersion effect opens up the possibility of determining the mechanical properties of thin films and coatings. This is one of the well-established applications of laser-based linear SAWs. Another one is the localization and sizing of surface-breaking cracks. The excitation of solitary waves in dispersive layered systems is possible with nonlinear SAW pulses, when dispersion and nonlinearity is balanced. For the generation of cracks and the measurement of the fracture strength of anisotropic crystals in defined crystallographic configurations, strongly nonlinear SAW pulses with steep shock fronts are needed [8].

Besides surfaces also solid wedges can function as waveguides. Here the edge of a wedge guides the elastic waves and the energy stays near the wedge tip during propagation. Therefore, these waves are called wedge waves (WWs), or sometimes edge waves, or line waves. Since most authors use now the term “wedge wave” this is employed throughout this review. The 1D acoustic WWs were first discovered by numerical calculations in 1972, independently by Lagasse [9] and by Maradudin and coworkers [10], as a new type of fundamental acoustic waves propagating dispersion-free along the perfect edge of a homogeneous wedge. This is true only, as long as no length scale is introduced into the wedge system. Any distortions of the ideal wedge geometry by modifications such as inhomogeneity, truncation, coatings lead to dispersion of
the phase velocity that can be used, of course, to evaluate the deviations from the ideal geometry of the particular wedge configuration.

Currently, the field of WWs is still in fast scientific and technical development and no industrial applications of these 1D guided waves are presently known. However, several feasibility studies have been performed in recent years pointing to interesting potential applications of these low-dimensional elastic waves in NDE, sensors, and actuators, e.g., in fluidic dynamics or ultrasonic motors.

2. Surface acoustic waves

2.1. Linear surface acoustic waves

2.1.1. Theory

Surface acoustic waves propagate along the interface between two elastic media, at least one of which is a solid. The long-wavelength elastic waves can be treated within the framework of classical continuum elasticity theory [8,11]. SAWs are solutions of the set of wave equations for the displacement components that usually satisfy the boundary conditions of a mechanically stress-free surface. The solutions can generally be presented as a linear combination of inhomogeneous plane waves with wavevectors parallel to the surface that propagate in a defined direction with the same phase velocity and that decay exponentially in depth. Typically more than 90% of the energy of the surface wave remains concentrated at a range of the order of one wavelength below the surface.

Following a standard approach, we start with the linear equation of motion for the displacement field \( u_i(x_1,x_2,x_3,t) \), \( i = 1, 2, 3 \) in Lagrangian coordinates \( x_i \), \( i = 1, 2, 3 \), which denote the position of a mass element in the undeformed state of the solid [12]. In the deformed state of the material, this mass element is displaced to its new position \( x \ast u \). The solid occupies the half-space \( x_3 \geq 0 \). In this notation the equation of motion is

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial x_j} \sigma_{ij},
\]

(2.1)

where \( \rho \) is the mass density and the stress tensor \( \sigma \) depends on the displacement gradients \( u_{ij} = \partial u_i / \partial x_j \) linearly: \( \sigma_{ij} = c_{ijkl} \partial u_k / \partial x_l \).

In Eq. (2.1) and in the following equations, summation over repeated Cartesian indices is implied. Substituting the stress tensor in this form in Eq. (2.1) provides the wave equation

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_j}{\partial x_k \partial x_l} u_k
\]

(2.2)

in the case of a homogeneous elastic medium.

Solutions to Eq. (2.2) are sought in the form of inhomogeneous plane waves

\[
u_i = a_i \exp(ik(1 \cdot x - ct)) \quad \text{or}
\]

\[
u_i = a_i \exp(ik(l_1 x_1 + l_2 x_2 + l_3 x_3 - ct)),
\]

(2.3)

where the first two components of the vector \( l = (l_1, l_2, l_3) \) are normally assumed to be real with \( l_1^2 + l_2^2 = 1 \) and determine the propagation direction parallel to the surface \( x_3 = 0 \).

The complex component \( l_3 \), having a positive imaginary part in the case of surface waves, describes the decay of the wave field in depth. Quantities \( a_i \) denote the polarization of the solution and its amplitude, \( c \) is the phase velocity.

Substitution of Eq. (2.3) into Eq. (2.2) yields

\[
\rho c^2 a_i = c_{ijkl} l_k a_k.
\]

(2.4)

Eq. (2.4) represents a system of three \( (i = 1, 2, 3) \) homogeneous linear equations for the quantities \( a_i \). It possesses a nontrivial solution only if its determinant equals zero:

\[
\det(c_{ijkl} l_k - \rho c^2 \delta_{ik}) = 0.
\]

(2.5)

For given \( l_1, l_2, \) and \( c \), the latter is an algebraic equation of sixth order in \( l_3 \) with real coefficients. It possesses either real roots or complex-conjugated pairs of roots. Real \( l_3 \) correspond to the bulk waves, which do not decay with depth. Thus Eqs. (2.4) and (2.5) can be used to find the bulk waves in anisotropic media. In fact, when specifying \( l = (l_1, l_2, l_3) \) as a real unit vector in the propagation direction of the bulk wave, Eq. (2.4) is the Christoffel equation, having the form of an eigenvalue problem. The eigenvalues \( c^2 \) of the Christoffel matrix, i.e., the three roots of Eq. (2.5), are the squares of the phase velocities of one (quasi-)longitudinal wave and two (quasi-)shear waves. The corresponding polarization vectors \( a \) follow from Eq. (2.4).

In order to find surface wave solutions, we treat \( c \) as a parameter in Eq. (2.5). For \( c \) below a certain threshold, none of the roots \( l_3 \) of Eq. (2.5) is real, and we select the three roots \( l^{(n)} \), \( n = 1, 2, 3 \), with positive imaginary parts to satisfy the condition of the wave field vanishing at \( x_3 \rightarrow -\infty \). Thus the general solution to Eq. (2.2) that decays in depth has the form of a linear combination of three partial waves:

\[
u_i = \sum_{n=1}^{3} C_n a_i^{(n)} \exp \left( ik \left( l_1^{(n)} x_1 + l_2 x_2 + l_3^{(n)} x_3 - ct \right) \right).
\]

(2.6)

Vectors \( a_i^{(n)} \) may be normalized in an arbitrary way, but it is preferable to normalize them to the \( a_i \) component, for example, set \( a_1 = 1 \) (or \( u_1 = 1 \)), because the \( u_2 \) and \( u_3 \) components may be zero for some configurations in crystals.

Now we make use of the fact that the surface of the solid is traction-free, i.e., \( \sigma_{3j} = c_{3ijkl} u_k = 0 \) at \( x_3 = 0 \) for \( j = 1, 2, 3 \). In order to satisfy this boundary condition we should find those coefficients \( C_n \) which make the wave field \( u \) in the form of Eq. (2.6) satisfy the traction-free boundary conditions.

Using the expression for \( u_i \) in the form of Eq. (2.6), the boundary condition is reduced to

\[
C_{nijkl} u_k^{(n)} = 0.
\]

(2.7)

Here, we have defined \( l^{(n)} = (l_1, l_2, l_3^{(n)}) \) and Eq. (2.7) represents a homogeneous system of three linear equations in \( C_n \) (\( j = 1, 2, 3 \)). This
system possesses a nontrivial solution if the determinant \( D(c) \) of the 3 \( \times \) 3 matrix \( (M_{jn}) \) with components \( M_{jn} = C_{ijkl} a^{(i)}_{k} l^{(j)}_{l} \) vanishes,

\[
D(c) = \det(M_{jn}) = 0. \tag{2.8}
\]

Note, that the quantities \( a^{(i)}_{k} \) and \( l^{(j)}_{l} \) are functions of the unknown value of the phase velocity \( c \) of the surface wave. A conventional numerical procedure to calculate \( c \) searches for the value that minimizes the absolute value of \( D(c) \). It starts from some guess value for \( c \), which is usually chosen somewhat smaller than the lowest bulk velocity, and varies it to find the absolute minimum (or zero) of the modulus of the determinant. The value of \( c \) that makes the determinant vanish is the velocity of the wave that satisfies the equation of motion Eq. (2.2), the traction-free boundary condition Eq. (2.7), and decays in depth. With this value of \( c \), \( a^{(i)}_{k} \) and \( l^{(j)}_{l} \) are also known, and Eq. (2.7) gives access to the three weighting coefficients \( C_{n} \), and finally the displacement field of the surface wave can be obtained in the form of Eq. (2.6).

A typical dependence of the determinant \( D(c) \) calculated for the silicon (001) plane is shown in Fig. 1 for the direction 40° relative to the (100) direction. The vectors \( a^{(i)}_{k} \) were normalized such that \( a^{(i)}_{0} = 1 \). Three local minima can be seen: the one with the lowest \( c \) corresponds to the SAW and the slowest shear wave (s-SW), the mid one is the leaky pseudo-surface acoustic wave (p-SAW), and the third one is associated with the fast shear wave (f-SW). Only the first minimum reaches zero, indicating that the boundary conditions can be satisfied. The one correspondent to the p-SAW retains a small nonzero value, so this wave can exist only in superposition with a slower bulk mode, which is the s-SW in this case. Physically, the p-SAW emits the s-SW as it propagates, and therefore loses its energy continuously. Since this energy loss is relatively small, the p-SAW can be observed experimentally.

In anisotropic media velocities of acoustic modes are in general dependent on the propagation direction. To find this dependence one must either rotate the coordinate system and convert the tensor \( (C_{ijkl}) \) correspondingly, or retain the coordinate system and specify the propagation direction by varying \( l_{1}, l_{2} \). Then the minima of \( D(c) \) vary and deliver the phase velocity as a function of the propagation direction, as shown in Fig. 1.

We consider here the displacement field of SAW pulses that depends on two spatial coordinates, namely the direction of SAW propagation and the depth coordinate, and on time. The dependence on the third spatial coordinate can be neglected for a SAW pulse that is excited by a homogenous line source, which can be realized by focusing the excitation laser pulse with a cylindrical lens to a line on the surface. For a sufficient length of the line source, effects such as diffraction and beam steering (group velocity and wavevector are not parallel) are negligible. The displacement components calculated by Eq. (2.6) for the p-SAW branch and the wavevector direction of 40° are shown in Fig. 2. Due to lack of symmetry, the transversal component \( u_{2} \) is nonzero, in other words the polarization plane is tilted away from the sagittal plane. The particle motion in the wave field is elliptic, but \( u_{1} \) becomes zero at the depth about \( kx_{3} = 1 \), so that the ellipse degenerates into a vertical line. At greater depths the elliptic motion resumes, but with the opposite rotation direction. The surface wave in this direction is strongly coupled with the s-SW, polarized parallel to the surface, as can be seen in Fig. 1. Correspondingly, polarization of the SAW in this direction is almost parallel to the surface, which makes it difficult to detect.

Besides Rayleigh waves in isotropic solids and generalized Rayleigh waves in anisotropic solids, SAWs in semi-infinite isotropic and anisotropic solids covered with a layer with different mechanical properties will be investigated. Usually it is assumed that the film consists of a homogeneous solid of uniform thickness in intimate contact with the substrate. The resulting dispersion of the phase velocity in such a system can be used to extract information on the mechanical properties of the layered system, especially on unknown mechanical properties of the layer itself.

2.1.2. Excitation and detection of SAWs

Purely laser-based experiments of the pump–probe type have important advantages. Such an all-optical noncontact technique provides a wide variety of excitation and detection conditions [8]. These include broadband excitation with pulsed laser radiation by a point or line source, narrowband excitation of wave trains with a mask or the interference of two short coherent laser pulses. The excitation bandwidth is controlled by the width of the line or point source. In linear experiments the pulse energy of the excitation laser is selected for thermoelastic excitation of SAW pulses, which have amplitudes in the few-nanometer range. Since the corresponding strain is in the \( 10^{-3} \text{–} 10^{-4} \) region, damage of the surface of the solid can be avoided under these conditions.

Detection is performed with a continuous wave (cw) laser beam that allows the absolute interferometric measurement of the surface displacement or the determination of the transient surface slope or velocity by laser-probe-beam deflection [8]. To record the surface displacement an actively stabilized Michelson or heterodyne interferometer is employed, while the surface velocity is registered by a knife edge or position-sensitive detector. The detection bandwidth is controlled by the size of the point focus of the cw probe laser. In Fig. 3 such a laser setup with line source and interferometric detection or laser-probe-beam deflection is shown.

In measurements of the elastic coefficients of films a two-point probe setup is used to determine the frequency dependence of the phase velocity. By fitting the normal or anomalous dispersion of one or several acoustic modes the Young’s modulus or the whole set of elastic coefficients can be obtained, respectively. The most
advanced laser scanning NDE method of surface-breaking cracks employs a coupled excitation-and-detection laser arrangement, and the crack is moved step-wise from one side to the other of the fixed pump–probe setup. This procedure yields the complete acoustic field around the crack and information on the stiffness reduction in the crack.

2.1.3. Determination of elastic coefficients

SAWs provide a versatile tool for the determination of the elastic coefficients of, for example, advanced materials and coatings. This has been demonstrated for synthetic single-crystal, micro-, and nanocrystalline diamond, deposited by chemical vapor deposition (CVD) [13]. For anisotropic single-crystal diamond the linear stiffness tensor has three independent components, for example, $C_{11}$, $C_{12}$, and $C_{44}$, instead of only two for an isotropic material. In most studies of micro- and nanocrystalline diamond, for example, the assumption is made that the material is elastically isotropic, reducing the number of independent elastic coefficients to only two, e.g., the Young’s modulus and the Poisson ratio.

Thin films and coatings are widely applied in industry to modify surface properties, such as antireflection coatings used in optics, protective films of hard materials such as diamond, and metallic coatings. A versatile tool for the measurement of the Young’s modulus of thin films is the dispersion of SAWs [13]. More than one film property, such as the Young’s modulus, film thickness, or density, may be extracted if the thickness is in the micrometer range as is usually the case for diamond layers. On the other hand, an advanced laser scanning NDE method of surface-breaking cracks on silicon (see Fig. 4a) and a silicon oxide layer on silicon (see Fig. 4b), respectively.

The coatings were assumed to be isotropic and the thickness of the diamond film was 700 nm and that of the silica film 1.4 μm. At the low frequency limit both curves tend to the phase velocity of pure silicon, which is 4.92 km/s. The high frequency limit is the Rayleigh velocity of the film. The principle difference between the two structures is that the transition between the two limiting cases is always monotonous for the loading case, whereas for the stiffening layer it can take a more complicated shape as shown in Fig. 4a. The slope of the dispersion curve at zero frequency can conveniently be obtained via an effective boundary condition at the surface of the substrate, correct to first order in $kh$. Dispersion curves can be obtained experimentally by comparing Fourier transforms of the waveforms detected at several distances. Fitting the theoretical dispersion curve to the experimental one allows the evaluation of unknown material parameters of the film, such as elastic moduli, density or thickness if the substrate properties are known.

A complete set of elastic moduli has been determined for a 110 μm thick anisotropic heteroepitaxial quasi-monocrystalline diamond layer [15]. In the special multimode photoacoustic technique employed for analysis, acoustic surface and interface modes and SAW propagation in different crystallographic directions have been taken into account in the analysis. In Fig. 5a and b the experimentally studied dispersive and nondispersive acoustic modes are displayed together with the corresponding theoretical analysis of

\[
\sum_{n=1}^{3} C_{mn} a_{n}^{(m)} = \sum_{n=1}^{6} C_{en} a_{n}^{(e)}
\]

and the components $\sigma_{ij}$ of stress are continuous as well:

\[
c_{ijkl} \sum_{n=1}^{3} C_{mn} a_{n}^{(m)} l_{l}^{(m)} = \tilde{c}_{ijkl} \sum_{n=1}^{6} C_{en} a_{n}^{(e)} l_{l}^{(e)}.
\]

Eqs. (2.9), (2.10), and (2.11) form a system of 9 linear equations in $C_{mn}$ and $C_{en}$. Its determinant must be zero to admit a nontrivial solution. The phase velocity of the SAW is sought in the same way as for the uncoated half-space, but now it is dependent on the wavenumber via $kh$ (see Eq. (2.9)). It is important to discriminate two types of coatings, acoustically “faster” or stiffening film on a “slower” substrate (“anomalous dispersion”), and the opposite case, when the film is “slower” as compared to the substrate (“normal dispersion”). As examples for these configurations we present dispersion curves for a diamond film on silicon (see Fig. 4a) and a silicon oxide layer on silicon (see Fig. 4b), respectively.

![Fig. 4. Dispersion curves for (a) the stiffening case (diamond film on silicon) and (b) silicon-dioxide film on silicon (normal dispersion).](image)
the mode structure. An advantage of this photoacoustic method is that the diamond film does not necessarily have to be removed from the Ir/YSZ (yttria-stabilized zirconia) Si(001) substrate, if the properties of the nucleation layer and substrate can be taken into consideration with reasonable accuracy. Analysis of the experimental data yielded the following stiffness coefficients: $C_{11} = 1146 \pm 4.8$ GPa, $C_{12} = 178 \pm 46$ GPa, and $C_{44} = 562 \pm 3.7$ GPa [15]. Similar to homoepitaxial diamond deposition, $C_{12}$ shows the largest deviations, but $C_{11}$ and $C_{44}$ also deviate, within experimental error, from the generally accepted set of elastic coefficients of $C_{11} = 1080.4 \pm 0.5$ GPa, $C_{12} = 127.0 \pm 1.0$ GPa, and $C_{44} = 576.6 \pm 0.5$ GPa. Possible systematic errors involved in the evaluation of the effects of the nucleation layer could not be excluded in this work.

The laser-based SAW dispersion technique is extensively used. An example is a study of the dependence of the Young’s modulus on the source gas composition. By varying the methane concentration in the source gas between 0.5% and 2.0%, it was shown that the Young’s modulus, $E$, increases from $562 \pm 4.8$ GPa at 0.5% CH$_4$ to $1143 \pm 4.8$ GPa at 2.0% CH$_4$ [16]. However, we have to bear in mind that the lower the methane content, the lower the growth rate and, of course, no growth occurs at a methane pressure of zero.

Another example is the observation of SAW dispersion generated by gradients in the mechanical and elastic properties of millimeter-thick microcrystalline diamond plates [17]. These measurements of the Rayleigh velocity clearly demonstrated the variation of the mechanical properties between growth and nucleation sides. A further application is the demonstration of the positive role played by oxygen on the growth rate and on the stiffness as a function of the oxygen content in the source gas [18].

Nanocrystalline diamond with a crystallite size <100 nm can reach a Young’s modulus of 1100 GPa for low methane concentration (0.5%) and high power densities in the range of about 25 W/cm$^2$, even when the film thickness is only 140 nm [19]. The importance of the nucleation density on the mechanical film quality has been demonstrated for columnar-structured diamond films with column diameters of less than 100 nm. The Young’s modulus obtained for low nucleation density ($\sim 10^{10}$ cm$^{-2}$) of 517 GPa was roughly half that of a film or plate grown with a much higher nucleation density ($\sim 10^{11}$ cm$^{-2}$) of 1120 GPa [20]. The finding of a measurable reduction of the film stiffness at a grain size below approximately 100 nm is in good agreement with SAW dispersion measurements on nanocrystalline samples with a grain size between 60 nm and 9 nm, where the Young’s modulus ranged from 1050 to 700 GPa [21]. However, it is important to note that quite different stiffnesses of 609 GPa [22] and 1160 GPa [23] have been reported for comparable grain sizes of approximately 150 nm.

### 2.1.4. Nondestructive evaluation

An important application of linear SAWs is NDE of surface-breaking cracks. Since the surface is often subjected to the highest stress levels, the degradation of materials usually starts at the surface. These irreversible degradation processes may lead to fatigue or stress corrosion cracking, generating partially closed cracks with

---

**Fig. 5.** (a) Measurement of two surface modes in the two crystallographic directions, [001] and [011], for a diamond layer on silicon. (b) Joint Green’s function calculations for a diamond layer on silicon in the [001] direction. Mode 1 is the Rayleigh wave propagating in diamond and mode 2 is the pseudo-surface mode propagating partly in silicon.

**Fig. 6.** Experimental arrangement of crack, probe spot, and SAW source (zone I). The configurations of zones II and III were realized by moving the sample with respect to the fixed optical pump–probe setup until the crack passed the source.

**Fig. 7.** Illustration of the ratios of SAW penetration depths and crack depth for the lowest and highest wavelength of a typical wideband SAW pulse and a crack depth of 50 μm.
a finite penetration depth. Linear SAWs or Rayleigh waves with suitable wavelengths are ideally suited to detecting such surface discontinuities because the penetration depth of these guided surface waves can be selected for optimum detection sensitivity.

The first purely laser-based NDE experiments were performed with the goal of localizing artificial slots and extracting their size [24,25]. Since a laser beam can be moved easily along the surface, the extracted information can be extended by shifting the laser source, the laser probe, or both with respect to a surface crack. For example, in the first investigation using the scanning laser method only the source was swept across the crack; the SAW was registered at a fixed surface position using a laser or conventional transducer [26]. In the most advanced scanning technique the pulsed excitation laser and the cw probe laser are coupled at fixed distance and the sample with the microcrack is scanned below the fixed pump-probe arrangement, as illustrated in Fig. 6 [27]. The information content increases dramatically if the acoustic field around the crack is studied for the following three configurations: the source and probe to the left of the crack (zone I), the crack between pump and probe (zone II), and the pump and probe to the right of the crack (zone III). This double scanning approach is limited to all-optical set-ups.

Laser-based SAW methods have been most frequently applied to artificial rectangular slots and V-shaped notches, whereas investigations on real cracks are still rare. The latter include stress corrosion cracks [28], cracks induced by fatigue processes [26], and finite-size cracks originating from impulsive elastic loading by nonlinear SAW pulses with steep shock fronts [29]. Usually realistic cracks are not completely open but have partially touching crack faces near the tip. These partially closed cracks are of enormous practical importance.

While most laser-based NDE experiments have been conducted between 1 and 20 MHz, it proved possible to extend laser ultrasonics into the frequency range of 10–200 MHz [27]. These two frequency regions correspond roughly to well-detectable crack depths of about 1 mm and 10–100 μm, respectively. It is important to note that with nanosecond laser pulses SAW pulses with a bandwidth extending over 1–2 orders of magnitude can be launched, probing simultaneously different depths in the micrometer-to-millimeter range with a single broadband SAW pulse. Fig. 7 compares the penetration depths of such a pulse with the typical depth of ~50 μm of a crack generated by impulsive load. The highest sensitivity can be achieved for a crack depth comparable to the mean wavelength of the interrogating SAW pulse. Some unique ways to extract information about cracks from laser scanning experiments include analysis of the reflection and transmission coefficients of the incoming SAW, determination of the reduced interfacial stress of partially closed cracks by the finite difference method (FDM), and signal enhancement effects occurring when the pump or probe laser hits the crack, as described in more detail in the following [27].

In Fig. 8 a time–distance plot of a laser scanning experiment is displayed. The 3D picture represents the complete scattered acoustic field in the vicinity of the crack, including the incoming, transmitted, and reflected Rayleigh waves. The gray scale shows the measured velocity profiles. The reflected SAW velocity profile observed in zone I consists of two positive peaks with a trough. Such a tripolar waveform is expected for partially closed cracks. Obviously, the motion of the rear face of the crack affects the wave profile to a comparable extent as the front face, indicating a substantial transient interaction of the two crack faces during reflection. The physical basis for the evaluation of the crack size is the comparison of the reflected pulse shape with the initial undisturbed Rayleigh pulse, which is easily accessible in zone I.

Let us consider the reflection coefficient, which depends on the actual shape of the defect. In the simplest case of a semicircular (“half-penny shaped”) crack extending normal to the surface, the depth also describes the size. The reflection coefficient was calculated for both the simulated waveforms and the corresponding measured ones as the ratio of the spectral amplitudes of the reflected and incident waves. The depth of the crack was evaluated by fitting the FDM-simulated reflection coefficient to the experimentally measured one by a least squares procedure. From this fit, a value of 34 μm was extracted for a semicircular microcrack. This crack depth is in reasonable agreement with the direct optical measurement of the crack radius of 49 μm. Note that the lower part of the partially closed crack may not be seen acoustically, whereas optically it still may scatter light, owing to the smaller wavelength of light involved in optical detection.

Another approach for crack sizing, which is independent of the SAW amplitude, is the measurement of the time needed by the SAW pulse to travel along the complete crack faces. For crack depths larger than the SAW wavelength the depth is proportional to the time delay. The bipolar SAW pulses transmitted through an impulsive shock-induced crack show a clear time delay or phase shift, as can be seen in Fig. 7 as small upward shift of the horizontal Rayleigh wave trace in zone II. In fact, the time lag of a transmitted broadband SAW pulse depends on the depth/wavelength ratio. This offers an alternative to evaluating crack sizes from the frequency dependence of the phase lag. In the present analysis the transmitted bipolar SAW was first simulated using the FDM, and then the phase of the transmission coefficient was estimated as a function of the depth/wavelength ratio. This was fitted to the experimentally measured frequency dependence of the phase shift with the crack size as the only free fitting parameter. The crack depth evaluated by the transmission method was 40 μm, in reasonable agreement with the reflection method.

To simulate the interaction at the crack interface a simple one-parameter model has been developed, which allows the interfacial interaction to be modified continuously between completely open and completely closed. This stress parameter ε reduces all stress-tensor components by the same scalar factor (0 ≤ ε ≤ 1). Within the framework of a mass–spring representation, this corresponds to a weakening of the spring’s stiffness by a factor of ε between the masses situated on each side of the crack opening. Consequently, this reduced-stress model describes the softening of the material’s stiffness by the crack defect. While FD calculations for an open crack (ε = 0) result in a bipolar shape of the reflected SAW, a value of ε = 0.08 reproduced the measured tripolar profile (see Fig. 9). The reduction of the interfacial stress-tensor...
components to about 8% of the value in the intact lattice of the ideal solid indicates a relatively open crack. Of course, such a single parameter cannot describe the real behavior of a partially closed crack. It must be considered as a mean interfacial stress or stiffness averaged over the whole length of the crack. Consistent with this finding a two-parameter model yields the correct tripolar shape for the reflected wave only when the tensile and shear stress components have the same value as fitted for the one-parameter case.

A quantitative description of the signal enhancement effects observed near the crack in laser scanning experiments is a difficult task, since microscopic properties of the crack and of the interaction of the excitation or probe laser with the crack faces are involved, and exact information on the crack opening and laser beam radius may not be available. Therefore, only a qualitative comparison between experiments and model calculations can be performed. The present FD simulations, using a step width of 1 μm, yield about a factor of two for the enhancement of the motion at the front edge. Values in this range have been reported by several authors for cracks extending perpendicular to the surface. When the spatial resolution of the probe beam is decreased from about 2 μm to about 5 μm the enhancement effect disappears. In agreement with these simulations only a moderate enhancement of about 20% has been found under comparable experimental conditions.

It is important to note that in real systems cracks also grow at angles different from 90°. For example, in the case of rolling contact fatigue in rails, initial crack growth occurs at angles of about 20–25° to the surface. A number of in depth theoretical investigations were carried out in the past on the diffraction of ultrasonic waves by an elastic edge or wedge [30–33]. In principle, a wedge with a stress-free boundary may be considered as an open crack with infinite length. Tilted defects have been inspected by bulk waves using a conventional transducer for measuring the amplitude and phase of the wedge diffraction coefficient [34], and good agreement has been found with results of diffraction theory. Detecting and sizing tilted cracks with a piezo-electric transducer is a difficult task since longitudinal, shear, and Rayleigh waves are involved. A simpler process, namely scattering of Rayleigh waves from a surface-breaking crack, several wave lengths long, has been treated to extract the Rayleigh reflection and transmission coefficients [35]. A detailed experimental study of the interaction of Rayleigh waves with cracks has been performed by all-optical laser excitation and detection [36]. The observed variation of the reflection and transmission coefficient has been compared with 3D finite element calculations for a wide range of angles and depths, which are in good agreement with experiments. These simulations allow new insight into the displacements within

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**Fig. 9.** Comparison of FD simulations with experiment: (a) thermoelastic excitation, (b) reflection from the crack, and (c) transmission through the crack. Experimental waveform: red, simulation with ε = 0; green, simulation with ε = 0.08; black.

**Fig. 10.** Dependence of kernel $F$ in evolution Eq. (2.12) (a) on its argument $\eta$ and (b) on the wavevector direction.
the sample and the wave-crack interaction processes such as mode-conversion at the bottom of the crack [36].

The term “edge waves” or “edge resonance” is frequently used for waves guided by the edge of a plate with parallel surfaces [37,38] or a resonance in the reflection of Lamb waves from the edge of a plate [39]. We would like to state here that the use of the term “edge wave” that sometimes is used instead of “wedge waves” does not comprise these highly dispersive guided waves, which are an interesting topic by itself.

2.2. Nonlinear surface acoustic waves

2.2.1. Theory

With the absence of dispersion and reduced diffraction losses for plane SAWs, these waves are of particular interest for studying nonlinear acoustic waves and their effects [40]. An essential prerequisite for nonlinear behavior is that the attenuation length due to intrinsic damping is longer than the length scale on which wave propagation takes place ("shock formation distance"). By pulsed laser irradiation, plane SAW pulses with finite amplitudes can be launched, which develop steep shock fronts by frequency-up conversion (simultaneously frequency-down conversion takes place) during wave propagation along an elastically nonlinear medium. A nonlinear evolution equation has been derived [41–45] describing the corresponding changes of the SAW pulse profile during propagation in a nonlinear homogeneous half-space. It has the form

\[
\frac{i\epsilon_r}{\partial}\frac{\partial}{\partial x}B(0, x, t) = \alpha\left\{\int_0^{-\infty} F(\omega')B(\omega', x_1)B(\omega - \omega', x_1) d\omega / 2\pi + \frac{1}{\epsilon_0} \int_0^{\infty} F(\omega')B(\omega', x_1)B'(\omega' - \omega', x_1) d\omega / 2\pi\right\}
\]  

(2.12)

Here, \(\epsilon_r\) is the phase velocity of the Rayleigh waves and the wavevector direction is the \(x_1\)-direction (possibly after a rotation of the coordinate system). In Eq. (2.12) \(B(\omega)\) is the Fourier transform of a displacement gradient \(u_{m1}, m = 1, 2, 3\) at the surface,

\[
u_{m1}(x_1, x_2, 0; t) = 2\Re\left\{\int_0^{\infty} B(\omega) \exp\left[i\frac{\epsilon_0}{\epsilon_k}(x_1 - c_k t)\right] \frac{d\omega}{2\pi}\right\}
\]

(2.13)

or a linear combination of such displacement gradients that are assumed to be approximately independent of \(x_2\). In Eq. (2.13) \(\Re\) denotes the real part. \(B(\omega)\) is allowed to vary gradually with \(x_1\). The kernel function \(F(\eta)\) depends on ratios of second-order and third-order elastic moduli of the substrate. It may be expressed as an overlap integral over the three components of displacement gradients associated with linear Rayleigh waves and the sixth-rank tensor \(S_{ijklmn}\). Its components are the coefficients of the third-order terms in an expansion of the density of potential energy \(\Phi\) in powers of the displacement gradients,

\[
\Phi = \frac{1}{2} \epsilon_{ijkl}u_{ik}u_{jk} + \frac{1}{6} S_{ijklmn}u_{ik}u_{jm}u_{mn} + O(u^4),
\]

(2.14)

and are linear combinations of second-order and third-order elastic moduli,

\[
S_{ijklmn} = \epsilon_{ijklmn} + \delta_{ik} \epsilon_{jkmn} + \delta_{im} \epsilon_{jklmn} + \delta_{jm} \epsilon_{ikln}.
\]

(2.15)

where \(\epsilon_{ijklmn}\) are third-order elastic moduli and \(\delta_{ij}\) is the Kronecker symbol. The kernel \(F(\eta)\) has a simple functional form,

\[
F(\eta) = \sum_{n=0}^{\infty} \frac{M(n; n', n'')}{n! \eta^n} + \eta \frac{M(n; n', n'')}{n!} + (1 - \eta) \frac{M(n; n', n'')}{n!},
\]

(2.16)

The coefficients \(M(n; n', n'')\) can be expressed in terms of the quantities \(\epsilon_{ijkl}, \alpha_{ijkl}, C_n, n = 1, 2, 3, \) and \(\rho c_k^2\) introduced in Section 2.1.1., and the tensor \(S_{ijklmn}\). Obviously, they strongly depend on the choice of the displacement gradient component in Eq. (2.13). In isotropic media, \(F(\eta)\) is purely real if \(B(\omega)\) is the Fourier transform of \(u_{11}\) and purely imaginary if \(B(\omega)\) is the Fourier transform of \(u_{31}\). In anisotropic solids and propagation geometries of low symmetry, \(F(\eta)\) may possess both imaginary and real parts.

The dependence of the function \(F(\eta)\) on the wavevector direction on the \((1 1 1)\) surface of silicon is shown in Fig. 10, corresponding to the Fourier transform of \(u_{31}\). The relative size of real and imaginary parts depends on \(\eta\) and on the wavevector direction. This has strong consequences for pulse shape evolution, especially on the shape of solitary pulses.

Taking into account weak dispersion results in an additional term in the evolution equation [40]. By controlling the dispersion effect, for example by the film thickness, it is possible to balance the nonlinear and dispersive effects, if they occur on the same length scale. In the case of dispersion of SAWs resulting from a thin film on the surface, the additional term on the right-hand side of the evolution Eq. (2.12) has the form \(\Delta \omega^3 B(\omega)\). The coefficient \(\Delta\) depends on the linear acoustic mismatch of film and substrate and may be negative or positive, corresponding to normal or anomalous dispersion, respectively. Normally, the exponent \(n\) is equal to 2 and \(\Delta\) is proportional to the film thickness. Consequently, the dispersion term in Eq. (2.12) is the same as the one in the Benjamin–Ono equation. In fact, the extended evolution equation provides solitary solutions that represent the typical pulse shapes observed under such dispersive conditions. The shape of the solitary pulses is strongly influenced by the kernel \(F(\eta)\) in the nonlinear evolution equation. It turns out that it is not so much the dependence of \(F\) on its argument but the average phase angle of the complex quantity \(F\) that is relevant here and that strongly depends on the anisotropy of the substrate material. With the exponent \(n = 2\) in the dispersion term, the solitary pulses give rise to the following forms of the displacement gradients at the surface:

\[
u_{m1}(x_1, x_2, 0; t) = \kappa \mathbf{S}_{\omega} |\kappa(1 + \kappa) x_1 - c_k t + x_0|,\]

(2.17)

where \(\kappa > 0\) and \(x_0\) are parameters. The upper sign in Eq. (2.17) is found in the case of normal dispersion, the lower sign in the case of anomalous dispersion. For isotropic substrates, \(S_1\) is an even function and \(S_3\) is an odd (bipolar) function of its argument. In anisotropic substrates, this is no longer the case, and \(S_3\) may have an almost even shape of the Mexican-hat type. The numerical simulations reveal that these are solitary pulses and not real solitons, since they do not survive collisions with each other. Only under exceptional conditions \((n = 3)\) could almost elastic collisions be seen in these simulations.

2.2.2. Experimental

As mentioned above, SAW pulses with finite amplitude must be launched at the source to realize nonlinear surface pulses. Stronger excitation with nanosecond laser pulses can be achieved by adding a highly absorbing layer at the source region that explosively
Since SAWs show no dispersion in a homogeneous half-space, a film was deposited on the substrate to generate solitary pulses. In a layered system the film thickness introduces a new length scale responsible for a geometric film-based dispersion effect. Since this length scale is much larger than the interatomic spacing, intrinsic lattice-based dispersion can be neglected. During propagation in the nonlinear and dispersive medium, stable bipolar or Mexican-hat-shaped solitary pulses are formed, which travel faster than the linear Rayleigh velocity, whereas the small oscillatory tail (so-called “radiation”) moves with lower velocity for a loaded system, or vice versa for the stiffening case [49–51].

Fig. 12a and b shows two stationary periodic solutions of the nonlinear SAW evolution equation with a linear dispersion term, namely a bipolar and a Mexican-hat profile. A bipolar pulse was observed for a NiCr film loading the silica substrate, leading to anomalous dispersion as depicted in Fig. 12c. On the other hand, an added TiN film stiffens silica, leading to normal dispersion (see Fig. 12d) [49]. In this system the theoretically predicted Mexican-hat-shaped profile was observed for the first time [50]. The observed bipolar and Mexican-hat-shaped pulses agree well with numerical solutions of the nonlinear evolution equation with an essentially nonlocal character of nonlinearity and a linear dispersion term of the Benjamin-Ono type. In this respect the nonlinear evolution of surface waves in solids differs from that in shallow fluids, which can be described by the Korteweg–de Vries (KdV) equation [51]. The realization of solitary waves at surfaces and solitons in crystals achieved by nanosecond and picosecond laser ultrasonics, respectively, has been reviewed in [52].

### 2.2.4. Shock waves and determination of fracture strength

As discussed before, nonlinear SAWs can develop stress peaks which grow with the propagation distance and thus generate cracks when the critical strain of the material is reached [29,53,54]. The crack size is in the region of 50–100 μm, determined by the length of the SAW pulse (tens of nanoseconds) and the crack speed that is on the order of the Rayleigh velocity of 3–5 km/s. In order to evaluate the stress of the SAW pulse, it was measured at two different distances from the source by laser-probe-beam deflection, first at 1–2 mm from the source and the other at a distance of about 15–20 mm. A stress calibration procedure was applied, based on numerical integration of the nonlinear and thermal generation of SAWs.

With the “absorption layer” method, strains in the range of 0.01–0.1% have been realized. Since the critical strain of many brittle materials is in this region, mechanical fracture can be achieved, e.g., in silica [47] and silicon [29,47]. Such fracture processes can occur repetitively because after fracture frequency-up conversion takes place in the advancing pulse until the critical strain is reached again. Therefore, a whole more-or-less regular field of spatially limited cracks may be generated along the SAW propagation direction with this impulsive failure method. It is important to point out that frequency-up conversion concentrates the elastic energy of the guided surface wave in an even smaller depth from the surface of the solid.

Fig. 11 shows the complete pump–probe setup, which is slightly modified for nonlinear excitation by adding the absorption layer. Furthermore, the two laser probe positions allow the evolution of the SAW pulse shape to be monitored directly [48]. Usually the pulse profile measured at the first probe spot is inserted into the nonlinear evolution equation as an initial condition and the profile calculated for the second probe spot is compared with experiment. This arrangement has been employed in all experiments on solitary waves and the measurement of the fracture strength of silicon presented here.

**Fig. 12.** Theoretical bipolar (a) and Mexican-hat-shaped (b) solitary SAW pulses calculated using a nonlinear evolution equation with linear dispersion term. Measured solitary pulses for the systems silica/NiCr film (c) and silicon/SiO2 film (d).
evolution equation Eq. (2.12). The waveform detected at the shorter distance was fed into Eq. (2.12) as an initial condition with the calibration factor \( a \), which relates the detected waveform \( U(t) \) and the gradient \( \partial U / \partial t \) so that \( U(t) = aU(t) \). The calibration factor \( a \) was found by fitting the result of integration to the waveform detected at the remote location. Then all the strain and stress components can be determined everywhere in the sample, in particular at the location, where a crack was located. This provides an estimate of the material strength.

The spectrum of the initial laser-excited transient was limited to about 200 MHz, mainly owing to the laser pulse duration of 8 ns. Since in a nonlinear medium, such as a silicon single crystal, both frequency-up conversion and frequency-down conversion processes are taking place, a lengthening of the pulse profile occurs simultaneously with shock formation. This effect is proportional to its magnitude, and therefore the pulse length can be used as a sensitive measure of the nonlinear increase of strain. In particular, when the shock fronts become steeper and steeper this quantity can be determined quite accurately [59].

For Si(111) we made the observation that counterpropagating nonlinear SAW pulses, i.e., those moving in opposite directions, for example in the \( (-1 - 12) \) and \( (11 - 2) \) directions on the Si(111) plane, develop completely different nonlinear pulse shapes [29]. In the easy-cracking geometry Si(111) \( (-1 - 12) \) the tensile fracture strength was about 4 GPa. The surface-nucleated cracks propagated into the bulk along the \( (11 - 1) \) cleavage plane, which is inclined by 19.5° to the normal of the free surface (see Fig. 13a). According to the boundary conditions for SAWs, only the tensile opening stress \( \sigma_{11} \) is nonzero directly at the surface in the initial coordinate system. This tensile stress of \( \sigma_{11} = 4 \) GPa can be represented by a set of orthogonal components in the coordinate system associated with the tilted \( (11 - 1) \) cleavage plane.

The calculated value of \( \sigma_{11} \) is equal to 3.6 GPa and can be considered as an estimate of the opening strength of silicon in this special geometry. In fact, a combination of mode I (tensile opening) and mode II (in-plane shear sliding) processes is expected to control the fracture geometry. The resulting stress components for a biaxial fracture mechanism in the tilted coordinate system are \( \sigma_{11}^T = 3.6 \) GPa and \( \sigma_{11}^T = -1.3 \) GPa. Fig. 13b illustrates the biaxial fracture components with respect to the \( (11 - 1) \) cleavage plane for this geometry.

With the extension of crystalline silicon devices to smaller and smaller sizes the dependence of the mechanical strength on the system size becomes an important issue. In micro-electro-mechanical-system (MEMS) and nano-electro-mechanical-system (NEMS) applications, the mechanical stability is essential for their manipulation, functionalization, and integration into more complex systems. In general it is expected that the strength increases with decreasing size of the system owing to the smaller number of crystal defects such as voids, microcracks, or dislocations in the smaller volume. In early work on the fracture strength of silicon whiskers with diameters on the micrometer scale (\( \sim 20 \) μm) tensile fracture strengths of 2–8 GPa were found. These results are in the same range as the values measured for well-defined geometries. This can be interpreted by the assumption that similar fracture geometries and failure mechanisms were involved in these failure processes [56].

In recent years, several techniques, such as the chemical-vapor-deposition (CVD), vapor–liquid–solid (VLS), and CVD-VLS methods, have been developed to grow silicon nanowires with diameters down to the few-nanometer range. Currently, however, it is very difficult to extract general conclusions from this pioneering work, since contradictory results have been reported for the size effects [56]. It seems that the strength of silicon can increase to about 12 GPa, as the nanowire diameter decreases to 100–200 nm [57] and 15–60 nm [58] in wires grown along the [111] direction. This value is already near to the theoretical strength for tensile cleavage of silicon along the [111] plane of 22 GPa obtained by ab initio calculations for an ideal defectfree silicon lattice [59].

2.2.5. Nonlinear SAWs by wave interaction with scatterers

The source of nonlinear elasticity can not only originate from the intrinsic nonlinearity of the perfect bulk material, as used in shock wave formation, but also from mechanically damaged samples or local defects. This includes, for example, dislocations or cracks. SAWs have several advantages to image scatterers located at the surface in comparison to Lamb waves. Up to now such scattering experiments have been performed mostly with conventional transducers, e.g., piezo-electric wedge transducers, whereas lasers have been used especially for detecting the second order harmonics generated by the material’s damage [60]. In this work, an increase in the second harmonic amplitude was detected that was induced during low cycle fatigue tests and by monotonic tensile load of the material above the yield stress of the nickel-base superalloy. A goal of these experiments is the development of more sensitive methods, allowing NDE in the early stages of fatigue prior
to crack formation, by observing the nonlinear features of the nonlinear interaction process.

Another nonlinear acoustic signature of a crack is frequency mixing. Here a pair of laser beams is intensity modulated at two different frequencies and is used to induce parametric wave interaction, which proves the elastic nonlinearity of the crack. Optical interferometry is applied to detect the acoustic waves at mixed frequencies [61]. The highest contrast in crack imaging is found for partially closed cracks. Alternatively, dual frequency mixing can be achieved with a transducer used as low frequency pump SAW and a laser employed as high frequency probe SAW [62]. The nonlinear interaction has been measured as a phase modulation of the probe SAW and equated to a velocity change. The velocity-stress relationship has been considered as a measure of the material nonlinearity.

3. Wedge waves

3.1. Linear wedge waves

3.1.1. Theoretical approaches

As stated above, linear WWs were first discovered by numerical calculations in 1972, independently by Lagasse [9] and by Maradudin and coworkers [10]. The semi-analytical finite element method (FEM), used originally by Lagasse, is still being used [9,63,64]. He assumed translational invariance along the apex and reduced the three-dimensional (3D) problem to a two-dimensional (2D) one by introducing a 1D wavevector. He found an empirical formula for the dependence of the phase velocity of the anti-symmetric flexural (ASF) modes on the wedge angle $\Theta$

$$C_W = c_R \sin(n\Theta),$$

(3.1)

where $c_R$ is the Rayleigh velocity of the isotropic material, and $n$ is the mode number [65]. The main advantage of this method, besides its fast convergence, is its flexibility concerning shape modifications. It leads directly to the dispersion relations for modified wedge waves in the limit of very small wedge angles on the basis of thin-plate theory or, equivalently, in an expansion of the velocity and displacement field in powers of the wedge angle [70,71].

$$u_m(x_1,x_2,x_3;t) = \exp(i(kx_1 - \omega t))U_m(x_2,x_3).$$

(3.3)

The $x_1$ axis is chosen along the tip of the wedge. Inserting this in the Lagrangian density, integrating over the cross section $A$ of the wedge in the $x_2-x_3$ plane, and averaging over a time period $2\pi/\omega$ leads to the functional

$$J = \int \int \{ \rho \omega^2 U_m'' - |D_l(k)U_m|^2 + |D_m(k)U_m| \} dx_2 dx_3,$$

(3.4)

where we have introduced the operator $D_n(k)$ with components $D_l(k) = ik$, $D_m(k) = \imath \omega / \partial x_3$, and $D_0(k) = \imath \omega / \partial x_1$. When applying the finite element method, the area $A$ is divided into (usually triangular) elements and $U_m(x_2,x_3)$ is expanded in shape functions $N_j(x_2,x_3)$.

$$U_m(x_2,x_3) = \sum_j N_j(x_2,x_3) U_j,$$

(3.5)

with the node variables $U_j$ as expansion coefficients. As shape functions, polynomials of at least second order were employed. In the case of second-order polynomials, there are six shape functions and six nodes for each element. Insertion of Eq. (3.5) in the functional $J$ and variation with respect to the node variables leads to a generalized eigenvalue problem of the form

$$\omega^2 \sum_{mn} M_{mn} U_n'' = \sum_{nm} K_{nm}(k) U_m''$$

(3.6)

with the symmetric mass matrix ($M_{mn}$) and Hermitian (generalized) stiffness matrix ($K_{nm}$). The latter depends on the wavenumber $k$. For the Laguerre function approach, a linear transformation (conformal mapping) is performed from the original coordinates $x_2, x_3$ to new dimensionless variables $\eta, \zeta$:

$$\begin{pmatrix} \eta \\ \zeta \end{pmatrix} = k \mathcal{R} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$$

(3.7)

The $2 \times 2$ matrix $\mathcal{R}$ maps the wedge of given wedge angle to a rectangular one and its two faces to the two planes $\eta = 0$ and $\zeta = 0$. The functions

$$U_m(x_1,x_2) = \bar{U}_m(\eta, \zeta)$$

(3.8)

are subsequently expanded in products of two Laguerre functions,

$$\bar{U}_m(\eta, \zeta) = \sum_{k,l} \phi_k(\eta) \phi_l(\zeta) a_{ml}^{(m)},$$

(3.9)

with (in general complex) expansion coefficients $a_{ml}^{(m)}$. The Laguerre functions are defined in the usual way,

$$\phi_l(\zeta) = \exp(-\zeta/2) L_l(\zeta),$$

(3.10)

where $L_l(\zeta)$ is the $l$th Laguerre polynomial. Insertion of Eqs. (3.8) and (3.9) in the functional $J$, truncation of the sums over $k$ and $l$, and variation with respect to the expansion coefficients yields a Hermitian eigenvalue problem,

$$\begin{pmatrix} \lambda \mathcal{R}^T & c \mathcal{R}^T \end{pmatrix} \begin{pmatrix} a_{ml}^{(m)} \\ a_{nl}^{(n)} \end{pmatrix} = \sum_{n,l} \mathcal{H}^{mn}_{nl} a_{nl}^{(n)}.$$

(3.11)

For sharp-angle wedges, convergence can be improved by introducing a scale factor in the arguments of the Laguerre functions [69].

In both approaches, the dispersions and phase velocities are extracted from the smallest eigenvalues and the corresponding eigenvectors that solve the (generalized) eigenvalue problem. Both methods deliver the frequencies corresponding to elastic waves that are totally localized at the wedge tip.

In addition to these numerical approaches, analytical results were obtained for the wave speed and displacement field of wedge waves in the limit of very small wedge angles on the basis of thin-plate theory or, equivalently, in an expansion of the velocity and displacement field in powers of the wedge angle [70,71].
Furthermore, a large number of analytical results referring to sharp-angle as well as obtuse-angle wedges have been developed within the approximation of geometrical acoustics. These approaches are described in more detailed reviews [72,73].

First we consider theoretical results on the number of wedge modes, their phase velocity, and their symmetry as a function of the wedge angle. Generally, the number of modes increases with decreasing wedge angle and the higher modes successively approach the Rayleigh velocity. In Fig. 14 two ASF wedge modes are shown. The large deviation of the velocity of fundamental modes with low wedge angle, with respect to SAWs, is sufficient to ensure strong guiding. On the other hand, the higher modes of larger wedge angles tend to couple effectively with SAWs. For this reason only one fundamental mode is expected to be detectable for wedge angles \( >45^\circ \). Similar to the waves guided by traction-free surfaces, “leaky” or pseudo-wedge waves (p-WWs) can exist, coupled to SAWs or even bulk acoustic waves. A rigorous proof of the existence of localized waveguide modes for isotropic wedges with angles \( <90^\circ \) has been presented [74]. However, a fundamental antisymmetric ASF mode may exist even for aperture angles up to \( 101.25^\circ \) [66,75]. A symmetric mode may propagate for some intervals of angles larger than \( 90^\circ \) [75]. There is also an overlapping region, where both anti-symmetric and symmetric modes coexist. However, the domain of existence of symmetric wedge modes refers to Poisson ratios smaller than 0.14 and hence covers mainly non-standard isotropic materials [74]. Results reported in [66,76] indicate that symmetric modes existing at larger Poisson ratios have phase velocities very close to the Rayleigh wave velocity of the isotropic medium. Currently, essentially all experiments have been performed and interpreted in terms of ASF wedge modes. It is important to note that classification of wedges by symmetry is quite often used, however, this may not be applicable in the case of anisotropic materials, since the mid-apex plane would have to be a symmetry plane.

3.1.2. Excitation and detection of wedge modes

In early work, interdigital structures placed across the edge of the piezoelectric wedge [77] or a pair of broadband shear transducers bonded to the ends of the wedge [78] were employed for the generation and detection of flexural WWs. Compared with these methods the application of a pulsed laser for excitation and a cw laser to monitor the surface displacement with an interferometer [79] or the surface velocity with a laser-based probe-beam-deflection setup [80,81] has several advantages. Therefore the laser techniques are mainly used in the more recent work on WWs (see Fig. 15). Most important is the high spatial resolution, which allows the pump and probe to be placed very near the tip, but also the accurate measurement of the phase velocity along the edge. Furthermore, to monitor the decay of the displacement perpendicular to the edge tip a high spatial resolution is needed. The high power achievable with laser radiation makes this approach most suitable for future studies on nonlinear WWs.

Recently, a novel device for the selective excitation of guided waves by a pulsed laser-operated angle-tunable wedge transducer has been introduced that allows, for example, the separate efficient excitation of WWs, SAWs, and pseudo-wedge waves (p-WWs) [81]. The separate detection of these modes can be seen in Fig. 16. The advantage of this laser-based transducer is that the desired wedge or surface mode can be launched with maximal efficiency, while excitation of all other guided modes is more or less suppressed. This compensates for a possible loss in simplicity and spatial resolution using this particular laser-based contact transducer.

### 3.1.3. Properties of perfect wedges

The requirements on the quality of wedges and especially the wedge tip vary with the wavelength employed but are generally high. It seems to be difficult to reach a sufficient wedge quality in most materials by conventional cutting and polishing of the material. Anisotropic single crystals with pronounced cleavage planes provide a unique possibility to prepare high quality samples by cleavage along the weakest cleavage plane [82,83]. In fact, up to now cleaved wedges have reached the highest perfection and essentially dispersion-free propagation of WWs could be observed. In general, even wedge modifications in the micrometer range may be critical and generate measurable dispersion effects, when the gigahertz frequency range is approached. On the other hand, such a sensitivity of the phase velocity to wedge modifications can be employed to assess the quality of edges in cutting tools and blades. The effort needed for the preparation of suitable samples is one reason why the field of dispersion-free wedges is not yet further advanced. The lack of versatile easy-to-apply excitation and detection methods is another reason.

As an example of a nearly perfect wedge we consider experiments with a cleaved silicon wedge consisting of two \((111)\) planes forming a wedge angle of 70.5° [81]. For selective wave excitation the optical variable-angle transducer and for detection the probe-beam-deflection setup were employed. With this arrangement the fundamental WW, the SAW, and a supersonic pseudo-wedge wave (p-WW) were excited separately. By measuring the propagation time of these waves between the source and various detection points along the edge the phase velocity was measured, clearly showing that the p-WW is faster than the SAW with its wavevector along the edge. The wavevector along one of the wedge faces is controlled by varying the angle \( \phi \) of the wedge transducer [81]. The slowest mode was the localized WW propagating at \( 4.2 \) km/s for \( \phi \approx 43^\circ \), then the SAW propagating along the tip.
was generated at $\varphi \approx 39^\circ$ with a velocity of 4.54 km/s. Further velocity scanning revealed a new wave at $\varphi \approx 32^\circ$, propagating with a velocity of 5.4 km/s, which is substantially above the SAW velocity of silicon in this direction. By shifting the probe laser perpendicular to the edge the strong signal decay with separation from the tip could be verified directly, as displayed in Fig. 17a. This decay is weaker for the p-WW and its coupling with the SAW is clearly visible at the surface, as can be seen in Fig. 17b. This observation clearly indicates that the plane wave front is lost with depth, contrary to the behavior of the localized WW. A more detailed analysis shows that the supersonic wave couples not only with the SAW but also with the slower bulk shear waves, which cannot be seen as easily.

3.1.4. Dispersive wedge waves

Currently, the vast majority of published papers deals with dispersive WWs. Reasons for dispersion are, for example, truncation of the wedge tip or coating of one or both wedge faces. Still, of course, the sample must be prepared with great care so that the registered dispersion effect originates only from the considered source and not from additional wedge deficiencies.

Let us consider a rounded wedge and continuous flaws along the ridge of a rectangular waveguide. An ideal sharp wedge has zero thickness at the tip and no characteristic dimension and consequently no dispersion. Truncation by rounding of the wedge tip increases the apex angle close to the tip, causing higher frequency components to travel with a higher speed (closer to the Rayleigh velocity), whereas low frequencies propagate essentially with the velocity of the rectangular wedge [84]. Conversely, a wedge with sharp protruding defects acts as a wedge with an effective apex angle smaller than 90° close to the tip. Thus, high-frequency waves concentrated near the tip have a slower velocity than the low-frequency components travelling with the 90°-wedge velocity. These qualitative explanations agree with the frequency dependence observed experimentally, where the normal and anomalous dispersions disappear with increasing acuteness and transition to the undistorted shape of the wedge tip, as expected [84].

As an example of the effect of a coated layer on one of the wedge faces we discuss the cases of an aluminum wedge coated with a copper film and a brass wedge coated with an aluminum film [85]. Acoustically the first system corresponds to the loading case on a plane surface with a slow coating and the second to the stiffening case with a fast coating. The corresponding normal and anomalous dispersion behaviors were studied for several wedge angles [85]. Interestingly, there is an important difference between SAW propagation on a coated plane surface and WW propagation on a wedge concerning sensitivity. While the influence of a micrometer-thick layer can be enormous at the very tip, it may become immeasurably small at larger distances from the wedge tip, where the wedge is broadened considerably. Furthermore, the distance that still has an influence on the dispersion behavior is, of course, greater in the long-wavelength regime than in the small-wavelength regime.

Wedge modes have also been used to detect the adsorption of moisture at the wedge faces in a humidity sensor. The applicability of the corresponding velocity effect has been demonstrated by coating the wedge tip with a hygroscopic film that adsorbs water effectively [86]. It has also been demonstrated that the velocity of the fundamental ASF mode decreases upon water loading for wedge angles of 20–90° [87]. This effect is more pronounced for a larger mass density ratio of liquid/solid. A reduction of the phase velocity by fluid loading was also found for wedges with very small wedge angles [88] and has been confirmed theoretically [89,90] and experimentally [91,92].

A first report on the visualization of an isolated artificial rectangular notch placed on the wedge tip has been published recently [93]. Here a movable pulsed laser was scanned over the area of one wedge face. The signals of the propagating waves were registered with a shear transducer fixed at the end of the wedge. The detected signals were piled up into a data cube $(x,y,t)$ and pictures were generated by time-gating at various elapsed times. With this visualization system the reflection and transmission of the ASF mode through the defect and mode conversion of the ASF mode into a SAW at the notch could be observed. Despite the fact that the notch was relatively large, with a size in the millimeter range, this technique opens up the way to dynamic visualization of isolated defects on edges using WWs.

3.2. Nonlinear wedge waves

3.2.1. Theory

One important impetus for the investigation of WWs is the expectation that strong nonlinearities and large strains, owing to the extreme localization of elastic energy in the apex of the wedge, can be realized experimentally. Early measurements, using a cleaved LiNbO$_3$ crystal, were rather disappointing, since harmonic generation and parametric conversion were even less effective than estimated for nonlinear SAWs. In fact, third-order generation
outperformed second-order generation above a certain excitation power, because generation of even harmonics and second-order nonlinear effects cancelled out for symmetry reasons [82]. Unfortunately, the role played by nonlinearity in isotropic and anisotropic wedges presently is not well understood, especially what quantitative aspects are concerned.

Nonlinear evolution equations for symmetric and for anti-symmetric wedge waves in isotropic media have been derived [94]. The equation for symmetric waves contains a nonlinearity of second-order and looks very similar to Eq. (2.12) valid for SAWs. In fact, when defining $B(\omega)$ as the Fourier transform of a nonzero displacement gradient component $u_{mn}(t)$ at the tip of the wedge,

$$u_{nm}(x, 0; t) = 2 \text{Re} \left\{ \int_0^\infty B(\omega) \exp \left[ i \omega t - \frac{\omega x}{c_W} \right] \frac{d\omega}{2\pi} \right\}$$

where $c_W$ is the phase velocity of a linear wedge wave, the following evolution equation is obtained:

$$k_c \frac{\partial}{\partial x} B(\omega, x) = \alpha \left\{ \int_0^\infty G(\omega') B(\omega', x) B(\omega - \omega', x) d\omega' - \frac{2}{\pi} \right\} + 2 \int_0^\infty \frac{\omega}{\omega^2} G(\omega) B(\omega, x) B'(\omega' - \omega, x) d\omega'$$

(3.12)

Like the function $F(\eta)$ in the evolution Eq. (2.12) for nonlinear SAWs, the kernel function $G(\eta)$ depends on ratios of second-order and third-order elastic moduli of the elastic medium the guided wave is propagating in.

This type of evolution equation also holds for anisotropic media in wedge geometries of sufficiently low symmetry such that a classification of the wedge modes into symmetric and anti-symmetric is not possible. The kernel function $G(\eta)$ was computed for various wedge geometries of silicon crystals [68,95]. Apart from quantitative differences the kernel functions $F$ and $G$, the main difference between the evolution Eq. (2.12) for SAWs and (3.13) for symmetric WWs is the power of the factor $\omega/\omega'$ in the second integral on the right-hand sides. The exponent 2 for WWs in comparison to 1 for SAWs indicates that frequency down-conversion is even less effective for wedge waves than it is for surface waves. Therefore, wave steepening and shock formation of wedge waves may be favored as compared to surface waves.

The nonlinear evolution equation for ASF waves in isotropic media contains an effective nonlinearity of third order. Little is known about its size apart from earlier work for wedges with very small wedge angle [96]. Here, the nonlinearity is of purely geometric character and therefore governed by the linear Lamé constants only, while the third-order and fourth-order elastic moduli are irrelevant in this case.

3.2.2. Experimental

For isotropic wedges experimental results on dispersion-free wedges are rare. It may be expected that the influence of anisotropy on the nonlinear behavior of 1D WWs can be pronounced than for 2D SAWs, at least for selected geometries or crystallographic configurations. For rectangular edges of silicon it was found theoretically that no edge mode may exist for the case of one surface normal being along the (001) direction and the other along the (110) direction, in agreement with preliminary experiments [68]. For a second geometry with the two surface normals along cubic axes, a well-localized edge mode was found theoretically and confirmed experimentally. The resonant second-order nonlinearity vanishes for this geometry and nonlinear wavefront evolution is governed by an effective nonlinearity of third order. In a third rectangular geometry with the surface normals (011) and (111), an edge mode was found by calculations, which is predicted to exhibit appreciable second-order nonlinear effects, including the formation of solitary pulses in the presence of weak dispersion [68]. Spiking and shock formation predicted on the basis of the evolution Eq. (3.13) [97] could be confirmed experimentally for this particular geometry. Note that these nonlinear features are strongly geometry-dependent and reflect the strong effects of anisotropy.

Despite the fact that the strong nonlinearity of certain edge-localized modes is expected from these theoretical considerations, 1D solitary waves, feasible after introducing weak dispersion by tip modification or coating of one or both wedge faces, have not yet been realized, whereas nonlinear wedge waves developing steep shock profiles during propagation have been seen.

3.3. Applications of wedge waves

A number of possible applications have been discussed for linear and nonlinear edge waves and selected feasibility studies have been carried out already for linear WWs, however, no devices are on the market, as in the case of the well-developed SAWs. Such envisaged future applications include NDE of defects in cutting tools or turbine blades, sensor applications measuring the velocity change upon wedge face and tip modifications, ultrasonic motors, stirring and acoustic streaming in fluidics, and aquatic propulsion. Nonlinear WWs would open the door to 1D solitary waves in NDE of absorbing media and the investigation of the mechanical fracture strength and failure behavior of wedges. With improved preparation techniques for high-quality samples and strongly improved computing power for simulations, 1D wedge waves may soon leave the shadow of the well-established 2D surface waves. Especially in the intricate situation of anisotropic wedges, with respect to experiments, the simultaneous support of the selection of sample configurations by theoretical simulations is indispensable, owing to the rather complex behavior of these systems and the large variety of geometrical possibilities.

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