Imitation – Theory and Experimental Evidence

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Imitation is relevant

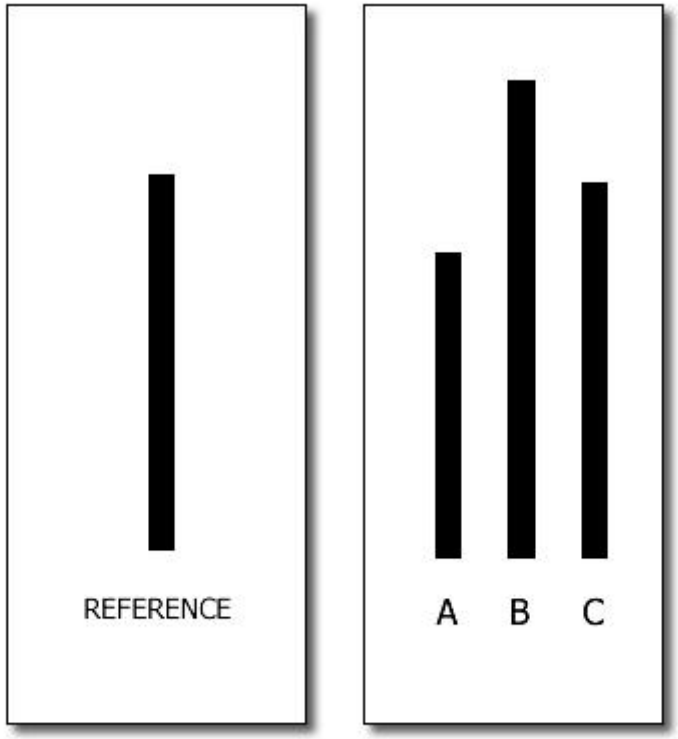
Neoplan (Germany) vs. Zhongwei (P.R. China)

Imitation is prevalent

Zeiss Ikon Contax II (1936) vs. Nikon I (1948)

→ Imitator can become more successful than the imitated

Imitation is prevalent: Asch Experiment 1951



A replication of the cards used during the experiment. The card on the left is for reference, the one on the right shows the comparison lines.

Faced with unanimous agreement by the group on the wrong answer, the test subject (#6) peers at the cards, deciding whether to trust his own eyes and his own confidence, or whether to instead change his mind and trust the group's opinion, or to simply pretend to agree with the group in order to avoid conflict or rejection.

Imitate-the-best

imitate-the-best =

imiate the action of the most sucessful player in the previous round.

in contrast to: unconditional imitation

Imitation in strategic settings

Vega-Redondo (1997): studies imitators in Cournot oligopoly (i.e. quantity competition)

Result: imitation converges to Walrasian (Bertrand) outcome, where p = MC (i.e. like a perfectly competitive market)

Intuition: in each round, firm with highest q has highest π (as long as p > MC) $\Rightarrow$ gets imitated

When p < MC, firm with lowest q gets imitated.
+ noise
$\Rightarrow$ total quantity approaches Walrasian quantity from below and above.

HNO (1999)

Experimental evidence

Huck, Normann, and Oechssler (*Econ J.* 1999)

(see also Offerman, Potters, and Sonnemans, *ReStud,* 2002)

- Cournot oligopoly with 4 firms
- 40 rounds, fixed matching
- linear demand and cost
- treatment Best: subjects know demand and cost, have access to profit calculator, get info on *total quantity of other firms* in previous round $\Rightarrow$ possible to calculate myopic best reply
- treatment Imit+: subject get info on *quantities and profits of all firms* from previous round

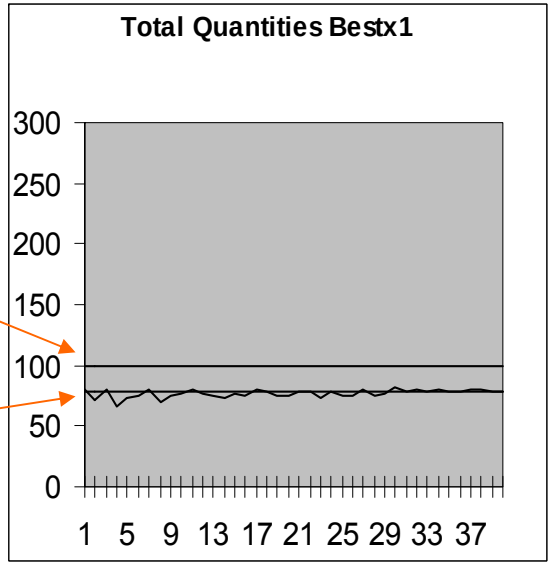
Treatment Best

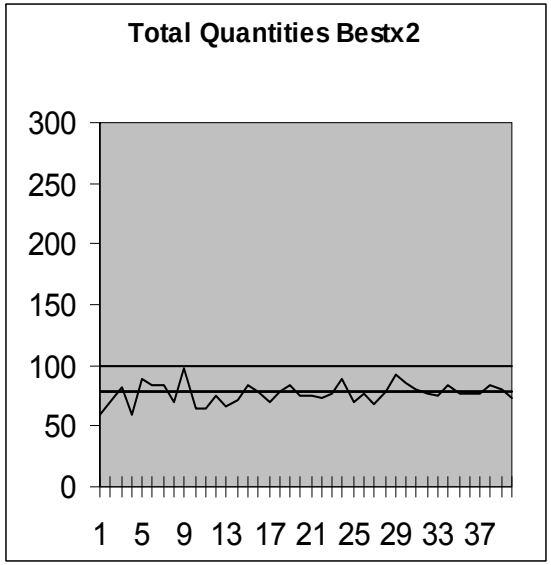
HNO (1999)

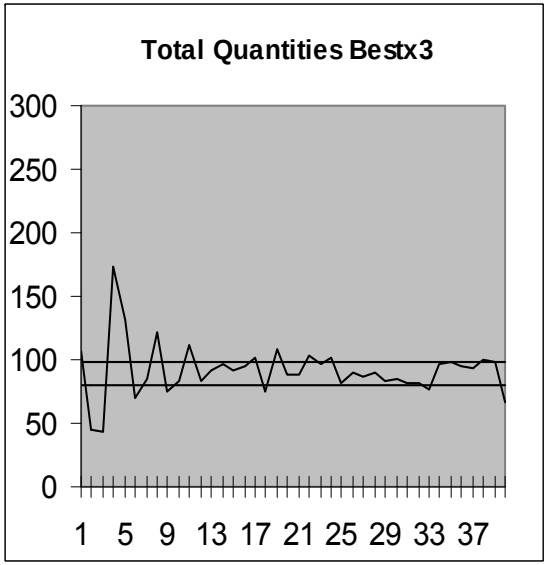
Information: only total quantity of others + demand, cost info.
Theoretical prediction: Nash eq. 79.2
mean quantity (last 20 rounds): 82.56

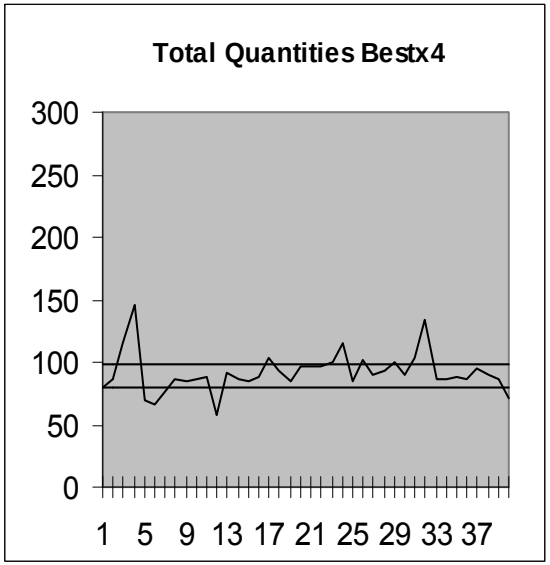
p=MC

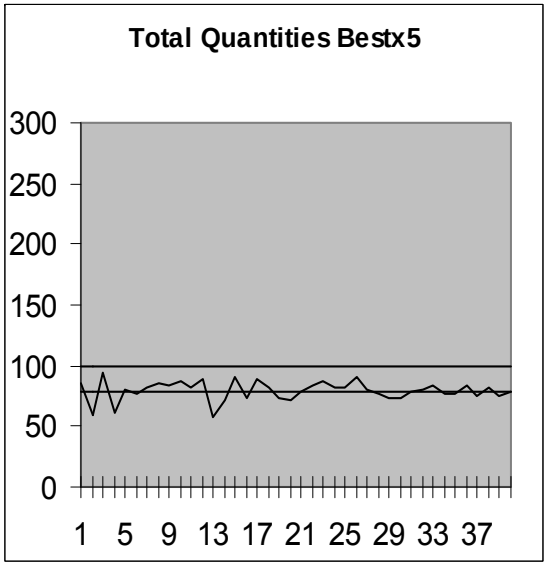
Nash

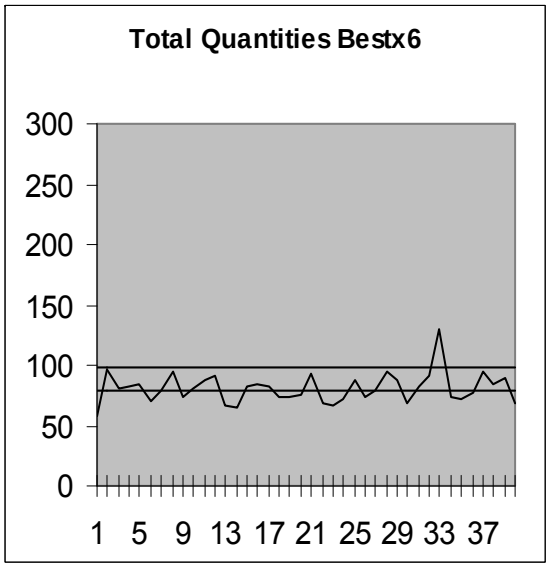








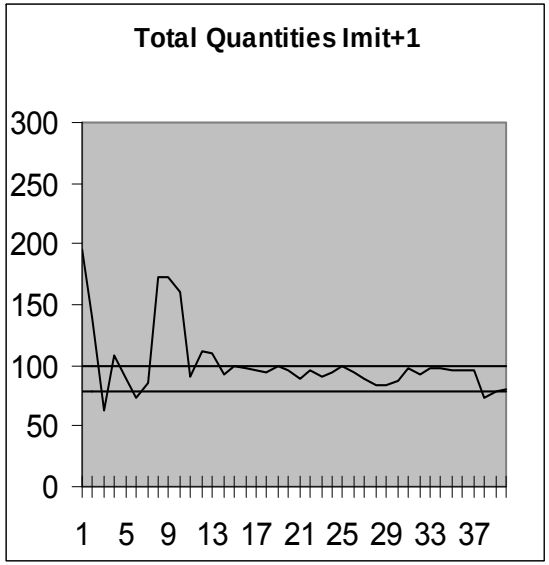


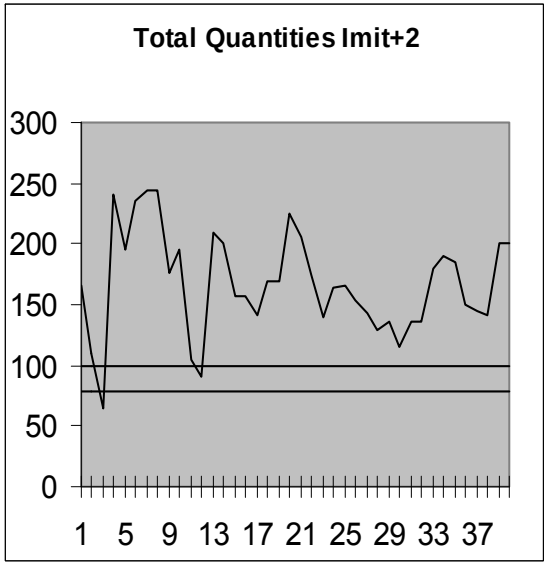


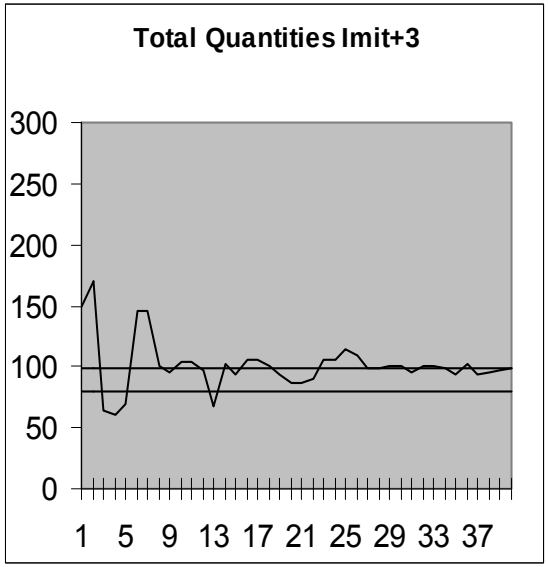
Treatment Imit+

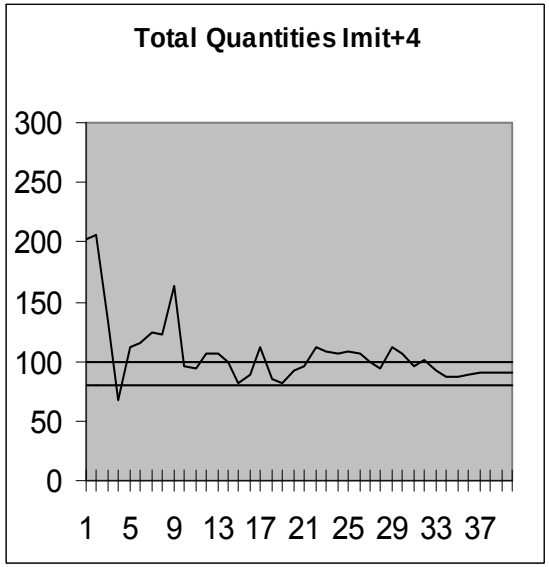
HNO (1999)

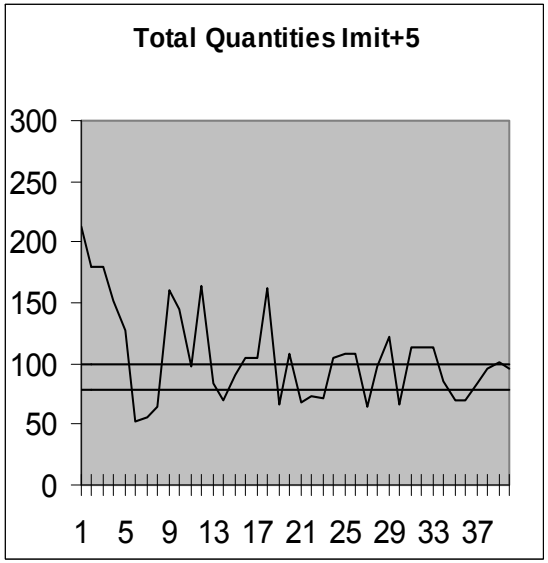
Information: quantity and profits of others.
Theoretical prediction: 99
mean quantity (last 20 rounds): 96.43

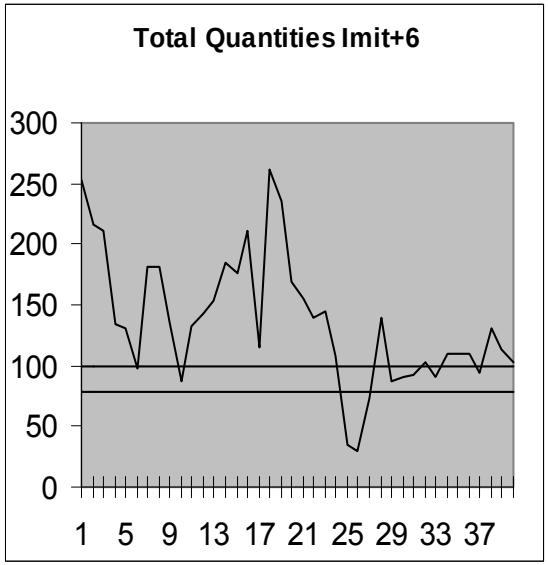












HNO (1999)

Message

- many subjects seem to imitate
- the more so, the less transparent the environment
- this has dramatic consequences in some games
- can become very competitive
- but…could also be relative payoff max.

AHO (2007)

Whom do you imitate? role vs. group

Apesteguia, Huck, Oechssler (*JET*, 2007)

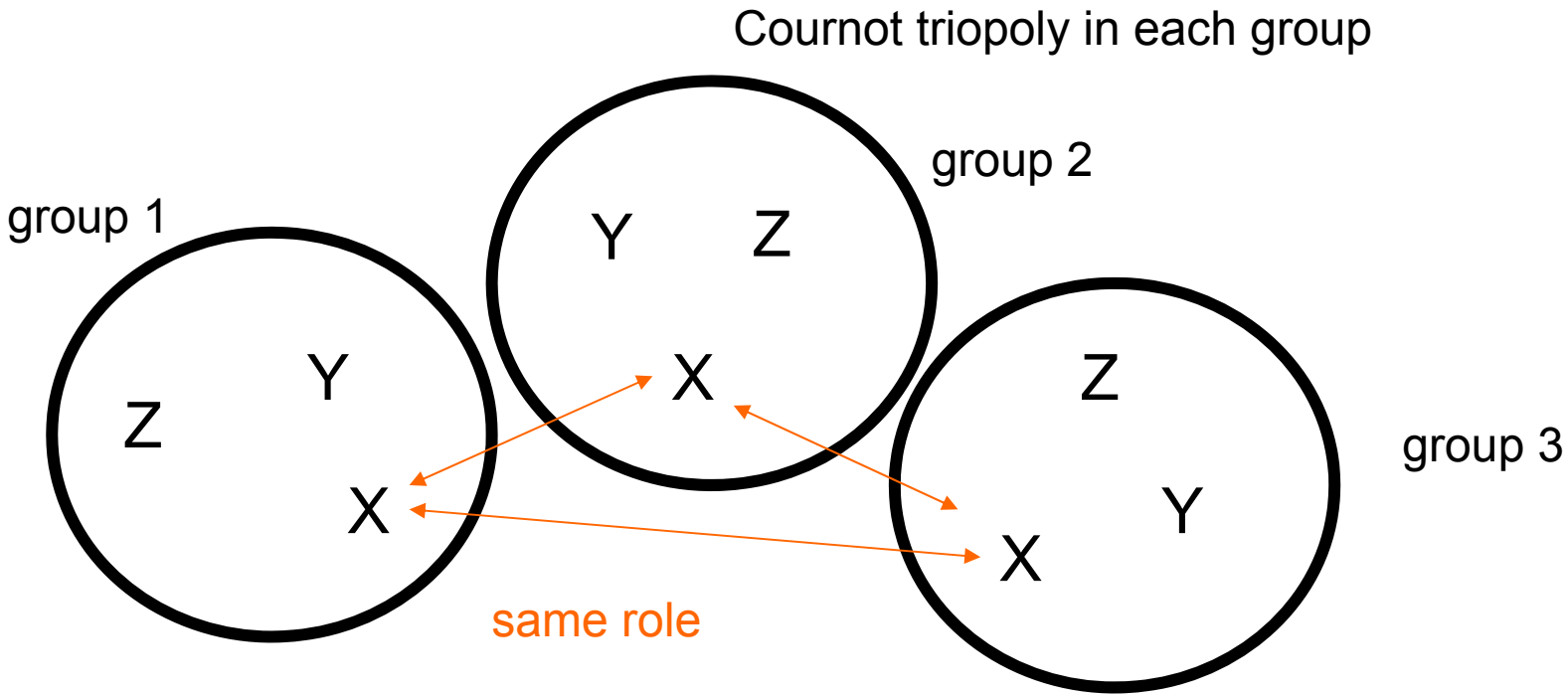
- show theoretically that it matters whether imitation of opponents or of others
  - imitation of opponents $\Rightarrow$ Walrasian outcome (p=MC) as in Vega-Redondo (1997)
  - imitation of others $\Rightarrow$ Cournot-Nash equilibrium as in Schlag (1998)

- confirm this experimentally in Cournot oligopoly

AHO (2007)

Whom do you imitate? role vs. group

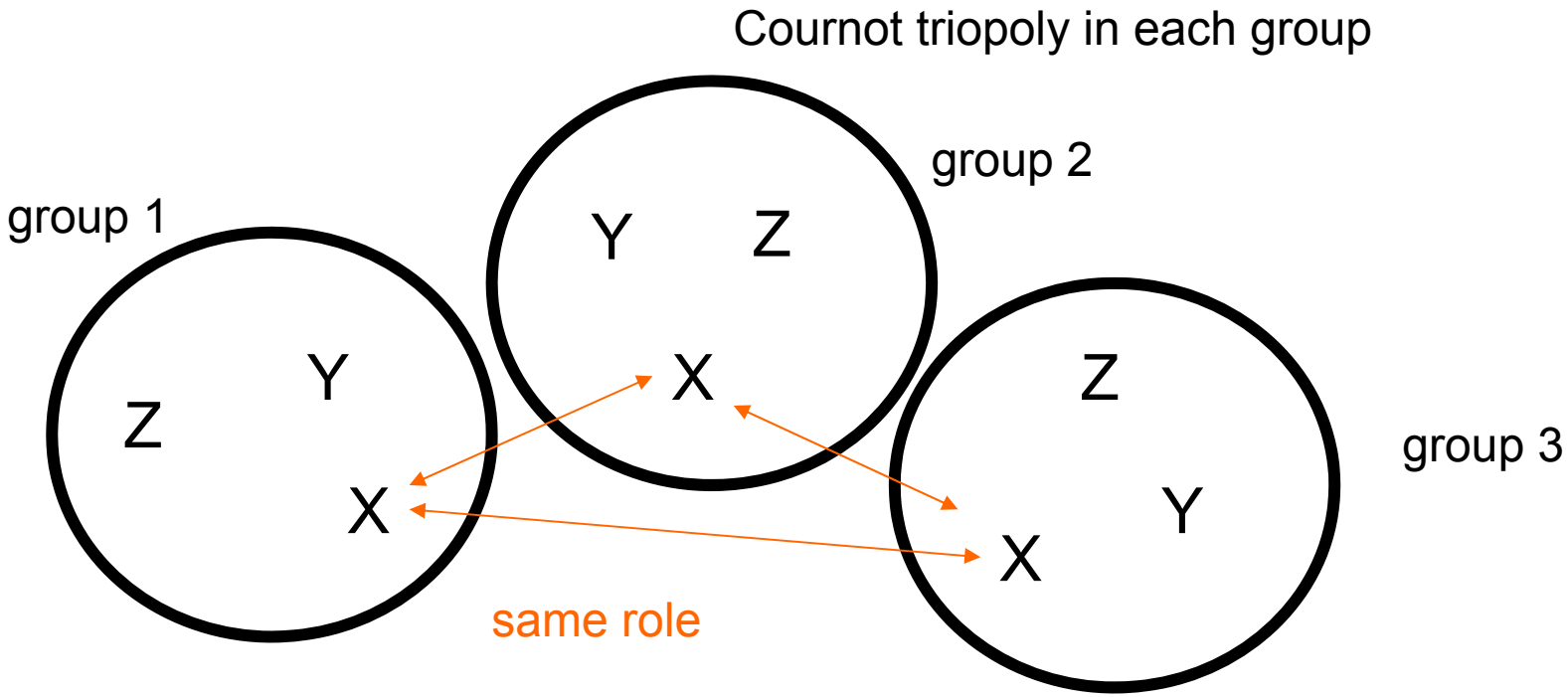
- population of 9 subjects
- roles $i \in \{X,Y,Z\}$, 3 subjects each, fixed at the start
- each period, subjects are randomly matched into 3 groups with one *X*-subject, one *Y*-subject, and one *Z*-subject.
- 60 rounds, 6 indep. observations per treatment



AHO (2007)

Whom do you imitate? role vs. group

- **ROLE**: A subject is informed of the actions and payoffs of subjects who have the same role (but play in different groups)
- **GROUP**: A subject is informed of the actions and payoffs of subjects in his own group.
- **FULL**: both kind of info



AHO (2007)

Whom do you imitate? role vs. group

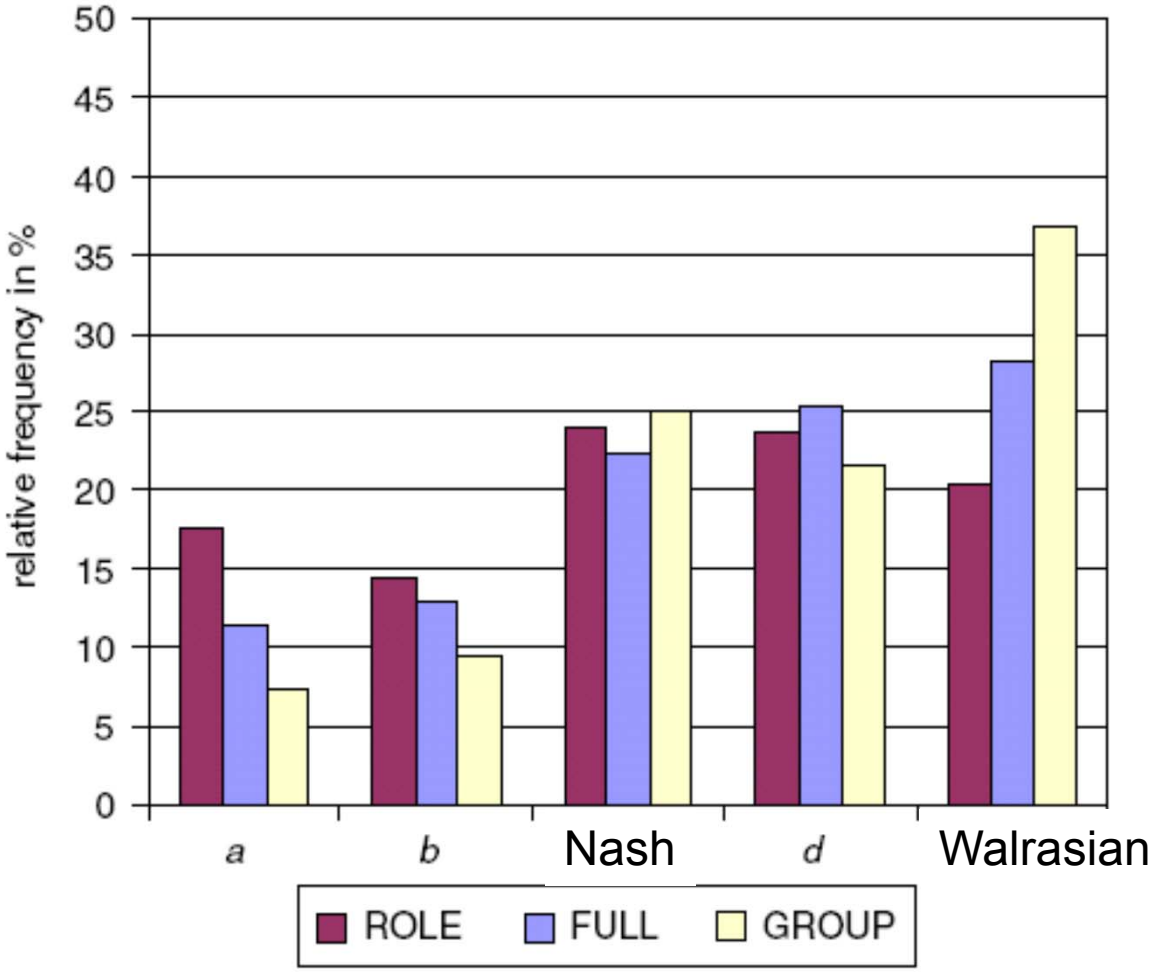


Fig. 2. Relative frequencies of actions, experimental data, rounds 31–60.

⇒ more competitive in GROUP

AHO (2007)

Message

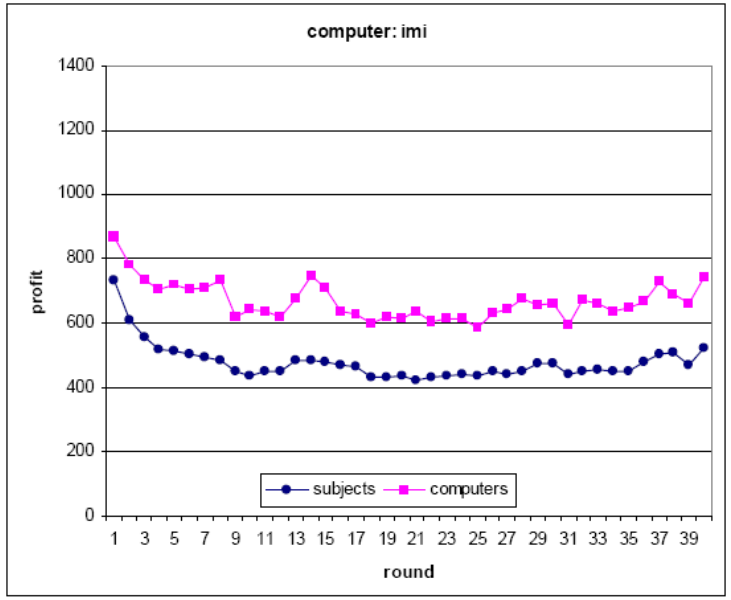
- it is more important whom do you imitate than how exactly (proportional imitation, best max./best avg. etc)
- regression analysis: imitation of actions more prevalent when subjects observe others with whom they interact as opposed to others who have the same role but play in different groups

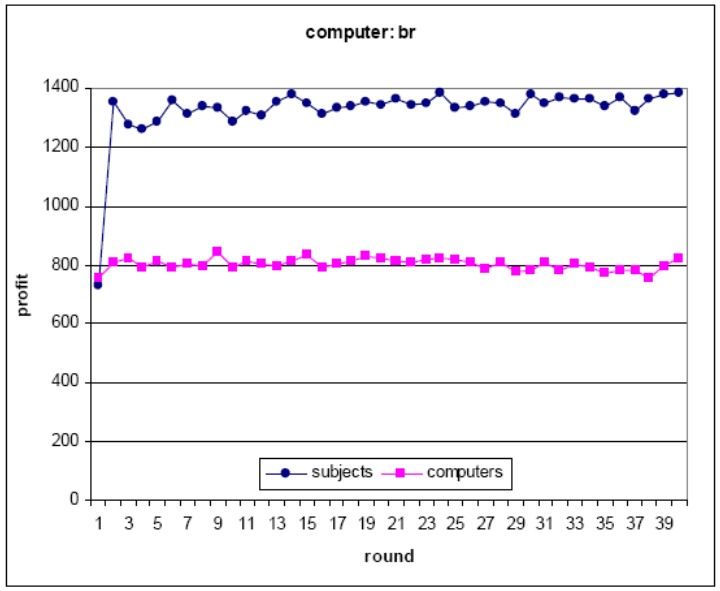
DKOS (2010)

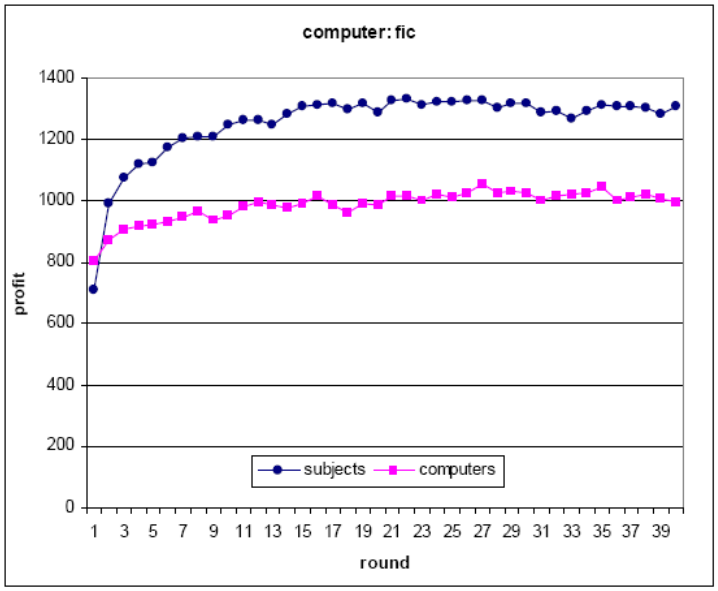
How do subjects play against imitators?

Duersch, Kolb, Oechssler, Schipper (*ET*, 2010)

- Linear Cournot duopoly
- Play as often as they like
- Know that opponent is an algorithm
- br, fic, imit...







Unbeatable Imitation

Games and Economic Behavior 76 (2012), 88-96.

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When is imitation unbeatable? – Some Intuition

Chicken Game

$$\begin{array}{cc} & \begin{array}{cc} \text{swerve} & \text{straight} \end{array} \\ \begin{array}{c} \text{swerve} \\ \text{straight} \end{array} & \begin{pmatrix} 3,3 & 1,4 \\ 4,1 & 0,0 \end{pmatrix} \end{array}$$

No one can beat the imitator by more than the one-period payoff differential of 3.

The games

Symmetric (finite) two-player game $\langle X, \pi \rangle$

- X is a (finite) set of pure actions,
- $\pi : X \times X \longrightarrow \mathbb{R}$ is a symmetric payoff function.

Relative payoff game $\langle X, \Delta \rangle$ with

- $\Delta(x, y) := \pi(x, y) - \pi(y, x)$ being the relative payoff function.

This is a symmetric zero sum game by construction.

The imitator

Imitator: The imitator adopts the other player's action if and only if in the previous round the other's payoff was strictly higher than that of the imitator.

Unbeatable 1

Definition

We say that *imitation is not subject to a money pump* if there exists a finite bound M such that for any initial action of the imitator y_0 and any sequence $\{x_t\}$ of actions of the opponent

$$\sum_{t=0}^{T} \Delta(x_t, y_t) \leq M, \quad \text{for all } T \geq 0, \tag{1}$$

where y_t is given by the imitator's decision rule.

Unbeatable 2

Definition

Imitation is essentially unbeatable if it can be beaten in total by at most the maximal one-period payoff differential, i.e., if

$$\sum_{t=0}^{\infty} \Delta(x_t, y_t) \leq \max_{x,y} \Delta(x, y).$$

Possible opponents

The opponent (“Maximizer”, she)

- may be infinitely patient and forward looking
- may know *exactly* what the imitator is going to do next
- may be able to commit to a strategy (closed loop)

In particular, may maximize her long-run absolute or relative payoff.

Symmetric 2x2 Games

Proposition 1 *In any symmetric 2x2 game, imitation is essentially unbeatable.*

Proof:

- Suppose in t, optimizer achieves $\Delta(x, x') > 0$.
- Imitator will follow in $t + 1$.
- For another period with strictly positive relative payoff, we must have $\Delta(x', x) > 0$.
- This yields a contradiction because zero sum, i.e., $\Delta(x', x) = -\Delta(x, x')$.

A full characterization

Paradigmatic example for exploitation:
Rock-paper-scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

A full characterization

Paradigmatic example for exploitation:
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A full characterization

Paradigmatic example for exploitation:
Rock-paper-scissors

Maximizer wins 1 in each period
Perfect money pump

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
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A full characterization

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- if RPS somewhere in the game as submatrix => money pump
- Def: generalized RPS (gRPS) game: if ∃sym. zero-sum submatrix in which in each column there is a positive entry

A full characterization

- if RPS somewhere in the game as submatrix => money pump
- Def: generalized RPS (gRPS) game: if ∃sym. zero-sum submatrix in which in each column there is a positive entry

Theorem

Imitation is subject to a money pump in the finite symmetric game (X, π) if and only if its relative payoff game (X, Δ) is a gRPS game.

Some Preliminaries

Definition 3 (Schaffer, 1988, 1989) *An action $x^* \in X$ is a finite population evolutionary stable strategy (fESS) of the game (X, π) if*

$$\pi(x^*, y) \;\geq\; \pi(y, x^*) \text{ for all } y \in X,$$

or, equivalently

$$\Delta(x^*, y) \;\geq\; 0, \text{ for all } y \in X.$$

Lemma 1 (Schaffer, 1988, 1989) *Let (X, Δ) be the relative payoff game derived from the symmetric game (X, π) by setting $\Delta(x, y) = \pi(x, y) - \pi(y, x)$. Then the following statements are equivalent:*

(i) x^* is a fESS of (X, π)

(ii) x^* is a symmetric pure strategy Nash equilibrium of (X, Δ)

(iii) x^* is a symmetric pure strategy saddle point (X, Δ)

Separable Relative Payoff Games

Definition

A relative payoff function Δ is additively separable if $\Delta(x, y) = f(x) + g(y)$ for some functions $f, g : X \longrightarrow \mathbb{R}$.

Theorem

Let (X, π) be a symmetric two-player game. Suppose that X is a compact and totally ordered set and π is continuous. Then imitation is essentially unbeatable if any of the following conditions holds:

(i) (X, π) is an exact potential game

(ii) (X, Δ) is an exact potential game

(iii) Δ has increasing differences

(iv) Δ has decreasing differences

(v) Δ is additively separable.

Separable Relative Payoff Games

Example 1 (Cournot Duopoly with Linear Demand)

$$\pi(x,y) = x(b - x - y) - c(x) = bx - bx^2 - c(x) - xy$$

Example 2 (Public Goods) $\pi(x,y) = g(x,y) - c(x)$

Example 3 (Common Pool Resources)

$$\pi(x,y) = \begin{cases} c(e-x) + \frac{x}{x+y}(a(x+y) - b(x+y)^2) & \textit{if } x,y > 0 \\ ce & \textit{otherwise} \end{cases}$$

Example 4 (Minimum Effort Coordination)

$$\pi(x,y) = \min\{x,y\} - c(x)$$

Quasiconcave Relative Payoff

Definition

(X, π) is quasiconcave (or single-peaked) if there exists a total order $<$ on X such that for each $x, x', x'', y \in X$ and $x' < x < x''$, we have that $\pi(x, y) \geq \min\{\pi(x', y), \pi(x'', y)\}$.

(Radzik, 1991)

Proposition 4 *Every finite quasiconcave symmetric two-player zero sum game has a symmetric pure strategy saddle point.*

(Duersch et al. 2010)

Quasiconcave Relative Payoff

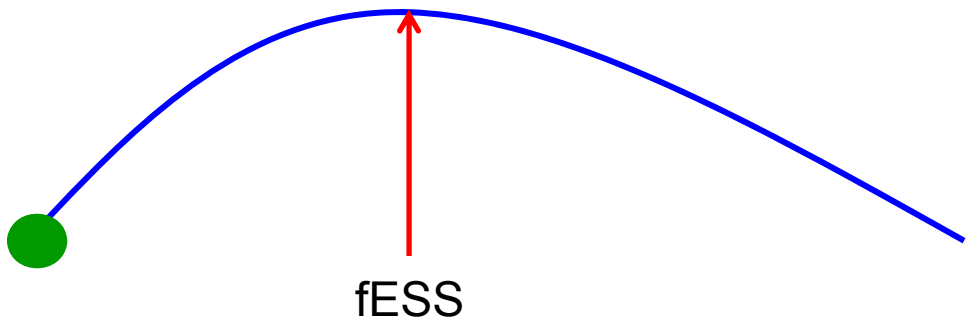
Proposition 5 *Let (X, π) be a finite game. If the relative payoff function $\Delta(x, y)$ is quasiconcave in x, then imitation is not subject to a money pump.*

Lemma 3 *Let (X, π) be a symmetric game and suppose $\Delta(x, y)$ is quasiconcave in x.*

1. *If x is between some y and some fESS x^*, then*

$$\Delta(x, y) = -\Delta(y, x) \geq 0.$$

2. *If x^* and x^{**} are fESS, then so are all x between x^* and x^{**}.*



Aggregative Games

Definition 6 *We say that* (X, π) *is an* aggregative game *if it satisfies the following properties:*

(i) X *is a totally ordered set of actions and* Z *is a totally ordered set.*

(ii) There exists an aggregator $a : X \times X \longrightarrow Z$ *that is*

- *monotone increasing in its arguments, i.e. if* $(x'', y'') > (x', y')$*, then* $a(x'', y'') > a(x', y')$*, and*
- *symmetric, i.e.,* $a(x, y) = a(y, x)$ *for all* $x, y \in X$*.*

(iii) π *is extendable to* $\Pi : X \times Z \longrightarrow \mathbb{R}$ *with* $\Pi(x, a(x, y)) = \pi(x, y)$ *for all* $x, y \in X$*.*

Aggregative Games

Definition 7 *We say that an aggregative game* (X, Π) *is quasisubmodular (resp. quasisupermodular) if* Π quasisubmodular (resp. quasisupermodular) *in* (x, y) *on* $X \times Z$*, i.e., for all* $z'' > z'$, $x'' > x'$,

$$\Pi(x'', z'') - \Pi(x', z'') \geq 0 \;\Rightarrow (\Leftarrow)\; \Pi(x'', z') - \Pi(x', z') \geq 0 \quad (3)$$

$$\Pi(x'', z'') - \Pi(x', z'') > 0 \;\Rightarrow (\Leftarrow)\; \Pi(x'', z') - \Pi(x', z') > 0 \quad (4)$$

An action $x^* \in X$ is a fESS of the aggregative game (X, Π) if

$$\Pi(x^*, a(x^*, x)) \geq \Pi(x, a(x^*, x)), \;\; \forall x \in X.$$

Aggregative Games

Proposition 6 *If* (X, Π) *is a finite quasiconcave quasisubmodular aggregative game for which a unique fESS exists, then imitation is not subject to a money pump.*

Example 7 (Cournot Duopoly) $\pi(x, y) = xp(x + y) - c(x)$ *with* $\pi(x, y)$ *is quasiconcave in* x*. Any symmetric Cournot oligopoly with an arbitrary decreasing inverse demand function* p *and arbitrary increasing cost function* c *is an aggregative quasisubmodular game (Schipper, 2009, Lemma 1).*

Example 8 (Rent Seeking) $\pi(x, y) = \frac{x}{x+y}v - x$ *(Tullock, 1980).*

Conclusions

Class of Games	Examples	Result
Symmetric 2x2 games	Prisoner's dilemma, Hawk-Dove, Coordination game …	Imitation is essentially unbeatable
Separable relative payoffs	Linear Cournot, Public good, Common pool resource, Minimum effort	Imitation is essentially unbeatable
Finite quasiconcave relative payoff games	Concave in first argument & convex in second	Imitation is not subject to a money pump
Finite quasiconcave quasisubmodular aggregative games	Cournot, rent seeking, Common pool resource	Imitation is not subject to a money pump

Main result: imitation subject to a money pump iff gRPS

Message

What to make of it?

- Since evolution cares about relative payoffs, imitation may be selected in many economically relevant contexts.

- Evolutionary advantage of being stubborn.

- However: Have examples that imitation can be beaten in games with 1 imitator and 2 maximizers.

Imitation vs. rel. payoff max.

Duersch, Oechssler, Schipper (new project)

- in all experiments so far:
- outcome of imitation = outcome of relative payoff max.
- Example: Cournot

- for more than 2 players: define relative payoff max as

$$\max \pi_i - 1/(n-1) \sum_{j \neq i} \pi_j$$

Imitation vs. rel. payoff max.

- Consider finite, symmetric two-player games.

Proposition:

(s,s) unique stochastically stable state of the imitation dynamic

$\Leftrightarrow$ (s,s) strict Nash eq. of the *relative* payoff game.

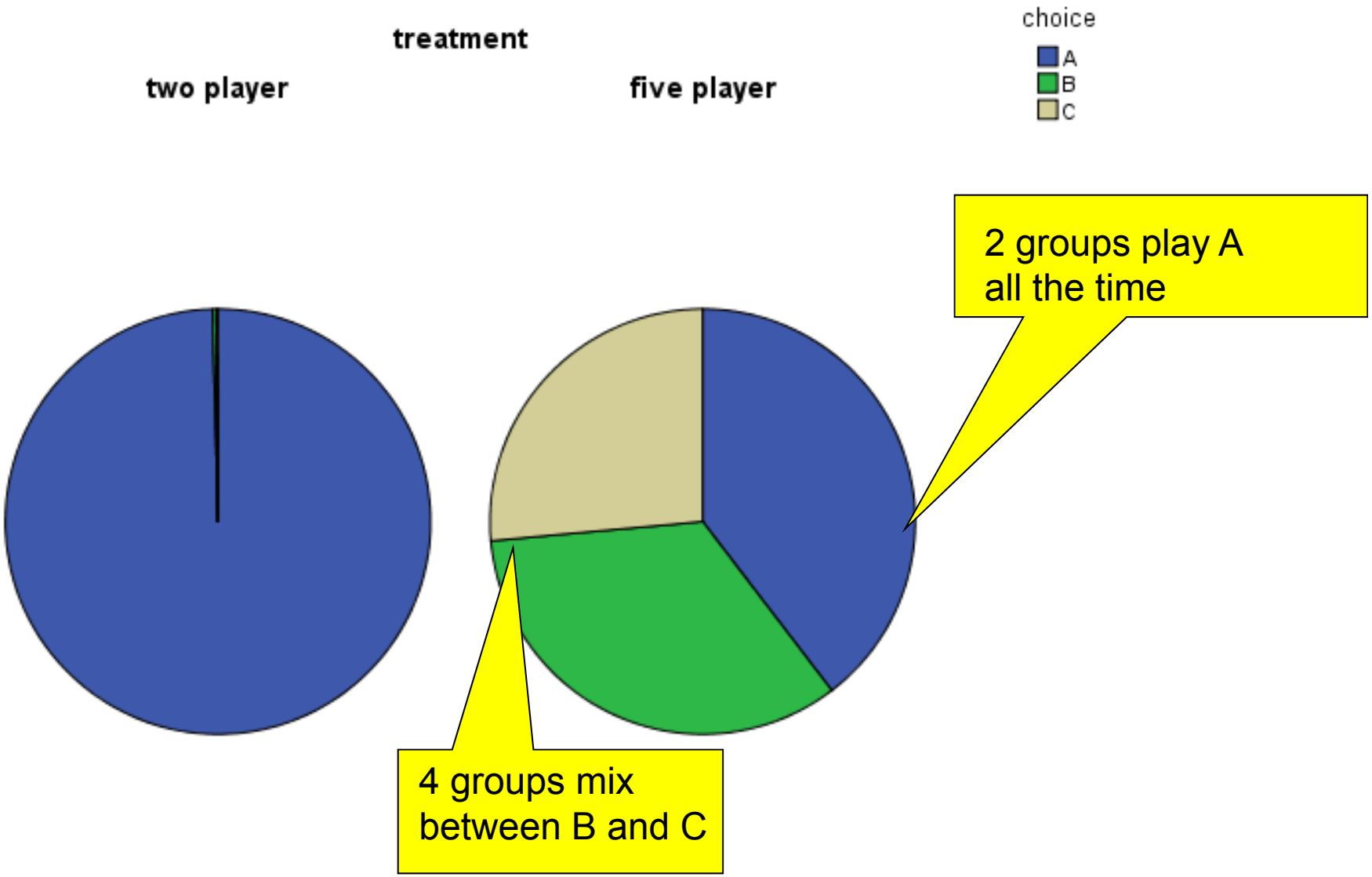
Imitation vs. rel. payoff max.

Design for new experiment

- artificial game with 2-player and 5-player version
- 2-player version: relative payoff max. and imitation predict action A
- 5-player version: relative payoff max: action A, imitation: actions B,C
- both outcomes yield almost same payoff
- provide subjects with payoff table
- feedback info: own payoff, action and payoffs of others in same matching group
- 60 rounds, fixed matching

Imitation vs. rel. payoff max.

Imitation vs. rel. payoff max.



Imitation vs. rel. payoff max.

Message

- imitation seems to dominate in 5-player game
- strongly driven by decisions in 1st round
- new treatment with **restart and rematching** after 20 rounds
- if one or two B/C players mixed in → whole group switches to B/C

Conclusion

- imitation can be an exciting research topic
- still many open questions
- imitation and stress
- imitation among kids
- „modest imitation“ in asymmetric situations

Thank you for your attention!