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# *Rotating Savings and Credit Associations when Participants are Risk Averse*

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Abstract:

Recent theoretical research on rotating savings and credit associations (Roscas) has assumed that Rosca participants do not face income uncertainty and save for a lumpy durable or an investment good. In this literature, no new information becomes available during the course of a Rosca and consequently all auctions which a bidding Rosca involves take place at the beginning of the Rosca. Here, in contrast, under the assumption that participants are risk averse and that their incomes are stochastic and independently drawn in each round, new information becomes available each time the Rosca participants meet. We model a bidding Rosca, where each auction is concurrent with the allotment of Rosca funds. When individual incomes are privately observed, it is shown that a random Rosca is not advantageous, while a bidding Rosca is if temporal risk aversion is less pronounced than static risk aversion. The payoff scheme of a bidding Rosca facilitates risk sharing in the presence of information asymmetries. The risk-sharing performance of a simple arrangement where a group of homogenous individuals runs several bidding Roscas simultaneously is as good as that of a linear risk-sharing contract, and is more enforceable because it carries a fixed rather than a variable contribution.

Keywords: Roscas; Informal Credit; Auctions; Risk Sharing; Insurance; Asymmetric and private information

JEL categories: D44; D82; G20; G22; O16; O17

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## 1. Introduction

The rotating savings and credit association (Rosca), an informal financial institution observed around the world, has attracted considerable theoretical and empirical research in recent years. Roscas are popular among high as well as low income households[2] and flourish in economic settings where formal financial institutions seem to fail to meet the needs of a large fraction of the population. In general terms, a Rosca can be defined as 'a voluntary grouping of individuals who agree to contribute financially at each of a set of uniformly-spaced dates towards the creation of a fund, which will then be allotted in accordance with some prearranged principle to each member of the group in turn' (Calomiris and Rajaraman, 1998). Once a member has received a fund, also called a pot, she is excluded from the allotment of future pots until the Rosca ends. In a so-called random Rosca, a lot determines each period's 'winner' of the pot. In a bidding Rosca, an auction is staged among the members who have not yet received a pot. The highest bid wins the pot and the price the winner pays is distributed among the members or added to future pots. In a third, empirically relevant, allocation mechanism, the decision on each period's allocation of the pot is left to the Rosca organizer.[3]

The name suggests that Roscas serve as a financial intermediary by transforming the bundled savings of a group into what might be considered a loan to one Rosca participant in each period. The theoretical literature on Roscas has focused exclusively on participants with non-stochastic incomes. Kuo (1993) analyses bidding Roscas when individuals differ with respect to their discount rates. These are drawn from a common distribution and each is private knowledge. In every period, moreover, each participant is assigned a new discount factor. Assuming that all participants share the same beliefs about the distribution of other participants' discount factors, the author derives Bayes-Nash equilibrium bidding strategies. In Kovsted and Lyk-Jensen (1999), each participant can engage in an investment project and has limited access to outside credit. The revenues of the projects differ among participants. The revenue yielded by each participant's project is his private information, but all participants have the same beliefs about the distribution

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[2] While Levenson and Besley (1996) find that participation is highest among high income households in Taiwan, Handa and Kirton (1999) report that, in Jamaica, low income households are most likely to join a Rosca.

[3] In Handa and Kirton's (1999) sample, 53 percent of the Roscas operated in this way.

of revenues of the other participants' projects. Deriving Bayes-Nash equilibrium bidding strategies, the authors find that when outside credit is not too costly, and when the distribution of revenues is sufficiently widely dispersed, a bidding Rosca is preferred to a random Rosca. Besley et al. (1993, 1995) assume that participants do not have access to outside credit and join a Rosca to finance a durable good whose costs require saving for more than one period. If participants have identical preferences and incomes, a random Rosca is preferred to the bidding arrangement. If, however, participants are sufficiently heterogeneous, a bidding Rosca can be preferred to a random Rosca. In the papers by Besley et al. and Kovsted and Lyk-Jensen, the auctions serve as a mechanism to allocate pots earlier to participants who have a higher willingness to pay and can therefore be advantageous if participants are not identical.

In many economic settings where Roscas are found, individuals are exposed to both idiosyncratic and aggregate risks. Examples are farmers' uncertainty about harvests, employment uncertainty among casual laborers and individual illness when no health insurance is available. There is a body of empirical evidence that, when participants are exposed to risk, Roscas can serve as a risk-sharing mechanism (Calomiris and Rajaraman, 1998). Calomiris and Rajaraman find that, except for one case[4], all of the empirical literature reports Rosca arrangements where bidding is concurrent with the allocation of pots. They argue that the auctions in the course of the Rosca allow the participants to respond to income shocks, which are not yet realized when they join the Rosca. In the approaches taken by Besley et al. (1993, 1994) and Kovsted and Lyk-Jensen (1999), however, random variables are either not existent or realized before the beginning of the Rosca. In Kovsted and Lyk-Jensen's model, the outcome of a bidding Rosca does not depend on whether the auctions for all future pots are staged at the beginning of the Rosca or concurrently with allotment. In Besley et al.'s model, all bidding even has to occur ex ante, that is before the first pot is allocated.

Another striking difference lies in the course of the winning bid from period to period. In their field studies on Roscas in India, Calomiris and Rajaraman (1998) and Klonner (2002) do not find that winning bids decrease steadily from auction to auction as predicted by the models of Besley et al. (1993) and Kovsted and Lyk-Jensen (1999). Instead, the sequence of winning bids exhibits considerable volatility. Calomiris and Rajaraman conclude that, at least for their

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[4] This is Campbell and Ahn (1962) for Korea.

particular Rosca, deterministic models with ex ante bids do not capture the observed features. Instead, they hypothesize that a bidding Rosca may serve as a risk-sharing device by allocating each period's pot to the bidder who has suffered the most severe shock.

Besley et al. (1993) argue that Roscas are not suited for insuring against risk because the fund can be obtained only once, whereas shocks might occur several times during the duration of the Rosca. Empirical evidence, however, shows that many individuals are members of several Roscas or hold more than one share in the same Rosca, thus being entitled to bid for more than one pot.[5] Of course, Roscas cannot effectively insure against aggregate shocks when participants belong to an economically and socially homogenous group like small farmers in a village whose harvests depend on the weather to a large extent. But even here, as Townsend's (1994) results suggest, a variety of mechanisms appear to be at work in providing substantial insurance against idiosyncratic risks like illness or the death of farm animals. We do not claim that Roscas never play a role for the accumulation of funds to finance lumpy goods. There is, however, a startling imbalance between the number of empirical studies which stress the risk-sharing aspect of Roscas and the focus of theoretical papers on Roscas, which so far have dealt only with the lumpy goods story. It is this imbalance which motivates the present paper.

We analyze how a bidding Rosca functions under the following assumptions, which are set out and discussed in detail in Section 2: First, participants are risk averse and use funds from the Rosca entirely for consumption, each participant's income being stochastic. Second, participants cannot observe other participants' incomes, but all share the same beliefs about the distribution from which the incomes are drawn. By assuming such a private information environment, the analysis focuses on urban Roscas among homogenous participants who do not observe each other's incomes, e. g. hawkers and shoeshine boys (see Nayar, 1983, and Calomiris and Rajaraman, 1998, for examples from India). Third, we employ the invariable assumption in the literature on risk sharing (see, among many others, Coate and Ravallion, 1993, and Wang, 1995) that transfers of income across periods are possible neither through storage nor through borrowing and lending. Fourth, the analysis of this paper is restricted to the case of only a small

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[5] Geertz (1962) reports that, in a sample of Roscas in Java, almost every woman belonged to at least two Roscas.

number of individuals joining either one or several Roscas at the same time.[6] Section 2 derives the bidding equilibrium of a two-participant bidding Rosca with an oral ascending price auction and compares the bidding equilibrium of such a Rosca auction with that of a standard oral ascending price auction. Section 3 investigates the relationship between preferences for risk bearing and preferences for random and bidding Roscas. In this connection, we make use of the concept of temporal risk aversion to determine under which conditions on individual preferences a bidding Rosca works as a risk-sharing device. It turns out that these conditions also play a crucial role for an individual's decision to join a bidding or a random Rosca. Section 4 develops a risk-sharing mechanism for two individuals based on simultaneous participation in several bidding Roscas and evaluates such an arrangement against the benchmark of a linear risk-sharing contract. The final section summarizes the findings and offers conclusions.

## 2. Risk Sharing and the Functioning of Bidding Roscas under Private Information

It is well known that in the absence of borrowing and savings opportunities, the optimal risk-sharing contract among $n$ ex ante identical individuals involves pooling all individual incomes and allocating one $n$'th thereof to each individual in each period. Such an arrangement, however, requires that, in any period, each individual's income is public knowledge. While this is a reasonable assumption for residents of a small rural village who generate their incomes within that village, it is less persuasive in an urban setting, where the income of a casual laborer may be exclusively his private knowledge. In such cases, the arrangement just outlined collapses, because there is an incentive to underreport one's income and thus contribute less to the income pool. Thus a risk-sharing mechanism under such informational constraints must give individuals an incentive to report their incomes truthfully. Specifically, in a two-individual-two-period context, such an incentive can be generated by intertemporal trade, compensating the individual who is a net payer in the first period with a positive net transfer in the second. If the world ends after two periods, then, in the second period, no further intertemporal trade can take place. Exactly these features can be found in a two-participant-two-period bidding Rosca: before the first period, the two participants $A$ and $B$ make an arrangement whereby each pays a stipulated amount $m$ into a pot in each period. In the first period, the participants bid for pot one. Assuming

[6] An economy where many individuals engage in many Roscas simultaneously is investigated in a companion paper (Klonner, 2000).

that half the price paid for this pot, $b$ say, is allocated to each participant, one would expect the participant with the higher current need for funds, $A$ say, to win this auction. In this context, 'higher need' is equivalent to 'lower first-period income'. Consequently, in the first period, $A$ receives a net transfer of $m - b/2$ from $B$. According to the rules of the Rosca, however, $B$ receives the pot and thus a net transfer of $m$ from $A$ in the second period. This latter transfer can be viewed as the 'price' $A$ has to pay for the transfer she received in the first period.

To set out the analytical framework, assume that each of the two participants[7] evaluates consumption levels in periods one and two $c_{i1}$ and $c_{i2}$, respectively, with a bivariate von-Neuman-Morgenstern utility function, $u(c_{i1}, c_{i2})$, which is strictly increasing and concave in each argument, and that, in period $t$, her income $y_{it}$[8] is drawn from a distribution characterized by the smooth distribution function $F$ on the domain $I = [y_l, y_u]$. All $Y_{it}$, $i, t = 1, 2$ are assumed to be independently and identically distributed according to $F$. Support for this assumption comes from the fact that Rosca participants typically belong to the same social and professional group (see, e. g., Bouman, 1979). It is further assumed that participants have access to neither credit nor savings opportunities outside the Rosca. Although the absence of savings opportunities in particular appears to be a very restrictive assumption, it is a fact that in many urban settings where Roscas are observed, it may be dangerous or even impossible to store money. A wife[9], for example, might not have control over money that is not invested in some fixed scheme because her husband may have different, likely more short-sighted, ideas about how to use the money.[10] In this section it is further assumed that each individual participates in only one Rosca and that

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[7] For ease of exposition, I restrict attention to two-period Roscas.

[8] Throughout the paper, random variables are denoted by capital letters, while lower case letters denote realizations.

[9] See, for example, Adams and Canavesi de Saherno (1989) for Roscas in Bolivia or Anderson and Balland (forthcoming), who find only women in their sample Roscas.

[10] Anderson and Baland (forthcoming) find strong support for this claim in their sample of low-income households in Nairobi. Thomas (1993) reports that income in the hands of women tends to increase the share of the household's budget spent on health, education and housing as well as improvements in child health.

the contribution to the Rosca each member makes in each period, $m$ say, has been agreed upon beforehand and can be considered fixed.

To avoid technical complications, we assume that participants can always pay their contribution $m$, even if they are hit by the most severe income shock possible. Formally, define $c_{\min} \equiv y_l - m$. We require that $c_{\min} \geq 0$ and that $u(x_1, x_2)$ is strictly bounded from below on the domain $D_u \equiv \{(x_1, x_2): x_1 \geq c_{\min}, x_2 \geq c_{\min}\}$.

Any problems of enforceability of contributions to the Rosca by members who have received a pot earlier, and are thus left with only obligations, are neglected. This can be justified by the fact that defaulting on contributions results in exclusion from future Roscas and by assuming that the disutility therefrom is prohibitively high.[11] Another important empirical feature, the remuneration of the Rosca organizer, is also excluded from this analysis.

In the literature, a variety of arrangements have been observed when it comes to the auctioning of the pot. There are various rules determining how the price for a period's pot is used. The most important issue is whether the said price is added to future pots (as in Calomiris and Rajaraman, 1998), or distributed at once. This paper concentrates on the latter system.

We will confine our analysis to oral ascending bid (OA) auctions, which are most frequently reported in empirical studies on bidding Roscas. In an OA-auction Rosca, the active participants meet and submit successive oral bids until only one bidder, the winner, remains. As in the analysis of standard auctions with symmetric, independent private value (SIPV) bidders,[12] one might ideally think of such an OA auction as a so called button auction where each bidder presses a button in front of her as the standing bid continuously increases. A bidder drops out of the bidding process once she releases her button. The auction is over once there is only one bidder still pressing her button. She receives the pot at a price equal to the standing bid at the

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[11] There is sufficient empirical evidence in support of this assumption. See, among others, Calomiris and Rajaraman (1998).

[12] In a standard SIPV bidder auction, there is one seller, who owns a single, indivisible item, and $n$ buyers. Each bidder knows $n$ and his own valuation (or value, in short) for the item, which is the maximum amount he would be willing to pay for the item, but none of the other bidders' values. The values are identically and independently distributed (see Matthews, 1990). It is further assumed that the seller cannot set a minimum price.

moment her last competitor dropped out. For the derivation of bidding equilibria in the OA auction, it is useful to consider a second-price, sealed bid (SPS) auction. In this auction, the active participants submit their bids in sealed envelopes. The highest bid wins and the winner pays a price equal to the second highest bid submitted. Although this type of auction is not reported in any of the Rosca literature, its equilibrium is also an equilibrium in the OA auction. In the button auction, each bidder's problem is to decide when to release her button. Suppose that each participant releases her button at a standing bid equal to her bid in the SPS auction. If participants follow this rule, the payoffs to all participants are equal in the SPS and the OA auction. In the language of game theory, the reduced normal form games corresponding to the second-price, sealed bid and the oral ascending bid auction are identical. Thus they are strategically equivalent, which implies that the equilibrium of the SPS auction is also an equilibrium of the OA auction.[13]

With this in hand, we can now embark on the strategic analysis of Rosca auctions. As a first step, it is useful to introduce the concept of the maximum willingness to pay for period one's pot. After a participant, $i$ say, has observed her first-period income, $y$ say, and if she receives pot one at price $b$, her consumption in the first period is given by $y - m + (2m - b + b/2) = y + m - b/2$, where $y - m$ is her first-period income minus her contribution to the Rosca and $(2m - b + b/2)$ is the pot she receives minus the price $b$, plus the half of this price that is redistributed to her according to the rules of the Rosca. In this case, $i$'s second-period consumption is $y_2 - m$, where $y_2$ denotes the realization of her second-period income. Accordingly, her expected utility after observing $y$ is $u^{\text{win}}(b, y) \equiv \tilde{u}(y + m - b/2, Y - m)$, where $Y$ denotes the random variable corresponding to $y_2$ and $\tilde{u}(\cdot, X) \equiv E_X[u(\cdot, X)] = \int_{y_l}^{y_u} u(\cdot, x)dF(x)$. If, on the other hand, the other participant receives pot one at price $b$, $i$'s expected utility is given by $u^{\text{lose}}(b, y) \equiv \tilde{u}(y - m + b/2, Y + m)$. Now consider a Rosca with an OA auction where the standing bid $b$ is raised subsequently. At low levels of $b$, a bidder with first-period income $y$ prefers winning pot one to losing it, formally $u^{\text{win}}(b, y) > u^{\text{lose}}(b, y)$ for sufficiently small $b$. Given the definition of $u(\cdot,\cdot)$, however, the said bidder's preference over winning or losing pot one is reversed at

[13] This result is in line with the literature on standard SIPV bidder auctions. See, e.g., Matthews (1990).

sufficiently high levels of the standing bid, formally $u^{\text{win}}(b, y) < u^{\text{lose}}(b, y)$ for sufficiently big $b$. We define the maximum willingness to pay for pot one, $b^0$ say, as that level of the standing bid at which a bidder is indifferent between winning and losing pot one. Formally, $b^0(y)$ is the value of $b$ that satisfies

(1) $$\tilde{u}(y - m + b/2, Y + m) = \tilde{u}(y + m - b/2, Y - m).$$

It is now argued that $b^0$ corresponds to a bidder's value in a standard (not a Rosca) auction with SIPV bidders. In such auctions, by definition, a bidder is indifferent between winning and not winning the item auctioned when she has to pay a price equal to her true value. This definition applies to $b^0(y)$ in the present case; for by (1), a bidder with first-period income $y$ is indifferent between receiving pot one and not receiving it at a level of the standing bid equal to $b^0(y)$. In what follows, it will be assumed that the participant with the more severe income shock in period one has a higher maximum willingness to pay for pot one:

*<u>Assumption 1:</u> $b^0$ is strictly decreasing in first-period income:*

(2) $$\frac{db^0(y)}{dy} = 2\,\frac{\tilde{u}_1(y + m - b^0(y)/2, Y - m) - \tilde{u}_1(y - m + b^0(y)/2, Y + m)}{\tilde{u}_1(y + m - b^0(y)/2, Y - m) + \tilde{u}_1(y - m + b^0(y)/2, Y + m)} < 0 \textit{ for all } y.$$

We now derive the symmetric Bayes-Nash bidding equilibrium of a SPS Rosca auction, which is also a symmetric equilibrium of an OA Rosca auction, as has been argued above. Towards this end, assume that $i$ conjectures that $j$ determines her, i.e. $j$'s, bid $b_j$ according to a smooth, strictly decreasing function $b(y_j)$, where $y_j$ denotes $j$'s first-period income. Because of the private information assumption, for $i$, $j$'s first-period income is a random variable distributed according to $F$. Therefore, from $i$'s perspective, the probability of losing the auction conditional on bidding $b_i$ is $\mathrm{P}(b_i < b(Y_j)) = \mathrm{P}(b^{-1}(b_i) > Y_j) = F(b^{-1}(b_i))$, where the events $b_i < b(Y_j)$ and $b^{-1}(b_i) > Y_j$ are identical by virtue of the assumption that $b(\cdot)$ is a strictly decreasing function. The probability that $i$ loses is, by definition, $1 - \mathrm{P}(b_i < b(Y_j)) = 1 - F(b^{-1}(b_i))$.

If $i$ loses the auction, her expected utility conditional on her first-period income $y_i$ and her bid $b_i$ is $\tilde{u}(y_i - m + b_i/2, Y + m)$. If, on the other hand, $i$ wins the auction, her expected utility

conditional on $y_i$ and $b_i$ is $E[\tilde{u}(y_i + m - b(Y_j)/2, Y - m) | Y_j > b^{-1}(b_i)]$. Consequently, $i$'s interim expected utility[14] as a function of her bid $b_i$ is given by

$$E[U(b_i)|\, y_i] = \tilde{u}(y_i - m + b_i/2, Y + m)\, F(b^{-1}(b_i))$$

(3)

$$+ E[\tilde{u}(y_i + m - b(Y_j)/2, Y - m) | Y_j > b^{-1}(b_i)]\,(1 - F(b^{-1}(b_i))),$$

and $i$'s task is to maximize $E[U(b_i)|\, y_i]$ by choice of $b_i$. The corresponding first-order condition reads

$$\frac{\partial E[U(b_i)|\, y_i]}{\partial b_i} = \tilde{u}_1\big(y_i - m + b_i/2, Y + m\big) F(b^{-1}(b_i))/2$$

(4)

$$+ \Big(\tilde{u}\big(y_i - m + b_i/2, Y + m\big) - \tilde{u}\big(y_i + m - b_i/2, Y - m\big)\Big) \frac{f(b^{-1}(b_i))}{b'(b^{-1}(b_i))} = 0.$$

The Bayes-Nash equilibrium bidding strategy, $b_s(\cdot)$ say, is obtained by substituting $b(y_i)$ for $b_i$ in the RHS of (4), where the subscript $s$ indicates that $b_s(\cdot)$ characterizes an equilibrium of a SPS and thus of an OA Rosca auction.

<u>*Proposition 1*</u>

*Consider a two-participant bidding Rosca with an oral ascending bid auction, in which assumption 1 holds. Then*

($i$) *in a symmetric Bayes-Nash equilibrium, each bidder quits the auction at a standing bid equal to $b_s(y)$, where $y$ is a bidder's privately observed first-period income and*

(5) $$b_s'(y) = 2\frac{f(y)}{F(y)}\left[\frac{\tilde{u}(y + m - b_s(y)/2, Y - m) - \tilde{u}(y - m + b_s(y)/2, Y + m)}{\tilde{u}_1(y - m + b_s(y)/2, Y + m)}\right],$$

(6) $$b_s(y_l) = b^0(y_l);$$

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[14] In accordance with the literature on SIPV auctions, at the interim stage, a bidder has observed her type (in the present case determined by $y_i$) but not yet submitted her bid.

(*ii*) *In such an equilibrium, bidders overbid relative to their maximum willingness to pay, i.e.* $b_s(y) > b^0(y)$ *for all* $y > y_l$*;*

(*iii*) *bids are strictly decreasing in income, i.e.* $b_s'(y) < 0$ *for all* $y$*.*

<u>Proof:</u>

(i) Necessity follows from (4). □

(ii) By applying L'Hôpital's rule to the RHS of (5) in view of the fact that $y_l$ constitutes a singularity for the differential equation (5), we obtain

$$(7) \qquad b_s'(y_l) = 2\frac{\tilde{u}_1(y_l + m - b^0(y_l)/2, Y - m) - \tilde{u}(y_l - m + b^0(y_l)/2, Y + m)}{\tilde{u}_1(y_l + m - b^0(y_l)/2, Y - m) + 2\tilde{u}_1(y_l - m + b^0(y_l)/2, Y + m)},$$

where, according to (6), $b^0(y_l)$ has been substituted for $b_s(y_l)$. Comparing (2) with (7), it follows that

$$(8) \qquad 0 > b_s'(y_l) > b^{0\prime}(y_l).$$

Combining (8) with (6), we obtain

<u>*Lemma 1:*</u> *There exists an* $\varepsilon > 0$ *such that* $b_s(y) > b^0(y)$ *for all* $y_l < y \leq y_l + \varepsilon$*.*

Now assume that

$$(9) \qquad b_s(y_0) = b^0(y_0) \text{ for some } y_0 > y_l.$$

By (1) and (5), (9) implies that

$$(10) \qquad b_s'(y_0) = 0.$$

By assumption 1, however,

$$(11) \qquad b^{0\prime}(y_0) < 0.$$

Hence, for any $y_0 > y_l$, $b_s$ intersects $b^0$ from below, which contradicts Lemma 1. Thus,

(12) $b_s$ and $b^0$ cannot intersect for any $y > y_l$.

Moreover, by combining (10) and (11), we find that (9) implies that $b_s'(y_0) > b^{0\prime}(y_0)$ and hence

(13) $b_s$ cannot touch $b^0$ for any $y > y_l$.

Combining (12) and (13) with Lemma 1 gives the desired result. □

(iii) Since, by virtue of (ii), $b_s(y) > b^0(y)$ for all $y > y_l$, it follows from (1) and the fact that $u$ is strictly increasing in its first argument that the RHS of (5) is negative. Together with the first inequality in (8), this gives the desired result. □

QED

Sufficient conditions for the existence of the Bayes-Nash equilibrium characterized by Proposition 1 are given in Section 1 of the Appendix.

Two things are particularly noteworthy in connection with this proposition. First, part (iii) states that, for any pair of realized first-period incomes, pot one is always allocated to the bidder with the lower first-period income, which means that, provided Assumption 1 holds, the auction serves its purpose and generates a net transfer from the better to the worse off bidder. It is thus a successful device to facilitate risk sharing in the presence of information asymmetries.

Second, the result of part (ii) that, in an OA Rosca auction, bidders overbid relative to their true valuation, is in marked contrast to the bidding equilibrium of a standard SIPV oral English auction, where bidding one's true value is a dominant strategy. As argued above, the analogue to a bidder's value in a standard SIPV auction is $b^0(y)$ in a Rosca auction. Suppose participant $i$ originally intends to quit the auction at a standing bid equal to $b^0(y_i)$ and the other participant, $j$ say, is also still in the auction at $b^0(y_i)$. By staying in the auction up to $b^0(y_i) + \varepsilon$ instead of $b^0(y_i)$, $i$ takes the chance of winning the pot at a price at which she prefers not to win the pot, because, by definition, $u^{\text{lose}}(b^0(y_i) + \varepsilon, y_i) > u^{\text{win}}(b^0(y_i) + \varepsilon, y_i)$. This happens whenever $j$ quits the bidding process before $b^0(y_i) + \varepsilon$. On the other hand, staying in the auction up to $b^0(y_i) + \varepsilon$ instead of $b^0(y_i)$ improves $i$'s situation whenever $j$ intends to quit the bidding process at a standing bid higher than $b^0(y_i) + \varepsilon$, because now, $i$ receives $(b^0(y_i) + \varepsilon)/2$ instead of $b^0(y_i)/2$ as her share of the price $j$ pays for the pot. In a standard SIPV auction, only the former of these two effects is present and therefore, in such auctions, there is no gain from overbidding. Proposition 1, however, shows that, in the equilibrium of a Rosca auction, the gains from overbidding exceed the losses except for a bidder with income $y_l$, who wins the pot with probability one. Thus $b_s(y_l) = b^0(y_l)$. The lesson from this is that, contrary to standard SIPV bidder oral English auctions, bidding in a Rosca auction is always strategic and equilibria in dominant strategies fail to exist. The reason for this arises from the fact that, in the terminology of Kovsted and Lyk-Jensen (1999), in a Rosca auction, the seller is internalized in the group of bidders. As a

consequence, the loser of a Rosca auction is not left with the same economic situation as before the beginning of the auction, but rather receives a gain from the share of the winning bid that is allocated to him.[15]

When the framework developed in this section is extended to model a bidding Rosca with $n > 2$ participants, it is clear that each of the $n - 1$ auctions has to take place concurrent with the allotment of a pot because, in each round, each participant draws her income anew. For that reason, the sequence of winning bids cannot be identified ex ante and, depending on the specification of preferences and income shocks, this sequence may exhibit considerable volatility. Such dynamics are well in accordance with empirical observation, where winning bids do not decrease monotonically from one auction to the next but instead fluctuate significantly (Calomiris and Rajaraman, 1998). This contrasts the predictions of Rosca models with ex ante bids, where the sequence of winning bids is always monotonically decreasing.

## 3. Preferences for Risk Bearing and Preferences for Random and Bidding Roscas

With the results of the previous section in hand, we can now ask: how do preferences for risk bearing influence the decision to participate in a random or a bidding Rosca? We shall make use of the concept of temporal risk aversion, which was first defined by Richard (1975) as follows: a decision maker is said to be multivariate risk averse if, for any pair $(x, y)$, $u_{12}(x, y) < 0$ and multivariate risk seeking if $u_{12}(x, y) > 0$. The case of $u_{12}(x, y) = 0$ is defined as multivariate risk neutrality. If $u$'s arguments refer to consumption at two points in time, 'multivariate' may be replaced by 'temporal' (see Ingersoll, 1987). This concept can be illustrated as follows: Consider two lotteries $L_1$ and $L_2$ that are both resolved in period zero. $L_1$ involves a consumption level of $x$ in both the first and the second period with probability 0.5 and a consumption level of $y$ in both periods with probability 0.5. $L_2$ involves a consumption level of $x$ in the first and $y$ in the second period with probability 0.5, and $y$ in the first and $x$ in the second period with probability 0.5. A temporal risk averse decision maker prefers $L_2$ to $L_1$, while a temporal risk seeking decision maker prefers $L_1$ to $L_2$ for any pair $(x, y)$. Thus, loosely speaking, a temporal risk seeking agent

[15] Rosca auctions share this feature with auctions where the bidders jointly own the item to be auctioned. See Cramton et al. (1987), Engelbrecht-Wiggans (1994), Güth and van Damme (1986), and McAfee and McMillan (1992).

has a preference for lotteries whose payoffs are positively correlated over time while a temporal risk averse agent prefers negatively correlated payoffs.[16]

Since, in reality, a Rosca is planned before the first meeting, it is appropriate to compare an individual's expected utility from consumption streams with and without a Rosca before she observes her first- and second-period income. We will refer to this stage as 'ex ante stage'. We begin with a random Rosca. While uncorrelated without such a Rosca, the consumption levels of participants in a random Rosca are negatively correlated. For a participant in a random Rosca, we can write ex ante expected utility as

(14) $$E[U^R] \equiv E_{Y_1}[\tilde{u}(Y_1+m, Y_2-m)+\tilde{u}(Y_1-m, Y_2+m)]/2\,.$$

For the sake of analytical tractability, we concentrate on Roscas with an infinitesimally small contribution $m$. Evaluating the derivative of (14) w.r.t. $m$ at $m = 0$ yields zero, while the second derivative is

(15) $$\frac{d^2E[U^R]}{dm^2}_{|m=0} = 2\,E_{Y_1}[\tilde{u}_{11}(Y_1,Y_2)+\tilde{u}_{22}(Y_1,Y_2)-2\tilde{u}_{12}(Y_1,Y_2)].$$

It is seen that, if $u_{12} \geq 0$, the said derivative is strictly negative. Thus, not participating in a random Rosca is the optimal decision for temporal risk seeking and temporal risk neutral agents. A simple continuity argument, moreover, establishes at once the result that agents who are sufficiently mildly temporal risk averse will not participate either. In general, however, if $u_{12} < 0$, the case is ambiguous. The question then is whether the effect of temporal risk aversion arising from the negative cross derivative outweighs the effect of static risk aversion arising from the concavity of $u$ in each argument. Formally, similar to Ronn (1988), define the coefficients of static and temporal risk aversion as

$$RA_t(x_1,X_2) \equiv -\frac{\tilde{u}_{tt}(x_1,X_2)}{\tilde{u}_t(x_1,X_2)} \text{ and } TRA_{kt}(x_1,X_2) \equiv -\frac{\tilde{u}_{kt}(x_1,X_2)}{\tilde{u}_t(x_1,X_2)},$$

respectively, and let $E[RA_t]$ and $E[TRA_{kt}]$ denote $-E_{X_1}[\tilde{u}_{tt}(X_1,X_2)]/E_{X_1}[\tilde{u}_t(X_1,X_2)]$ and $-E_{X_1}[\tilde{u}_{kt}(X_1,X_2)]/E_{X_1}[\tilde{u}_t(X_1,X_2)]$, respectively. We can now rewrite (15) as

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[16] Ronn (1988) argues that for a temporal risk averse agent, consumption levels in any two periods are 'substitutes through time', whereas they are complements for a temporal risk seeker.

$$\frac{d^2E[U^R]}{dm^2}_{|m=0} = 2\left(E_{Y_1}[\tilde{u}_1(Y_1,Y_2)]\left(E[TRA_{21}]-E[RA_1]\right)+E_{Y_1}[\tilde{u}_2(Y_1,Y_2)]\left(E[TRA_{12}]-E[RA_2]\right)\right).$$

Defining autarky as not participating in a Rosca, we have

<u>*Proposition 2:*</u> *If*

(16) $E[TRA_{21}] \leq E[RA_1]$ *and* $E[TRA_{12}] \leq E[RA_2]$,

*then autarky is preferred to participation in a random Rosca with a small contribution m.*

A borderline case arises when $u(x_1, x_2) = v(x_1 + x_2)$ for some strictly increasing and concave function $v$.[17] Then $TRA_{tk} = RA_t = RA_k$ and such individuals are indifferent between participating in a random Rosca or not.

Turning to bidding Roscas, before we can make statements about ex ante expected utility, we need to deal with the fact that bidding involves a strategic element, which arises at the interim stage. Since, as for random Roscas, the goal of this section is to decide how a bidding Rosca with an infinitesimally small contribution $m$ affects ex ante expected utility with autarky as the reference point, we first need to analyze the bidding equilibrium of a Rosca auction when $m$ is arbitrarily small. For any fixed $m$, this was accomplished in the previous section. One important result was that the auction always generates a net transfer from the better to the worse off bidder by allocating the first-period pot to the bidder with the lower realized first-period income (see Proposition 1 (iii)). As a sufficient condition, however, Assumption 1 was needed, which states that the maximum willingness to pay for pot one is decreasing in first-period income. When $m$ is chosen sufficiently small, it turns out that Assumption 1 can be replaced by sufficient and necessary conditions.

<u>*Proposition 3*</u>

*In the auction of a two-participant bidding Rosca with an infinitesimally small contribution, a bidding equilibrium which allocates the first-period pot to the participant with the lower first-period income exists if, and only if,*

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[17] If $v(x) = x$, this is the case of risk neutral agents who do not discount future consumption.

(17) $RA_1(y,Y) \geq TRA_{12}(y,Y)$ *for all* $y \in I$ *and*

(18) *there exists an* $\varepsilon > 0$ *such that* $RA_1(y,Y) > TRA_{12}(y,Y)$ *for all* $y_l < y < y_l + \varepsilon$.

<u>Proof (Sketch):</u>

*A Sufficiency*

Sufficiency requires that it follows from (17) and (18) that, for arbitrarily small $m$, there exists an equilibrium bidding function $b_s(y)$ which is strictly decreasing in first-period income $y$. It is easily verified that $b_s(y)_{|m=0}$ and thus $b_s'(y)_{|m=0}$ is equal to zero for all $y$. It is, moreover, established in Section 2 of the Appendix that

(19) $$\frac{\partial b_s'(y)}{\partial m}_{|m=0} = -4\frac{f(y)}{\left(F(y)\right)^3}\int_{y_l}^{y}\frac{\tilde{u}_2(\rho,Y)}{\tilde{u}_1(\rho,Y)}\left(RA_1(\rho,Y) - TRA_{12}(\rho,Y)\right)\left(F(\rho)\right)^2 d\rho.$$

By L'Hôpital's rule, $\frac{\partial b_s'(y_l)}{\partial m}_{|m=0}$ is always zero, whereas, if (17) and (18) hold, the RHS of (19) is strictly negative for all $y > y_l$, which implies that, for small $m$, $b_s(y)$ is strictly decreasing for all $y$.

To verify that bidding according to $b_s(y)$ is a best response, Section 2 of the Appendix establishes that, for all $y$, (17) is sufficient for the pseudoconcavity of $\frac{\partial E[U_s(b)|\, y]}{\partial m}_{|m=0}$ in $b$, where $E[U_s(b)|\, y]$ is equal to $E[U(b)|\, y]$ as defined in (3) with $b_s(\cdot)$ substituted for $b(\cdot)$. □

*B Necessity*

(i) Necessity of (17): Assume that

(20) $RA_1(y',Y) < TRA_{12}(y',Y)$ for some $y'$.

It is established in Section 2 of the Appendix that (20) implies that, evaluated at $b = b_s(y')$, $\frac{\partial E[U_s(b)|\, y']}{\partial m}_{|m=0}$ is strictly convex in $b$. Thus $b_s(y') \neq \arg\max_b E[U_s(b)|\, y']$ for small values of $m$ and so the strictly decreasing function $b_s(\cdot)$ is not an equilibrium bidding function. □

(ii) Necessity of (18): Assume that there exists no $\varepsilon > 0$ such that $RA_1(y,Y) > TRA_{12}(y,Y)$ for all $y_l < y < y_l + \varepsilon$. Then it follows from (19) that, in some neighborhood of $y_l$, $\frac{\partial b_s'(y)}{\partial m}_{|m=0} \geq 0$, which violates the requirement that $b_s(y)$ be strictly decreasing for all $y$. □

QED

With Proposition 3 in hand, we can now analyze whether participation in a bidding Rosca is advantageous from an ex ante perspective, provided that (17) and (18) hold. The following proposition is based on the change in ex ante expected utility arising from participation in a bidding Rosca evaluated at $m = 0$. As for random Roscas, we thus compare autarky with participation in a bidding Rosca with a small contribution.

*Proposition 4*

*Suppose individual preferences satisfy equations* (17) *and* (18)*. Then participation in a two-participant bidding Rosca with an infinitesimally small contribution is strictly preferred to autarky.*

Proof:

Consider a bidder with realized first-period income $y$ at the interim stage. Define interim expected utility in equilibrium $E[U|\, y]$ as $E[U(b_i)|\, y_i]$ in (3) with $y$ and $b_s(y)$ substituted for $y_i$ and $b_i$, respectively. Given that $b_s(y)$ is strictly decreasing, it is established in Section 2 of the Appendix that

$$\frac{\partial E[U|\, y]}{\partial m}_{|m=0} = \tilde{u}_1(y, Y)$$

(21)

$$*\left( \int_{y}^{y_u} \frac{\tilde{u}_2(\rho, Y)}{\tilde{u}_1(\rho, Y)} \left( RA_1(\rho, Y) - TRA_{12}(\rho, Y) \right) \left(1 - F(\rho)\right)^2 d\rho + \int_{y_l}^{y} \frac{\tilde{u}_2(\rho, Y)}{\tilde{u}_1(\rho, Y)} \left( RA_1(\rho, Y) - TRA_{12}(\rho, Y) \right) \left(F(\rho)\right)^2 d\rho \right),$$

which is clearly positive for all $y$ if (17) and (18) hold. If participation in a bidding Rosca is advantageous for all possible values of $y$ at the interim stage, then participation is also advantageous ex ante because, by the law of iterated expectations, ex ante expected utility with a bidding Rosca is simply $E[U] \equiv E_Y[E[U|\, Y]]$. QED

It can be shown that, for preferences whose coefficient of temporal risk aversion is uniformly higher than the coefficient of static first-period risk aversion, participation in a bidding

Rosca with a strictly increasing equilibrium bidding function is advantageous. All of the qualitative empirical evidence (see, e.g., Calomiris and Rajaraman, 1998), however, suggests that such bidding behavior does not occur in reality and is therefore not discussed further in this paper. A particularly important specification of intertemporal utility involves additive separability of the utility contributions from the first and second period, $u(x_1, x_2) = v_1(x_1) + v_2(x_2)$, with $v_t' > 0$ and $v_t'' < 0$, $t = 1, 2$. For all such utility functions, $u_{12} = 0$ and thus, within the present framework, additively separable utility functions induce participation exclusively in bidding Roscas.

Empirically, participation in random and bidding Roscas is observed among ex ante identical individuals (Gugerty, 2000). Our framework can explain the co-existence of both of these allocation mechanisms when preferences differ between groups of individuals who join Roscas. To give an example, consider the family of utility functions $u(x_1, x_2) = -v(x_1)v(x_2)$ with $v(x_t) = -\exp[-x_t^a]$, where $a > 0$. If $a < 1$, equations (17) and (18) hold while the RHS of (15) is smaller than zero. Such individuals will thus find participation in a bidding Rosca with a decreasing equilibrium bidding function advantageous while they prefer autarky to participation in a random Rosca. On the other hand, for $a > 1$, we find that the RHS of (15) is positive while (17) and (18) do not hold. Such individuals find participation in a random Rosca advantageous, while, by virtue of (19), a bidding equilibrium with a decreasing bidding function and thus a bidding Rosca which facilitates risk sharing does not exist.[18] Figure 1 depicts the change in ex ante expected utility with a bidding and a random Rosca, respectively, in response to the preference parameter $a$. Note that, within this example, no value of $a$ exists which makes both $\frac{dE[U^R]}{dm}$ and $\frac{dE[U]}{dm}$ bigger than zero at the same time. Thus a group of identical individuals that chooses to start a bidding Rosca which facilitates risk sharing will not consider to start a random Rosca and vice versa.[19]

[Figure 1 about here]

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[18] Theoretically, such individuals may join a bidding Rosca with a strictly increasing equilibrium bidding function.

[19] Without further restrictions on preferences, this is not true in general.

Propositions 2, 3 and 4 highlight that the relationship between static and temporal risk aversion plays a crucial role for the participation in Roscas. Although the example of the previous paragraph illustrates that, within our framework, different groups of identical individuals may choose different types of Roscas, indirect evidence for what drives observed Rosca participation might be gained from the literature on the relationship between temporal and static risk aversion. For example, if there is empirical evidence against condition (16), which is necessary for the participation in a random Rosca, then the motives for joining a random Rosca have to be sought in other approaches than the present one. To the author's knowledge, only one study has addressed the relationship between static and temporal risk aversion empirically. In a data set of US consumers, Epstein and Zin (1991) find a statistically significant positive intertemporal elasticity of substitution, which, in their framework, implies that static risk aversion is more pronounced than temporal risk aversion. In the present context, this finding implies that such consumers would choose to join a bidding Rosca with a decreasing equilibrium bidding function rather than a random Rosca.

Following this argument, it is apparent that our approach is complementary to the deterministic Rosca models presented by Besley et al. (1993) and Kovsted and Lyk-Jensen (1999), who show that, when incomes are deterministic and Rosca funds are used for the purchase of an investment or a durable consumption good, identical individuals prefer a random to a bidding Rosca. This complementary relationship is well in accordance with empirical observation. While most field studies on random Roscas within groups of homogenous individuals report that the pots are used for purchasing a durable good (see Anderson and Balland, forthcoming, and Gugerty, 2000, for examples from Africa), Calomiris and Rajaraman (1998) argue that their sample bidding Rosca among casual laborers in an Indian city serves as a risk-sharing device.

To conclude this section, a remark on the optimal value of $m$, $m^*$ say, in the case of a bidding Rosca is in order. At the ex ante stage, the participants' problem is to maximize $E[U]$ by choice of $m$. Since this problem has no explicit solution, we consider a numerical example where $u(x_1, x_2) = log(x_1) + \delta\, log(x_2)$ and income within each period is uniformly distributed on the interval [1, 2]. If there is no discounting, i.e. $\delta = 1$, the optimum contribution is 0.083. For strong

discounting, that is $\delta = 0.5$, the corresponding value is 0.109. Thus, about six to eight percent of the expected income is contributed to the Rosca in each period.[20]

## 4. A Rosca-Based Mechanism for Improved Risk Sharing

In the preceding two sections we have shown that, provided some empirically plausible restrictions on preferences apply, a Rosca auction does succeed in identifying the bidder with the highest concurrent need for funds in an environment of private information and that a bidding Rosca thus facilitates risk sharing. A single Rosca participation, however, has the drawback of a somewhat rigid payoff pattern. It is on these grounds that Besley et al. (1993) argue against the insurance function of Roscas. Empirically, however, multiple Rosca participation is frequently observed (see Footnote 5), and often the same individuals are found in different Roscas. The latter scenario is the point of departure for this section. We compare the performance of a multiple Rosca arrangement with that of a linear risk-sharing contract.

As in section 2, assume that two individuals, whose incomes are iid distributed, engage in $n$ simultaneous two-participant Roscas only formed by themselves. For all $n$ Roscas, each contributes a total amount of $m$ in each of the two periods. If the same amount is contributed to all Roscas, the contribution to each Rosca is $m/n$. For the sake of analytical tractability, we simplify the bidding procedure by assuming that the participants agree beforehand on some bid, $\alpha$ say, such that, in each of the $n$ Roscas, each of them can either submit zero or $\alpha/n$. As $n$ tends to infinity and assuming that ties are resolved by throwing a coin, it is easy to show that each participant's only relevant choice variable is the fraction of Roscas in which she submits $\alpha/n$ instead of zero. Moreover, in the limit, the said choice variable, $x$ say, is continuous on the unit interval. W.l.o.g. we assume that, in the first $n x_k$ auctions, participant $k$, $k = 1, 2$, submits $\alpha/n$, and zero in the following $n(1-x_k)$ auctions. In a Bayes-Nash-equilibrium, participant $i$ observes her first-period income $y_i$ and then chooses $x_i$ such that it is a best response to the other participant's strategy, $x(y_j)$ say, where, for $i$, $Y_j$ is a random variable. Her problem is thus to maximize

$$(22) \qquad E[U(x_i)|\, y_i] \equiv E_{Y_j}[\tilde{u}(y_i + (x_i - x(Y_j))(m - \frac{\alpha}{2}), Y - (x_i - x(Y_j))m)]$$

[20] Handa and Kirton (1999) in their sample of Jamaican Roscas estimate this figure at about 19%.

by choice of $x_i$. Note that $(x_i - x(Y_j))$ is the fraction of first-period auctions in which $i$ submits a higher bid than $j$. From each of these auctions, $i$ receives a net transfer of $(m - \alpha/2)/n$ in the first period and the obligation to pay $m/n$ in the second period. In the remaining auctions, both participants submit the same bid and, because of the law of large numbers, the transfers resulting from these auctions cancel out. Taking the derivative of (22) w.r.t. $x_i$ and substituting $x(y_i)$ for $x_i$ characterizes the symmetric equilibrium:

$$E_{Y_j}[\tilde{u}_1(c_1,c_2)\left(m-\frac{\alpha}{2}\right)-\tilde{u}_2(c_1,c_2)\,m]=0 \text{ for all } y_i, \text{ where}$$

(23)

$$c_1 = y_i + \left(x(y_i)-x(Y_j)\right)\left(m-\frac{\alpha}{2}\right),\ c_2 = Y-\left(x(y_i)-x(Y_j)\right)m\,.$$

Assuming that the terms of the Roscas are agreed upon before first-period incomes are observed, the participants' task ex ante is to choose the contribution $m$ and the bid $\alpha$ such that ex ante expected utility is maximized given that both participants follow the equilibrium bidding strategy:

(24) $$\max_{x(\cdot),m,\alpha} E_{Y_i,Y_j}[\tilde{u}(Y_i+\left(x(Y_i)-x(Y_j)\right)(m-\frac{\alpha}{2}),Y-\left(x(Y_i)-x(Y_j)\right)m)] \text{ s.t. (23).}$$

Note that, in general, optimality of a positive $\alpha$ may require a sufficiently high degree of impatience. To give an example, for the CARA specification $u(z_1, z_2) = v(z_1) + \delta v(z_2)$ with $v(y) \equiv -\exp[-ay]$, and incomes distributed uniformly on the unit interval, we find that $\alpha > 0$ requires that

$$\delta < \frac{v(E[Y])}{E[v(Y)]} = \frac{a}{2}\operatorname{csch}\left[\frac{a}{2}\right],$$

which is a strictly decreasing function of $a$ and, by Jensen's inequality, smaller than unity. E.g. for $a = 1$, the term on the RHS is equal to 0.96.

Now consider a linear risk-sharing contract between two individuals. Under this alternative arrangement, the individuals agree in period zero on the values of two parameters, $\Delta_1$ and $\Delta_2$ say, that determine the extent of transfers between agents in period one and two, respectively. In the first period, each agent announces his demand for first-period funds, $\sigma_i$ say, $i = 1, 2$. In period $t$, the resulting transfer is $\Delta_t$ times the excess demand of the first period, that is,

for $i$, $\Delta_1(\sigma_i - \sigma_j)$ in the first and $-\Delta_2(\sigma_i - \sigma_j)$ in the second period. In a Bayes-Nash equilibrium, agent $i$ chooses his announcement, $\sigma_i$ say, such that it maximizes interim expected utility defined by

(25) $$E[U_\Delta(\sigma_i)|\, y_i] \equiv E_{Y_j}[\tilde{u}\big(y_i + \Delta_1(\sigma_i - \sigma(Y_j)), Y - \Delta_2(\sigma_i - \sigma(Y_j))\big)],$$

where we have substituted $\sigma(Y_j)$ for $\sigma_j$ because $i$ conjectures that $j$ determines his, i.e. $j$'s, announcement according to a strictly monotonic function $\sigma(\cdot)$ of his first-period income $y_j$ and because $j$'s income is a random variable for $i$. Taking the derivative of (25) w.r.t. $\sigma_i$ and substituting $\sigma(y_i)$ for $\sigma_i$ gives the necessary condition for a symmetric equilibrium,

(26)
$$E_{Y_j}[\tilde{u}_1(\hat{c}_1,\hat{c}_2)\Delta_1 - \tilde{u}_2(\hat{c}_1,\hat{c}_2)\,\Delta_2] = 0 \text{ for all } y_i, \text{ where}$$
$$\hat{c}_1 = y_i + \Delta_1(\sigma(y_i) - \sigma(Y_j)), \hat{c}_2 = Y - \Delta_2(\sigma(y_i) - \sigma(Y_j)),$$

and the agents' task ex ante is to

(27) $$\max_{\sigma(\cdot),\Delta_1,\Delta_2} E_{Y_i,Y_j}[\tilde{u}\big(Y_i + \Delta_1(\sigma(Y_i) - \sigma(Y_j)), Y - \Delta_2(\sigma(Y_i) - \sigma(Y_j))\big)] \text{ s.t. (26).}$$

Note that $\Delta_1$ can be normalized to unity because, by (26), the equilibrium demand adjusts correspondingly to yield equivalent payoffs. Comparing (27) with (24) and setting $(m - \alpha/2) = \Delta_1$ and $m = \Delta_2$, we obtain

<u>Proposition 5:</u> *Consider two individuals who run n two-participant bidding Roscas simultaneously. As $n \to \infty$, in equilibrium, such an arrangement is payoff-equivalent to a bilateral linear risk-sharing contract, where each agent announces his demand for first-period funds.*

Note that, within the present set-up, linear risk sharing is simply a credit contract involving a predetermined interest rate of $\Delta_2 - 1$. Thus, within this framework, the performance of Rosca-participation as a risk-sharing device is as good as, but not better than, that of a credit contract with a fixed, predetermined interest rate.

We now address the question if Rosca participation can be considered an optimal feasible risk-sharing mechanism in environments characterized by low incomes and illiteracy. With the result in hand that Rosca participation does not better than a linear contract, we first have to ask

why this latter contract could be the optimal feasible contract when a non-linear, more flexible contract is, in principle, available. In a setting where contracts cannot be written because of agents' illiteracy, anything more than a linear contract appears hardly enforceable because complicated terms negotiated only verbally are easily subject to intentional or unintentional misinterpretation. Further, non-linear contracts may exceed the intellectual capacity of such individuals. Such 'bounded rationality' arguments have been advanced in the context of sharecropping where, in practice, only linear contracts are observed (see Singh, 1989). Fixing the interest rate before first-period incomes are observed simplifies the bargaining problem considerably because, by doing so, ex ante identical individuals agree unanimously on an interest rate and, in the first period, each of the agents needs to report only a single number, his demand for credit. If, in contrast, the interest rate is not fixed beforehand, each agent has to report an entire schedule of interest rate-credit demand pairs in the first period. This, again, may well exceed the contractual and intellectual capacity available in many low-income environments.

Having thus established the linear risk-sharing contract as a benchmark, it needs to be asked why individuals should join Roscas instead of entering a linear contract directly. In this connection, it can be argued that a fixed contribution in each period is easier to enforce than a variable contribution, because the former is unambiguously determined before incomes are observed and leaves no room for negotiation. This is especially important when more than two individuals engage in risk sharing, which is always the case in actual Roscas. In addition, participation in several Roscas, as outlined in this section, further simplifies the procedure determining the allocation of funds. Each auction requires only a binary choice by each participant while, with the linear contract, each of them has to report a real number.

## 5. Concluding Remarks

All existing theoretical research on rotating savings and credit associations (Roscas) has assumed that Rosca participants do not face income uncertainty and are risk neutral. Further, in most of this literature (Besley et al., 1993; Kovsted and Lyk-Jensen, 1999), no new information becomes available during the course of a Rosca and consequently all auctions that a bidding Rosca involves can take place at the beginning of the Rosca. Such ex ante bidding contradicts the findings of much of the empirical literature on bidding Roscas, where auctions are concurrent with allotment. Here, in contrast, under the assumption that participants are risk averse and that their incomes are stochastic and independently drawn in each round, new information becomes

available each time the Rosca participants meet. Accordingly, we have modeled a bidding Rosca in which each auction is concurrent with the allotment of a pot.

It has been established that, under the assumptions set out above, participation in a single bidding Rosca with an oral ascending price auction is advantageous for a wide class of preferences, namely, when temporal risk aversion is less pronounced than static risk aversion. Under this assumption, participation in a random Rosca does not occur while, in a bidding Rosca, a bidding equilibrium exists in which each pot is allocated to the bidder who has suffered the most severe income shock. In contrast to standard SIPV oral ascending bid auctions, where truth-telling is an equilibrium, the bidding equilibrium of a Rosca auction involves strategic overbidding. Numerical computations with logarithmic utility suggest that, depending on the extent of intertemporal discounting, an individual optimally contributes six to eight percent of her expected income to such a bidding Rosca. The transactions observed in many actual bidding Roscas are better explained by the present approach, where, in contrast to the predictions of Rosca models with ex ante bids, the price paid for a period's pot does not decrease monotonically with the number of rounds played, but instead fluctuates considerably.

Roscas impose severe restrictions on the set of feasible allocations among participants within each period, which arise from a fixed transfer in the last period and the strategic behavior of bidders in prior periods. By doing this, however, they bring about a net transfer from the better to the worse-off participant each time a pot is auctioned and thereby facilitate risk sharing among homogenous, risk averse individuals in the presence of information asymmetries. These rigidities can be overcome once individuals participate in several Roscas simultaneously. A simple mechanism involving multiple Roscas among the same individuals is as good as that of a linear risk-sharing contract, and is more enforceable because it carries a fixed rather than a variable contribution.

The present results suggest that, if reasonable restrictions on preferences are imposed, homogenous individuals prefer a bidding to a random Rosca because, in equilibrium, the former allocates each pot auctioned to the bidder with the most urgent current need. This finding is supported by empirical studies where bidding Roscas are observed among ex ante identical individuals for whom there is scope for the sharing of income risk. Our approach is thus complementary to the existing deterministic Rosca models with ex ante bids, which show that, when incomes are deterministic and Rosca funds are used for the purchase of an investment or a

durable consumption good, homogenous individuals prefer a random to a bidding Rosca while a bidding Roscas is only preferred when individuals are sufficiently heterogeneous. In these deterministic models, heterogeneity is a permanent individual characteristic and bidding serves to accommodate those with the highest willingness to pay first, which in turn generates a gain for the other members through the distribution of the winning bid. In the model presented in this paper, in contrast, individuals are identical ex ante and it is individual-specific uncertainty that generates gains from intertemporal trade.

**References**

Adams, D. W., and M. L. Canavesi de Sahonero, "Rotating Savings and Credit Associations in Bolivia," *Savings and Development*, 13 (1989), 219-235.

Anderson, S., and J. M. Balland, "The Economics of Roscas and Intra-household Resource Allocation," *Quarterly Journal of Economics*, forthcoming.

Besley, T., S., Coate, and G. Loury, "The Economics of Rotating Savings and Credit Associations," *American Economic Review*, 83 (1993), 792-810.

Besley, T., S. Coate, and G. Loury, "Rotating Savings and Credit Associations, Credit Markets and Efficiency," *Review of Economic Studies* 61 (1994), 701-720.

Bouman, F.J.A., "The Rosca: Financial Technology of an Informal Savings and Credit Institution in Developing Economies," *Savings and Development* 4 (1979), 253-276.

Calomiris, C. W., and I. Rajaraman, "The Role of ROSCAs: Lumpy Durables or Event Insurance?" *Journal of Development Economics* 56 (1998), 207-216.

Campbell, C. D., and C. S. Ahn, "Kyes and Mujins, Financial Intermediaries in South Korea," *Economic development and cultural change* 11 (1962), 55-68.

Coate, S., and M. Ravallion, "Reciprocity without Commitment: Characterization and Performance of Informal Insurance Arrangements," *Journal of Development Economics* 20 (1993), 1-24.

Cramton, P., R. Gibbons, and P. Klemperer, " Dissolving a Partnership Efficiently," *Econometrica* 55 (1987), 615-632.

Engelbrecht-Wiggans, R., "Auctions with Price-Proportional Benefits to Bidders," *Games and Economic Behavior* 6 (1994), 339-346.

Epstein, G. E., and S. E. Zin, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: an Empirical Analysis," *Journal of Political Economy* 99 (1991), 263-286.

Geertz, C., "The Rotating Credit Association: a Middle Rung in Development," *Economic Development and Cultural Change* 10 (1962), 241-263.

Gugerty, M. K., "You Can't Save Alone: Testing Theories of Rotating Savings and Credit Associations," mimeo, Kennedy School of Government, Harvard University, 2000.

Güth, W, and E. van Damme, "A Comparison of Pricing Rules for Auctions and Fair Division Games," *Social Choice and Welfare* 3 (1986), 177-198.

Handa, S., and C. Kirton, "The economics of Rotating Savings and Credit Associations: Evidence from the Jamaican 'Partner'," *Journal of Development Economics* 60 (1999), 173-194.

Ingersoll, J.E., *Theory of Financial Decision Making* (Savage, Maryland: Rowman & Littlefield, 1987).

Kovsted, J. and P. Lyk-Jensen, "Rotating Savings and Credit Associations: the Choice between Random and Bidding Allocation of Funds," *Journal of Development Economics* 60 (1999), 143-172.

Klonner, S., "How Roscas Perform as Insurance," mimeo, Südasien Institut, Universität Heidelberg, 2000.

Klonner, S., "Buying Fields and Marrying Daughters: an Empirical Study of Rosca Auctions in a South Indian Village," mimeo, Südasien Institut, Universität Heidelberg, 2002.

Kuo, P.-S., "Loans, bidding strategies and equilibrium in the discount bid rotating credit association," *Academia Economic Papers* 21 (1993), 261-303.

Levenson, A. R., and T. Besley, "The Anatomy of an Informal Financial Market: Rosca Participation in Taiwan," *Journal of Development Economics* 51 (1996), 45-68.

McAfee, R. P., and J. McMillan, "Bidding Rings," *American Economic Review* 82 (1992), 579-599.

Matthews, S. A., "A Technical Primer on Auction Theory," mimeo, Northwestern University, 1990.

Nayar, C. P. S., *Chit Finance* (Bombay: Vora and Co., 1983).

Richard, S. F., "Multivariate Risk Aversion, Utility Independence and Separable Utility Functions," *Management Science* 22 (1975), 12-21.

Ronn, E. I., "Nonadditive Preferences and the Marginal Propensity to Consume," *American Economic Review* 78 (1988), 216-223.

Singh, N., "Theories of Sharecropping," in P. Bardhan, ed., *The Economic Theory of Agrarian Institutions* (New York: Oxford University Press, 1989).

Thomas, D., "The Distribution of Income and Expenditure within the Household," *Annales d'Économie et de Statistique* 29 (1993), 109-136.

Townsend, R. M., "Risk and Insurance in Village India," *Econometrica* 62 (1994), 539-591.

Wang, C., "Dynamic Insurance with Private Information and Balanced Budgets," *Review of Economic Studies* 62 (1995), 577-595.

## APPENDIX

### Section 1

This appendix discusses sufficient conditions for the optimality of bidding $b_s(y)$ as defined by (5) and (6), given that the other participant bids according to $b_s(\cdot)$. Towards this end, we substitute $b_s(\cdot)$ for $b(\cdot)$ and $b_s(\psi)$ for $b$ in $E[U(b)|\, y]$ as defined by (3) to obtain

(28) $E[U|\ y,\ \psi] \equiv \tilde{u}(y-m+b_s(\psi)/2, Y+m)F(\psi) + E[\tilde{u}(y+m-b_s(Y_j)/2, Y-m)|Y_j > \psi](1\text{-}F(\psi))$,

which is the equilibrium interim expected utility of a participant who actually observes first-period income $y$ but acts in the auction as if his first-period income was $\psi$ instead. Note that, by the necessary conditions (5) and (6), $b_s(\cdot)$ is such that $\left.\dfrac{\partial E[U|\, y,\psi]}{\partial \psi}\right|_{\psi=y} = 0$ for all $y$. Consequently,

as in the analysis of standard SIPV auctions, pseudoconcavity of $E[U|\, y,\, \psi]$ in $\psi$ for all $y$ is sufficient for the optimality of bidding $b_s(y)$, given that the other participant bids according to $b_s(\cdot)$. Formally, the said pseudoconcavity requires that

(29) $$\frac{\partial E[U|\, y,\psi]}{\partial \psi} \geq 0 \text{ for all } \psi < y \text{ and } \frac{\partial E[U|\, y,\psi]}{\partial \psi} \leq 0 \text{ for all } \psi > y.$$

For notational convenience, we define

(30) $\Delta(y) \equiv m - b_s(y)/2.$

Now, differentiating $E[U|\, y,\, \psi]$ as given by (28) w.r.t $\psi$ gives

$$\frac{\partial E[U|\, y,\psi]}{\partial \psi} = f(\psi)\ \tilde{u}_1(y - \Delta(\psi), Y + m)\, g(\psi, y) \text{ with}$$

$$g(\psi, y) \equiv \frac{\tilde{u}(y - \Delta(\psi), Y + m) - \tilde{u}(y + \Delta(\psi), Y - m)}{\tilde{u}_1(y - \Delta(\psi), Y + m)} - \frac{\tilde{u}(\psi - \Delta(\psi), Y + m) - \tilde{u}(\psi + \Delta(\psi), Y - m)}{\tilde{u}_1(\psi - \Delta(\psi), Y + m)},$$

where the RHS of (5) evaluated at $\psi$ has been substituted for $b_s'(\psi)$. Since $f(\psi)\, \tilde{u}_1(y - \Delta(\psi), Y + m)$ is always positive, (29) is equivalent to

(31) $g(\psi, y) \geq 0$ for all $\psi < y$ and $g(\psi, y) \leq 0$ for all $\psi > y$.

Using line integral techniques, we obtain

$$g(\psi, y) = \int_{-1}^{1} \left\{ \Delta(\psi) \left( \frac{\tilde{u}_1(\psi - \gamma\Delta(\psi), Y + \gamma m)}{\tilde{u}_1(\psi - \Delta(\psi), Y + m)} - \frac{\tilde{u}_1(y - \gamma\Delta(\psi), Y + \gamma m)}{\tilde{u}_1(y - \Delta(\psi), Y + m)} \right) + m \left( \frac{\tilde{u}_2(y - \gamma\Delta(\psi), Y + \gamma m)}{\tilde{u}_1(y - \Delta(\psi), Y + m)} - \frac{\tilde{u}_2(\psi - \gamma\Delta(\psi), Y + \gamma m)}{\tilde{u}_1(\psi - \Delta(\psi), Y + m)} \right) \right\} d\gamma$$

$$= \int_{-1}^{1} \int_{\psi}^{y} \left\{ \frac{\tilde{u}_1(\delta - \gamma\Delta(\psi), Y + \gamma m)}{\tilde{u}_1(\delta - \Delta(\psi), Y + m)} \Delta(\psi) \left[ RA_1(\delta - \gamma\Delta(\psi), Y + \gamma m) - RA_1(\delta - \Delta(\psi), Y + m) \right] \right.$$

$$\left. + \frac{\tilde{u}_2(\delta - \gamma\Delta(\psi), Y + \gamma m)}{\tilde{u}_1(\delta - \Delta(\psi), Y + m)} m \left[ RA_1(\delta - \gamma\Delta(\psi), Y + \gamma m) - TRA_{12}(\delta - \Delta(\psi), Y + m) \right] \right\} d\delta\, d\gamma,$$

where the coefficients of static and temporal risk aversion, $RA_1$ and $TRA_{12}$, are defined in section 3. A set of sufficient conditions for (31) and thus for (29) is

(i) $RA_1(\delta - \Delta(\psi), Y + m) \leq RA_1(\delta + \gamma\Delta(\psi), Y - \gamma m)$ for all $\gamma \in$ [-1,1] and all $\delta,\ \psi \in [y_l, y_u]$,

(ii) $TRA_{12}(\delta - \Delta(\psi), Y + m) \leq RA_1(\delta + \gamma\Delta(\psi), Y - \gamma m)$ for all $\gamma \in$ [-1,1] and all $\delta,\ \psi \in [y_l, y_u]$.

If the utility function exhibits utility independence[21], the coefficients of static and temporal risk aversion depend on first-period consumption only. In this case, (i) and (ii) respectively become

(i)' $RA_1(\delta - \Delta(\psi)) \leq RA_1(\delta + \gamma\Delta(\psi))$ for all $\gamma \in$ [-1,1] and all $\delta$, $\psi \in [y_l, y_u]$

(ii)' $TRA_{12}(\delta - \Delta(\psi)) \leq RA_1(\delta + \gamma\Delta(\psi))$ for all $\gamma \in$ [-1,1] and all $\delta$, $\psi \in [y_l, y_u]$.

(i)' is implied by non-decreasing absolute risk aversion while (ii)' holds when temporal risk aversion is less pronounced than static risk aversion. Note, however, that non-decreasing absolute risk aversion does not need to hold when (ii)' holds with strict inequality.

**Section 2**

Defining $\Delta(y)$ as in (30), it is shown below that

$$\frac{\partial \Delta(y)}{\partial m}_{|m=0} = \int_{y_l}^{y} \frac{\tilde{u}_2(\rho, Y)}{\tilde{u}_1(\rho, Y)} \frac{2F(\rho)f(\rho)}{\left(F(y)\right)^2} d\rho. \tag{32}$$

Differentiating (32) w.r.t. $y$ gives

$$\frac{\partial \Delta'(y)}{\partial m}_{|m=0} = 2\frac{f(y)}{F(y)}\left( \frac{\tilde{u}_2(y, Y)}{\tilde{u}_1(y, Y)} - \int_{y_l}^{y} \frac{\tilde{u}_2(\rho, Y)}{\tilde{u}_1(\rho, Y)} \frac{2F(\rho)f(\rho)}{\left(F(y)\right)^2} d\rho \right). \tag{33}$$

Integrating the integral term in (33) by parts gives

$$\frac{\partial \Delta'(y)}{\partial m}_{|m=0} = 2\frac{f(y)}{F(y)} \int_{y_l}^{y} h(\rho) \left( \frac{F(\rho)}{F(y)} \right)^2 d\rho, \text{ where}$$

(34)

$$h(y) \equiv \frac{d}{dy}\left( \frac{\tilde{u}_2(y, Y)}{\tilde{u}_1(y, Y)} \right) = \frac{\tilde{u}_2(y, Y)}{\tilde{u}_1(y, Y)} \left( RA_1(y, Y) - TRA_{12}(y, Y) \right).$$

From (34) and (30) we immediately obtain

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[21] Utility independence means that the decision concerning consumption in period $t$ conditional on a consumption level $c_k$ in period $k \neq t$ remains the same for all possible values of $c_k$. With utility independence, $u(x_1, x_2)$ is either additively or multiplicatively separable, i.e. $u(x_1, x_2) = v_1(x_1) + v_2(x_2)$ or $u(x_1, x_2) = v_1(x_1)\, v_2(x_2)$ when $v_1, v_2 > 0$ or $u(x_1, x_2) = -\, v_1(x_1)\, v_2(x_2)$, when $v_1, v_2 < 0$. See Richard (1975).

$$\frac{\partial b_s'(y)}{\partial m}_{|m=0} = -2\frac{\partial \Delta'(y)}{\partial m}_{|m=0} = -4\frac{f(y)}{\left(F(y)\right)^3}\int_{y_l}^{y}\frac{\widetilde{u}_2(\rho,Y)}{\widetilde{u}_1(\rho,Y)}\left(RA_1(\rho,Y)-TRA_{12}(\rho,Y)\right)\left(F(\rho)\right)^2 d\rho.$$

<u>Proof of (32):</u>

For any fixed positive $m$, it follows from (5) and (6) that $\Delta(y)$ satisfies the following differential equation and boundary condition

(35) $$\Delta'(y) = \frac{f(y)}{F(y)}\left[\frac{\widetilde{u}(y+\Delta(y),Y-m)-\widetilde{u}(y-\Delta(y),Y+m)}{\widetilde{u}_1(y-\Delta(y),Y+m)}\right],$$

$$\Delta(y_l) = \Delta^0(y_l),$$

where, similar to (30), $\Delta^0(y) \equiv m - b^0(y)/2$. Evaluating the derivative of (35) w.r.t. $m$ at $m = 0$ gives

(36) $$\frac{\partial \Delta'(y)}{\partial m}_{|m=0} = 2\frac{f(y)}{F(y)}\left[\frac{\widetilde{u}_2(y,Y)}{\widetilde{u}_1(y,Y)} - \frac{\partial \Delta(y)}{\partial m}_{|m=0}\right].$$

Further, since $\Delta(y_l) = \Delta^0(y_l)$ for all values of $m$, $\frac{\partial \Delta(y_l)}{\partial m}_{|m=0} = \frac{\partial \Delta^0(y_l)}{\partial m}_{|m=0}$. The latter term is obtained by differentiating (1) totally. We thus have

(37) $$\frac{\partial \Delta(y_l)}{\partial m}_{|m=0} = \frac{\widetilde{u}_2(y_l,Y)}{\widetilde{u}_1(y_l,Y)}.$$

The unique solution to the boundary value problem (36), (37) is (32). QED

To establish the pseudoconcavity of $\frac{\partial E[U(b)|\,y]}{\partial m}_{|m=0}$ in $b$, where $E[U(b)|\,y]$ is defined in (3), we proceed as in Section 1 of the Appendix. The (w.r.t. $m$) infinitesimal version of (29) is

$$\frac{\partial^2 E[U|\,y,\psi]}{\partial m \partial \psi}_{|m=0} \geq 0 \text{ for all } \psi < y \text{ and } \frac{\partial^2 E[U|\,y,\psi]}{\partial m \partial \psi}_{|m=0} \leq 0 \text{ for all } \psi > y.$$

Going through some algebra, from (28) we obtain

$$\frac{\partial^2 E[U|\,y,\psi]}{\partial m \partial \psi}_{|m=0} = 2f(\psi)\tilde{u}_1(y,Y)\left(\frac{\tilde{u}_2(y,Y)}{\tilde{u}_1(y,Y)} - \frac{\tilde{u}_2(\psi,Y)}{\tilde{u}_1(\psi,Y)}\right)$$

(38)

$$= 2f(\psi)\tilde{u}_1(y,Y)\int_{\psi}^{y} \frac{\tilde{u}_2(\rho,Y)}{\tilde{u}_1(\rho,Y)}\left(RA_1(\rho,Y) - TRA_{12}(\rho,Y)\right)d\rho\,,$$

which, given (17) holds, is clearly positive (negative) whenever $\psi$ is smaller (bigger) than $y$.

<u>Proof of (21):</u>

*<u>Step 1:</u>*

Evaluated at $m = 0$, the derivative of $E[U|\,y]$ w.r.t. $m$ can be written as

$$\frac{\partial E[U|\,y]}{\partial m}_{|m=0} = u_2(y,Y)(2F(y)-1) + 2u_1(y,Y)g(y)\text{, where}$$

(39)

$$g(y) \equiv \int_{y}^{y_u} \frac{\tilde{u}_2(\rho,Y)}{\tilde{u}_1(\rho,Y)} f(\rho)d\rho - \int_{y_l}^{y_u} \frac{\tilde{u}_2(\rho,Y)}{\tilde{u}_1(\rho,Y)} F(\rho)f(\rho)d\rho\,.$$

<u>Proof of Step 1:</u>

First, rewrite interim expected utility as

(40) $$E[U|\,y] = \tilde{u}(y-\Delta(y)/2, Y+m)\,F(y) + \int_{y}^{y_u} \tilde{u}(y+\Delta(\rho), Y-m)f(\rho)d\rho\,.$$

Evaluating the derivative of (40) w.r.t. $m$ at $m = 0$ gives

(41) $$\frac{\partial E[U\mid y]}{\partial m}_{|m=0} = u_2(y,Y)(2F(y)-1) + 2u_1(y,Y)\hat{g}(y)\text{, where}$$

(42) $$\hat{g}(y) \equiv \frac{1}{2}\left(\int_{y}^{y_u} \frac{\partial \Delta(\rho)}{\partial m}_{|m=0} f(\rho)d\rho - \frac{\partial \Delta(y)}{\partial m}_{|m=0}\right).$$

Substituting $\frac{\partial \Delta(\cdot)}{\partial m}_{|m=0}$ from (32) into (42) gives

(43) $$\hat{g}(y) = \int_{y}^{y_u}\int_{y_l}^{\rho} \frac{\tilde{u}_2(\gamma,Y)}{\tilde{u}_1(\gamma,Y)}\frac{F(\gamma)}{F^2(\rho)} f(\gamma)d\gamma\; f(\rho)d\rho - \int_{y_l}^{y} \frac{\tilde{u}_2(\gamma,Y)}{\tilde{u}_1(\gamma,Y)}\frac{F(\gamma)}{F(y)} f(\gamma)d\gamma\,.$$

Applying Fubini's Theorem, the double integral can be written as

$$\int_{y_l}^{y_u}\int_{\gamma}^{y_u}\frac{f(\rho)}{F^2(\rho)}d\rho\frac{\widetilde{u}_2(\gamma,Y)}{\widetilde{u}_1(\gamma,Y)}F(\gamma)f(\gamma)d\gamma-\int_{y_l}^{y}\int_{\gamma}^{y}\frac{f(\rho)}{F^2(\rho)}d\rho\frac{\widetilde{u}_2(\gamma,Y)}{\widetilde{u}_1(\gamma,Y)}F(\gamma)f(\gamma)d\gamma$$

(44)

$$=\int_{y}^{y_u}\frac{\widetilde{u}_2(\gamma,Y)}{\widetilde{u}_1(\gamma,Y)}f(\gamma)d\gamma-\int_{y_l}^{y_u}\frac{\widetilde{u}_2(\gamma,Y)}{\widetilde{u}_1(\gamma,Y)}F(\gamma)f(\gamma)d\gamma+\int_{y_l}^{y}\frac{\widetilde{u}_2(\gamma,Y)}{\widetilde{u}_1(\gamma,Y)}\frac{F(\gamma)}{F(y)}f(\gamma)d\gamma,$$

Substituting (44) into (43), we find that $\hat{g}(y)=g(y)$, and so (41) is equivalent to (39). □

<u>*Step 2:*</u> Establish that

(45) $$g(y)=\frac{1}{2}\left(\frac{\widetilde{u}_2(y,Y)}{\widetilde{u}_1(y,Y)}(1-2F(y))+\int_{y}^{y_u}h(\rho)(1-F(\rho))^2d\rho+\int_{y_l}^{y}h(\rho)(F(\rho))^2d\rho\right),$$ where

$h(\cdot)$ is defined as in (34).

<u>Proof of Step 2:</u>

Integrating $g(y)$, as given by (39), by parts and collecting terms gives

$$g(y)=\frac{1}{2}\frac{\widetilde{u}_2(y_u,Y)}{\widetilde{u}_1(y_u,Y)}-\frac{\widetilde{u}_2(y,Y)}{\widetilde{u}_1(y,Y)}F(y)-\int_{y}^{y_u}h(\rho)F(\rho)d\rho+\frac{1}{2}\int_{y_l}^{y_u}h(\rho)F^2(\rho)d\rho$$

$$=\frac{1}{2}\left(\left[\frac{\widetilde{u}_2(y_u,Y)}{\widetilde{u}_1(y_u,Y)}-\frac{\widetilde{u}_2(y,Y)}{\widetilde{u}_1(y,Y)}\right]+\frac{\widetilde{u}_2(y,Y)}{\widetilde{u}_1(y,Y)}(1-2F(y))+\int_{y}^{y_u}h(\rho)\left((F(\rho))^2-2F(\rho)\right)d\rho+\int_{y_l}^{y}h(\rho)(F(\rho))^2d\rho\right).$$

Substituting $\int_{y}^{y_u}h(\rho)d\rho$ for the term in square-brackets gives

$$g(y)=\frac{1}{2}\left(\frac{\widetilde{u}_2(y,Y)}{\widetilde{u}_1(y,Y)}(1-2F(y))+\int_{y}^{y_u}h(\rho)\left[(F(\rho))^2-2F(\rho)+1\right]d\rho+\int_{y_l}^{y}h(\rho)(F(\rho))^2d\rho\right).$$

Applying the second binomial formula to the term in square brackets establishes (45). □

<u>*Step 3:*</u> Substituting (45) into (39) we obtain

$$\frac{\partial E[U|y]}{\partial m}_{|m=0}=\widetilde{u}_1(y,Y)\left(\int_{y}^{y_u}h(\rho)(1-F(\rho))^2d\rho+\int_{y_l}^{y}h(\rho)(F(\rho))^2d\rho\right).$$

Applying the definition of $h(\cdot)$ from (34) completes the proof of (21). QED

It follows from the discussion in Section 1 of the Appendix that the following two statements are equivalent.

'Evaluated at $b = b_s(y')$, $\dfrac{\partial E[U(b)|\, y']}{\partial m}_{|m=0}$ is convex in $b$' and

'Evaluated at $\psi = y'$, $\dfrac{\partial E[U|\, y', \psi]}{\partial m}_{|m=0}$ is convex in $\psi$'.

From (38) we obtain

$$\frac{\partial^3 E[U|\, y', \psi]}{\partial m \partial^2 \psi}_{\substack{|m=0 \\ |\psi=y'}} = 2 f(y') \tilde{u}_2(y', Y)\big(TRA_{12}(y', Y) - RA_1(y', Y)\big),$$

which is strictly positive if (20) holds. Thus, evaluated at $\psi = y'$, $\dfrac{\partial E[U|\, y', \psi]}{\partial m}_{|m=0}$ is strictly convex in $\psi$.

Figure 1. Change in ex ante expected utility with a random ($dE[U^R]/dm$, solid line) and a bidding Rosca with a decreasing equilibrium bidding function ($dE[U]/dm$, dashed line) in response to $a$ evaluated at $m = 0$.