Note on Exclusive Distribution

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October 29, 2010

This version: November 29, 2010

Abstract

The literature studying vertical trade relationships seems to assume that an upstream monopolist would prefer downstream competition over exclusive distribution arrangements. In this paper, we derive precise conditions for when an upstream monopolist will prefer a competing distribution system over an exclusive distribution arrangement in the downstream market. We investigate also the welfare properties of these distribution systems.

JEL Classification: D43; L13; L40

Keywords: Exclusive distribution, competing distribution, Cournot competition

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1 Introduction

There is a large literature on anti-competitive behaviour in vertically organised markets. One puzzle in this literature is the challenge to explain the common use of exclusive contracts in order to sell homogeneous products in downstream markets despite the obvious disadvantages of double marginalisation for an upstream monopolist. This is even more surprising since also welfare may decline under such contractual arrangements.

From the point of view of the upstream monopolist, exclusive contracts are favourable if the monopoly rents of the downstream monopolist can be extracted, e.g., by auctioning off the rights to the exclusive dealings. Vertical restraints, however, such as non-linear (two-part) tariffs, resale price maintenance or tie-in provisions are often illegal as they either restrict competition or constitute an abuse of dominant position, e.g., according to Sections 1 and 2 of the U.S. Sherman Antitrust Act or Articles 81 and 82 of the European Communities Treaty, respectively. If complete extraction of monopoly rents is impossible, perfect competition in the downstream market appears to be the best alternative, since downstream sellers would have to charge prices at the marginal costs determined by the upstream monopolist's prices.

Most of the literature on vertical market integration seems to (implicitly) assume that an upstream monopolist would prefer downstream competition over exclusive distribution arrangements under linear pricing. For instance, REY AND VERGÉ (2008), in a recent overview of the literature on the economics of vertical restraints, state on page 361:

> *"Notice that vertical restraints are not necessarily needed to solve the double-marginalization problem. Introducing strong intra-brand competition (using several perfectly substitutable retailers) would remove the retail markup. The manufacturer could then set the wholesale price equal to (...) the monopoly price (...)."*

Similarly, modelling oligopolistic intrabrand and interbrand competition with two upstream firms and two-part tariffs consisting of a fixed fee and a per-unit payment (royalty), SAGGI AND VETTAS (2002, P. 198) suggest that

> *"In the case of royalty-only contracts, each upstream firm prefers to increase its number of downstream firms, as long as the rival number of firms are not too large."*

In a model of exclusive territories with two upstream producers and price competition between

retailers on the downstream market, REY AND STIGLITZ (1995, P. 440) emphasise

> *"... that in this model, in the absence of imperfect competition at the upper level, there is no motivation for using exclusive territories: a monopoly producer (...) would achieve the integrated optimum by dealing with competitive retailers."*

In a two-stage divisionalization model with Cournot quantity competition, BAYE ET AL. (1996) also find that oligopolistic upstream firms opt for competition between divisions on the downstream market as this allows them to commit to a greater level of output[1].

To the best of our knowledge, there is no formal paper that explicitly investigates this intuition. In this paper, we will study Cournot competition in the downstream market. We will derive precise conditions for when a downstream oligopoly of traders yields higher profits for the upstream producer than a downstream monopoly. Moreover, we will show that welfare effects of downstream competition are less clear cut than expected. In particular, a downstream oligopoly may have negative welfare properties compared to a downstream monopoly.

2 A simple model

There is a single producer P who can produce a product x with a linear cost function,

$$c(x) = c \cdot x.$$

Demand for the product is described by the following inverse demand function:

$$P(x) = a - bx.$$

Suppose the producer cannot access the market directly but has to sell the product through several dealers. We will consider two cases:

- *Exclusive distribution:* a single trader, Trader α, and
- *Competing distribution:* there is a (finite) set of competing traders $J = \{1, 2, ..., n\}$.

Traders $j \in J$ may differ in their constant marginal cost of delivery k_j. Without loss of generality for our problem, we will assume that traders in J can be ranked by their trading costs: $k_1 \leq k_2 \leq ...k_j \leq k_{j+1} \leq ... \leq k_n$. Moreover, since the producer's supply prices cannot be

[1] Similar views can be found in DURHAM (2000, P. 210FF), POLASKY (1992) and CORCHON (1991).

lower than the marginal cost of production c, a trader with local costs $k_j \geq a - c$ would never be able to supply in the downstream market. Hence, we will assume that $a - c - k_n > 0$ holds.

Assumption 2.1

(a) increasing local costs: $k_1 \leq k_2 \leq ...k_j \leq k_{j+1} \leq ... \leq k_n$;

(b) potential traders: $a - c - k_n > 0$.

Notice that Assumption 2.1 implies that all traders provide some scope for profits of the producer.

3 Exclusive distribution: double marginalization

Consider first the case of *exclusive rights* for trading the product. Trader α has a monopoly in the downstream market[2]. We will assume that production costs c and local trading costs k_α allow for a monopoly in the downstream market, $a > c + k_\alpha$. Given the price q charged by the monopolistic producer, Trader α chooses x to maximize profit:

$$\pi_\alpha = P(x) \cdot x - (q + k_\alpha) \cdot x.$$

It is straightforward to derive the optimal supply policy of the downstream exclusive trader:

- $x_\alpha(q) := \max\left\{0, \frac{1}{2b}(a - q - k_\alpha)\right\}$,
- $p_\alpha(q) := \frac{1}{2}(a + q + k_\alpha)$.
- $\pi_\alpha(q) := \frac{1}{4b}(a - q - k_\alpha)^2$.

The producer will set a price q to maximize profit, $\pi = (q - c) \cdot x_\alpha(q)$, yielding the optimal strategy

$$q^M := \frac{a + c - k_\alpha}{2}.$$

Summarizing, if the downstream monopoly is viable, $a > c + k_\alpha$, then $q^M > c$ and we obtain:

- supply price: $q^M = \frac{1}{2}(a + c - k_\alpha)$,
- quantity supplied: $x^M := x_\alpha(q^M) = \frac{1}{4b}(a - c - k_\alpha)$,
- price: $p^M := p_\alpha(q^M) = \frac{1}{4}(3a + c + k_\alpha)$,
- profit of Trader α: $\pi_\alpha^M := \pi_\alpha(q^M) = \frac{1}{16b}(a - c - k_\alpha)^2$,
- profit of the Producer: $\pi^M := \pi(q^M) = \frac{1}{8b}(a - c - k_\alpha)^2$.

[2] For a more detailed analysis of this case, see, e.g., GAL-OR (1991), p. 1241ff.

Finally, consumer surplus $S(p)$, which is easily derived from the linear demand function,

$$S(q^M) := \frac{1}{2}\left[a - p^M\right] \cdot x^M,$$

and aggregate profits provide us with a measure of welfare:

- consumer surplus: $S^M := S(q^M) = \frac{1}{32b}\left(a - c - k_\alpha\right)^2,$
- aggregate profit: $\Pi^M := \pi^M + \pi^M_\alpha = \frac{3}{16b}\left(a - c - k_\alpha\right)^2,$
- welfare: $W^M := \Pi^M + S^M = \frac{7}{32b}\left(a - c - k_\alpha\right)^2.$

4 Cournot competition in the downstream market

Before turning to the case of an arbitrary number of potential traders, it is instructive to consider the duopoly case.

4.1 The two-trader case

We will denote the two potential traders as Trader α and Trader β with local trading costs $k_\alpha \leq k_\beta$, respectively. Figure 1 illustrates this situation.Each trader $j \in \{\alpha, \beta\}$ chooses x_j to

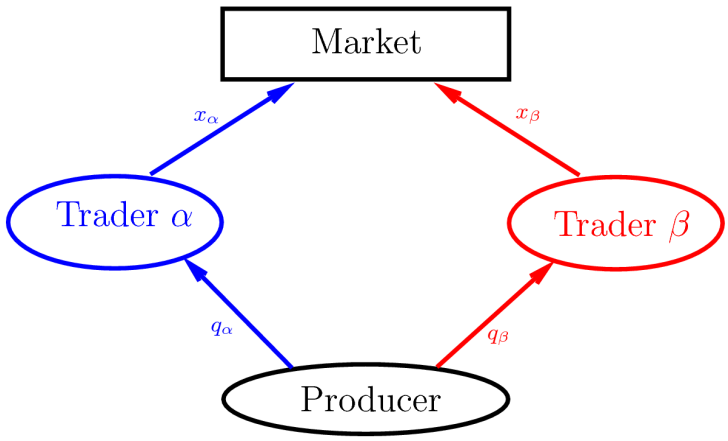

Figure 1. Traders and market

maximize

$$\pi_j = P(x_j + x_{-j}) \cdot x_j - (q_j + k_j) \cdot x_j.$$

Given a supply price policy (q_α, q_β) of the producer and a quantity x_{-j} brought to market by Trader $-j$, we obtain the best reply of Trader j,

$$r_j(x_{-j}) = \max\left\{0, \frac{a - bx_{-j} - q_j - k_j}{2b}\right\}.$$

The Nash equilibrium quantities $x_\alpha(q_\alpha, q_\beta)$ and $x_\alpha(q_\alpha, q_\beta)$ depending on the supply price policy of the producer are easily derived as

$$\begin{aligned} x_\alpha(q_\alpha, q_\beta) &= \min\left\{\max\left\{0, \frac{a + (q_\beta + k_\beta) - 2\,(q_\alpha + k_\alpha)}{3b}\right\}, \max\left\{0, \frac{a - (q_\alpha + k_\alpha)}{2b}\right\}\right\}, \\ x_\beta(q_\alpha, q_\beta) &= \min\left\{\max\left\{0, \frac{a + (q_\alpha + k_\alpha) - 2\,(q_\beta + k_\beta)}{3b}\right\}, \max\left\{0, \frac{a - (q_\beta + k_\beta)}{2b}\right\}\right\}. \end{aligned}$$

Notice that the supply price policy of the producer could be such that only one trader serves the market. Hence, exclusive trading can be induced by the producer via price differentiation between the two traders. Figure 2 illustrates the two cases.The producer will set prices (q_α, q_β)

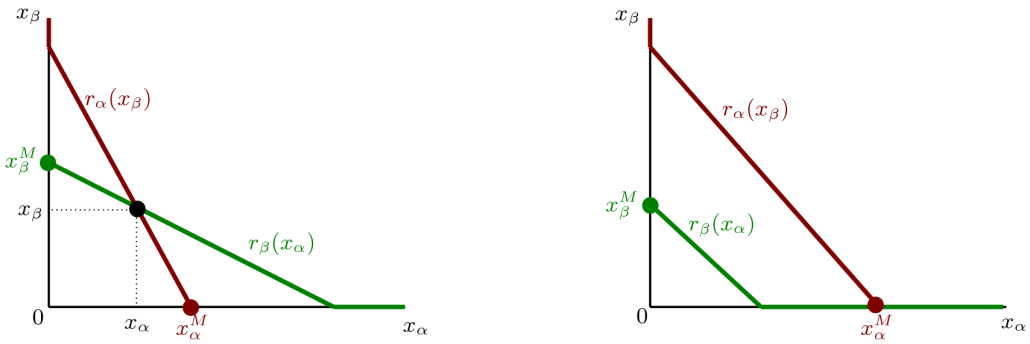

Figure 2. Types of Nash equilibria in the downstream market

to maximize profit

$$\pi = (q_\alpha - c) \cdot x_\alpha(q_\alpha, q_\beta) + (q_\beta - c) \cdot x_\beta(q_\alpha, q_\beta). \tag{1}$$

In general, the Nash equilibrium quantities $x_\alpha(q_\alpha, q_\beta)$ and $x_\alpha(q_\alpha, q_\beta)$ are neither concave nor convex functions. This complicates the solution of the producer's problem considerably. The Nash equilibrium quantities $x_\alpha(q_\alpha, q_\beta)$ and $x_\beta(q_\alpha, q_\beta)$ are, however, piecewise linear functions of (q_α, q_β). Moreover, for any $j = \alpha, \beta$, one checks easily that $x_j(q_\alpha, q_\beta) = x_j(q_j)$ if and only if $x_{-j}(q_\alpha, q_\beta) = 0$ holds. Indeed, this is obvious from Figure 2. Hence, one can partition the set of supply prices (q_α, q_β) into four regions:

(i)	$x_\alpha(q_\alpha, q_\beta) \leq x_\alpha(q_\alpha)$	and	$x_\beta(q_\alpha, q_\beta) \leq x_\beta(q_\beta)$,
(ii)	$x_\alpha(q_\alpha, q_\beta) = x_\alpha(q_\alpha)$	and	$x_\beta(q_\alpha, q_\beta) = 0$,
(iii)	$x_\alpha(q_\alpha, q_\beta) = 0$	and	$x_\beta(q_\alpha, q_\beta) = x_\beta(q_\beta)$,
(iv)	$x_\alpha(q_\alpha, q_\beta) = 0$	and	$x_\beta(q_\alpha, q_\beta) = 0$.

Figure 3 shows iso-profit lines for the profit function of the producer (Equation 1). The blue

line satisfies $x_\alpha(q_\alpha, q_\beta) = x_\alpha(q_\alpha)$ and the red line $x_\beta(q_\alpha, q_\beta) = x_\beta(q_\beta)$. The four regions are indicated in Figure 3.

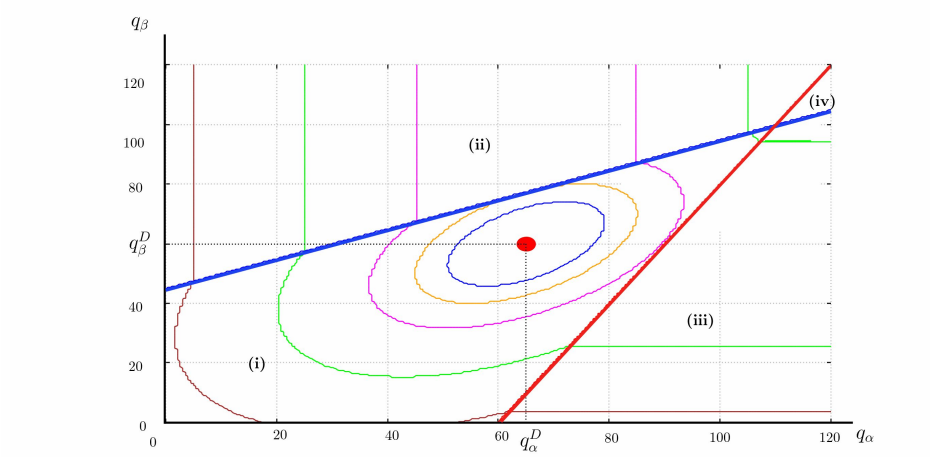

Figure 3. Iso-profit lines of the producer

It is easy to see that the profit function is concave within each region. In Region (ii) Trader α holds exclusive rights, while in Region (iii) Trader β is the monopolistic trader[3]. For these cases, the optimal profit of the producer has already been derived in Section 3. The optimal solution for case (i) can be derived from the first order conditions of the following problem:

$$\begin{aligned}\max_{q_\alpha, q_\beta} \pi \quad &= \quad \frac{1}{3b}\left\{(q_\alpha - c)\cdot\left[a + (q_\beta + k_\beta) - 2\left(q_\alpha + k_\alpha\right)\right]\right. \\ &\qquad \left. + (q_\beta - c)\cdot\left[a + (q_\alpha + k_\alpha) - 2\left(q_\beta + k_\beta\right)\right]\right\}.\end{aligned}$$

Straightforward computations yield the solution which is summarised in Lemma 4.1.

Lemma 4.1 *Suppose there is an equilibrium where the optimal supply price policy* $\left(q_\alpha^D, q_\beta^D\right)$ *is such that both traders supply the commodity, then the following solution holds:*

- *supply prices of the traders:* $q_\alpha^D = \frac{1}{2}\left(a + c - k_\alpha\right), \qquad q_\beta^D = \frac{1}{2}\left(a + c - k_\beta\right),$
- *quantities in the downstream market:* $x_\alpha^D = \max\{0, \frac{1}{6b}\left(a - c - 2k_\alpha + k_\beta\right)\},$
  $x_\beta^D = \max\{0, \frac{1}{6b}\left(a - c + k_\alpha - 2k_\beta\right)\},$
- *aggregate supply:* $x^D := x_\alpha^D + x_\beta^D = \frac{1}{6b}\left(2\left(a - c\right) - k_\alpha - k_\beta\right),$
- *price in the downstream market:* $p^D = \frac{1}{6}\left(4a + 2c + k_\alpha + k_\beta\right),$
- *profit of the traders:* $\pi_\alpha^D = \frac{1}{36b}\left[\left(a - c - k_\alpha\right) - \left(k_\alpha - k_\beta\right)\right]^2,$
  $\pi_\beta^D = \frac{1}{36b}\left[\left(a - c - k_\beta\right) + \left(k_\alpha - k_\beta\right)\right]^2,$
- *profit of the Producer:* $\pi^D = \frac{1}{12b}\left[\left(a - c - k_\alpha\right)^2 + \left(a - c - k_\beta\right)^2 + \left(k_\alpha - k_\beta\right)^2\right].$

[3] In Region (iv), the supply prices are such that no trader wants to supply the commodity.

Comparing the profits obtained from exclusive contracts reveals that an exclusive contract with Trader α, the trader with low local costs, is more profitable than an exclusive contract with Trader β. Hence, the producer will choose a supply price policy $\left(q_\alpha^D, q_\beta^D\right)$ which

- either makes it unprofitable for the trader with the high local costs to participate in the downstream market, which corresponds to an exclusive contract with Trader α,
- or which has both players supply the commodity, a non-exclusive trade arrangement.

In Figure 3, the producer will choose supply prices in the regions (i) or (ii), but never in Region (iii).

The following proposition characterises these cases.

Proposition 4.2 Non-exclusive trade condition
An equilibrium with non-exclusive trade in the downstream market is optimal for the producer if

$$(a - c - k_\beta) > (k_\beta - k_\alpha) \tag{2}$$

holds.

Proposition 4.2 shows that the producer will choose non-excluding supply prices if the costs of the high-local-cost trader β do not exceed the costs of the low-cost trader by too much. Indeed, for differing local costs $k_\beta > k_\alpha$, the upstream producer will subsidise the high-cost trader in order to keep him in the market,

$$q_\alpha^D = \frac{1}{2}\left(a + c - k_\alpha\right) > \frac{1}{2}\left(a + c - k_\beta\right) = q_\beta^D.$$

Condition 2 of Proposition 4.2 can be viewed as an upper limit for such subsidisation.

The following proposition summarises the welfare implications of non-exclusive trade.

Proposition 4.3 *Suppose there is an equilibrium where the optimal supply price policy $\left(q_\alpha^D, q_\beta^D\right)$ is such that both traders supply the commodity, then one obtains the following welfare level:*

- *aggregate profit:* $\Pi^D = \pi^D + \pi_\alpha^D + \pi_\beta^D$
  $= \frac{1}{36b}\left[4\left(a - c - k_\alpha\right)^2 + 4\left(a - c - k_\beta\right)^2 + 7\left(k_\alpha - k_\beta\right)^2\right],$
- *consumer surplus:* $S^D = \frac{1}{72b}\left[\left(a - c - k_\alpha\right) + \left(a - c - k_\beta\right)\right]^2,$
- *welfare:* $W^D = \Pi^D + S^D$
  $\frac{1}{72b}\left[20\left(a - c\right)^2 - 20(k_\alpha + k_\beta)\left(a - c\right)\right.$
  $\left. +10\left(\left(k_\alpha\right)^2 + \left(k_\beta\right)^2\right) + 13\left(k_\alpha - k_\beta\right)^2\right].$

Welfare conditions are harder to interpret since there is a trade-off between more efficient trade in the downstream market and inefficient cost effects.

Proposition 4.4 Welfare effects of non-exclusive trade
Non-exclusive trade will
(i) increase consumer surplus if

$$a - c > 2k_\beta - k_\alpha$$

holds;
(ii) increase aggregate profits if

$$16\left(a - c - k_\beta\right)^2 + 28\left(k_\alpha - k_\beta\right)^2 > 11\left(a - c - k_\alpha\right)^2;$$

(iii) increase welfare if

$$136\left(a - c\right)^2 + 368\left(a - c\right)k_\alpha + 320\left(k_\beta\right)^2 + 416\left(k_\alpha - k_\beta\right) > 640\left(a - c\right)k_\beta + 184\left(k_\alpha\right)^2.$$

The conditions on the exogenous parameters $(a, c, k_\alpha, k_\beta)$ given in Proposition 4.4 are necessary and sufficient for welfare gains from non-exclusive trade. Most of them are are not intuitive, since they reflect the trade-off between gains in consumer surplus and production inefficiencies from including traders with higher local costs. By Proposition 4.2, gains in consumer surplus will be achieved whenever there are non-exclusive trade contracts in the downstream market. This will be true only if the high local costs of trading in the downstream market do not exceed the low costs by too much. In contrast, if $k_\alpha = k_\beta$ holds, Proposition 4.4 and Proposition 4.2 show that non-exclusive trade will increase consumer surplus. Indeed, the highest gain in consumer surplus will be realised if there are no local trading costs for both traders.

Similarly, aggregate profits will increase for $k_\alpha = k_\beta$, as one can see immediately from Proposition 4.4 (ii). Hence, we can conclude that non-exclusive trade is unambiguously welfare increasing if local costs of the traders are equal. Indeed, by continuity of the conditions in Proposition 4.4 this will remain true for local costs which do not differ by too much.

4.2 The n-traders case

In this section we will extend the analysis of the duopoly case to the case of n traders in the downstream market. Except for the last result, this section will provide straightforward generalisations of the duopoly case.

Consider a set of n potential traders in the downstream market who differ in their individual

trading costs k_j. Denoting by $X = \sum_{k \in J} x_k$ the aggregate quantity supplied in the downstream market and by $X_{-j} := \sum_{j \neq k \in J} x_k$ the aggregate supply of the competing traders other than j, one can write the profit of Trader j as

$$\pi_j = P(X_{-j} + x_j) \cdot x_j - (q_j + k_j) \cdot x_j.$$

Straightforward optimisation with respect to the quantity x_j yields $r_j(X_{-j}) = \max\left\{0, \frac{a - bX_{-j} - q_j - k_j}{2b}\right\}$.

Lemma 4.5 *Given an array of prices $q = (q_1, ..., q_n)$ charged by the producer, a* Cournot equilibrium with n potential traders *is given as*

- *aggregate supply:* $X^C(q) = \frac{1}{(n+1)b}\left(na - \sum_{j=1}^{n}(q_j + k_j)\right)$,
- *market price:* $p^C(q) = \frac{1}{(n+1)}\left(a + \sum_{k=1}^{n}(q_k + k_k)\right)$,
- *supply of firm $j \in J$:* $x_j^C(q) = \max\left\{0, \frac{1}{(n+1)b}\left(a + \sum_{\substack{k=1 \\ k \neq j}}^{n}(q_k + k_k) - n(q_j + k_j)\right)\right\}$,
- *profit of firm $j \in J$:* $\pi_j^C(q) = \frac{1}{(n+1)^2 b}\left(a + \sum_{\substack{k=1 \\ k \neq j}}^{n}(q_k + k_k) - n(q_j + k_j)\right)^2$.

The producer will choose prices $(q_j)_{j \in J}$ to maximize profit π,

$$\begin{aligned} \pi(q) &:= \sum_{j=1}^{n}(q_j - c)\, x_j^C(q) \\ &= \frac{1}{(n+1)b}\sum_{j=1}^{n}(q_j - c)\left(a + \sum_{\substack{k=1 \\ k \neq j}}^{n}(q_k + k_k) - n(q_j + k_j)\right). \end{aligned}$$

Proposition 4.6 shows the solution to this problem.

Proposition 4.6 *For a set of n potential traders, the following table contains the producer's optimal supply prices and the associated Nash equilibrium:*

- *supply prices of the traders:* $q_j^C = \frac{1}{2}\left(a + c - k_j\right),$
- *quantities in the downstream market:* $x_j^C = \max\left\{0, \frac{1}{2(n+1)b}\left(a - c + \sum_{k=1}^{n} k_k - (n+1)\, k_j\right)\right\},$
- *aggregate supply:* $X^C = \frac{1}{2(n+1)b}\left(n\,(a-c) - \sum_{k=1}^{n} k_k\right),$
- *price in the downstream market:* $p^C = \frac{1}{2(n+1)}\left((n+2)a + nc + \sum_{k=1}^{n} k_k\right),$
- *profit of the traders:* $\pi_j^C = \frac{1}{4(n+1)^2 b}\left(a - c + \sum_{k=1}^{n} k_k - (n+1)\, k_j\right)^2,$
- *profit of the Producer:* $\pi^C = \frac{1}{4(n+1)b}\left[\sum_{j=1}^{n}\left(a - c - k_j\right)^2 + n\sum_{k=1}^{n} k_k^2 - \left(\sum_{k=1}^{n} k_k\right)^2\right].$

Notice that the optimal supply prices are the same as the ones set for the respective monopolies[4]. The number of active traders in a Nash equilibrium will depend on the supply price vector $q = (q_1, ..., q_n)$. Suppose there is a trader $m \leq n$ in the downstream market who will not offer the producer's product given the equilibrium supply prices, then, by Assumption 2.1(a), it is clear that traders $m + i$ for $i = 1, .., n - m$ will also not trade the product. The following proposition gives the condition for traders to be active in the downstream market.

Proposition 4.7 *There exists a trader $m \in I$ such that all traders $j < m$ will be active in the downstream market. All traders $j \geq m$ will not supply in the downstream market. Trader m will be determined by the inequality*

$$k_{m+1} > k_m \geq \frac{1}{n+1}\left(a - c + \sum_{k=1}^{n} k_k\right) > k_{m-1}.$$

Proposition 4.7 is the analogue of Proposition 4.2 for the n-trader case. It shows that, with differing local costs of trading, not all traders will be active in the downstream market at the Nash equilibrium supply prices $q^C = (q_1^C, ..., q_n^C)$ chosen by the producer. Given Assumption 2.1 (b), it is possible that $k_j < \frac{1}{n+1}\left(a - c + \sum_{k=1}^{n} k_k\right)$ for all $j \in J$ holds. In this case all n traders will be active.

It is also possible to give sufficient conditions for the welfare effects of an increase in the number

[4] This is a useful property of the linear demand function in combination with linear cost functions, which makes explicit derivation of a solution possible.

of active traders in the downstream market. These conditions are, however, not easy to interpret. Consumer surplus is strictly increasing in the amount of the product traded in the downstream market. By setting $X_{n+1}^C \geq X_n^C$, it is straightforward to derive sufficient conditions under which the aggregate quantity traded X^C increases if the number of active traders in the downstream market increases:

$$a - c \geq (n+1)\, k_{n+1} - \sum_{j=1}^{n} k_j.$$

It is also clear from the aggregate quantity traded X^C, which is given in Proposition 4.6, that it will depend on whether the demand effect $(a - c)$ of an additional trader outweighs the additional costs k_{n+1} introduced by this trader. A more clear-cut result is possible in the case of symmetric local costs which is treated next.

4.2.1 Symmetric local costs

Consider the special case where all local traders have the same local costs $k_j = k$ for all $j \in J$. From Proposition 4.7 it follows immediately that it is optimal for the producer to have all n traders active. In this special case, we can consider the limit if the number of traders increases.

Proposition 4.8 *Assume $k_j = k$ for all $j = 1, 2, ..., n$. As $n \to \infty$, we obtain the following equilibrium allocation:*

- *supply prices of the traders:* $q_j^C = \frac{1}{2}(a + c - k)$,
- *quantities in the downstream market:* $\lim_{n\to\infty} x_j^C(n) = \lim_{n\to\infty} \max\left\{0, \frac{1}{2(n+1)b}(a - c - k)\right\} = 0$,
- *aggregate supply:* $\lim_{n\to\infty} X^C(n) = \lim_{n\to\infty} \frac{n}{2(n+1)b}(a - c - k) = \frac{1}{2b}(a - c - k)$,
- *price in the downstream market:* $\lim_{n\to\infty} p^C(n) = \lim_{n\to\infty} \frac{n}{2(n+1)}\left(\frac{(n+2)}{n}a + c + k\right) = \frac{1}{2}(a + c + k)$,
- *profit of the traders:* $\lim_{n\to\infty} \pi_j^C(n) = \lim_{n\to\infty} \frac{1}{4(n+1)^2 b}(a - c - k)^2 = 0$,
- *profit of the Producer:* $\lim_{n\to\infty} \pi^C(n) = \lim_{n\to\infty} \frac{n}{4(n+1)b}(a - c - k)^2 = \frac{1}{4b}(a - c - k)^2$.

The proof is obvious and, therefore, omitted.

In this case individual traders' supply will vanish in the limit, their profit will be driven to zero and the producer can extract the full monopoly rent. This appears to be the scenario which most authors cited in the introduction appear to have in mind.

Note that, in the symmetric case, each additional trader will increase aggregate supply in the

downstream market and hence social surplus. The limiting maximal aggregate supply corresponds to the case where the producer acts like a direct monopoly in the downstream market. This maximal supply defines the upper limit of social surplus.

In conclusion, for symmetric local costs, each additional trader will reduce the double marginalisation problem unambiguously and lead to a higher profit of the monopolist and a higher consumer surplus.

5 Conclusion

In this note, we have studied Cournot quantity setting competition among traders of a homogeneous product in the downstream market of a single producer. Without detailed analysis, the literature seems to assume that downstream competition is always to the benefit of an upstream monopolist because it counterbalances the double marginalisation problem which the upstream monopolist would face otherwise. Indeed, this is what our analysis confirms for the case of a large number of downstream traders *with identical local costs*. If downstream traders have differing local costs, such a result will not hold in general. We investigate also the welfare effects of non-exclusive contracts and find that the upstream monopolist will differentiate supply prices in order to keep as many traders as feasible in the downstream market. This practice may, however, reduce consumer surplus compared to the exclusive-trade contract in the downstream market.

Appendix A. Proofs

Proof of Lemma 4.1. The first-order conditions of the optimisation problem

$$\max_{q_\alpha, q_\beta} \pi = \frac{1}{3b}(q_\alpha - c)\cdot[a + (q_\beta + k_\beta) - 2(q_\alpha + k_\alpha)] + (q_\beta - c)\cdot[a + (q_\alpha + k_\alpha) - 2(q_\beta + k_\beta)]$$

can be solved easily for the equilibrium supply prices:

$$q_\alpha^D = \frac{1}{2}(a + c - k_\alpha), \quad q_\beta^D = \frac{1}{2}(a + c - k_\beta).$$

Substituting into $x_\alpha(q_\alpha, q_\beta)$ and $x_\beta(q_\alpha, q_\beta)$, one obtains

$$x_\alpha^D = \frac{1}{6b}(a - c + k_\beta - 2k_\alpha), \quad x_\beta^D = \frac{1}{6b}(a - c + k_\alpha - 2k_\beta),$$

and the equilibrium price in the downstream market

$$p^D = a - b\left(x_\alpha^D + x_\beta^D\right) = \frac{1}{6}(4a + 2c + k_\alpha + k_\beta).$$

Substituting into the producer's profit function, one has

$$\pi^D = (q_\alpha - c)\cdot x_\alpha^D + (q_\beta - c)\cdot x_\beta^D = \frac{1}{12b}\left[(a - c - k_\alpha)^2 + (a - c - k_\beta)^2 + (k_\alpha - k_\beta)^2\right].$$

■

Proof of Proposition 4.2. The profit function of the producer is concave in (q_α, q_β) within each of the four regions. Comparing equilibrium profit of the monopoly cases reveals that a monopoly of the low-cost trader α , $\frac{1}{8b}(a - c - k_\alpha)^2$, yields higher profits for the producer than a monopoly of Trader β, $\frac{1}{8b}(a - c - k_\beta)^2$. Hence, it suffices to compare equilibrium profits of the producer from a monopoly of Trader α with the producer's profits from a duopoly:

$$\begin{aligned}\pi^D - \pi^M &= \frac{1}{12b}\left[(a - c - k_\alpha)^2 + (a - c - k_\beta)^2 + (k_\alpha - k_\beta)^2\right] - \frac{1}{8b}(a - c - k_\alpha)^2 \\ &= \frac{1}{24b}(a - c + k_\alpha - 2k_\beta)^2 \geq 0.\end{aligned}$$

Continuity of the producer's profit function guarantees that the duopoly profit of the producer will be equal to its profit from a monopoly of Trader α, when $x_\beta^D = 0$ holds. For $\pi^D = \pi^M$, i.e., $\alpha - c + k_\alpha - 2k_\beta = 0$, Trader α will be a monopolist in the downstream market,

$$x_\beta^D = \max\{0, \frac{1}{6b}(a - c + k_\alpha - 2k_\beta)\} = 0.$$

Such an exclusive trade arrangement will obtain if and only if $a - c + k_\alpha - 2k_\beta \leq 0$ holds. Hence, there will be non-exclusive trade in the downstream market if and only if $a - c + k_\alpha - 2k_\beta > 0$.

■

Proof of Proposition 4.3.

(i) Aggregate profit, $\Pi^D = \pi^D + \pi_\alpha^D + \pi_\beta^D$:

$$\begin{aligned}
\Pi^D &= \left[\frac{1}{12b}\left[(a-c-k_\alpha)^2 + (a-c-k_\beta)^2 + (k_\alpha - k_\beta)^2\right]\right] \\
&\qquad + \left[\frac{1}{36b}\left[(a-c-k_\alpha) - (k_\alpha - k_\beta)\right]^2\right] + \left[\frac{1}{36b}\left[(a-c-k_\beta) + (k_\alpha - k_\beta)\right]^2\right] \\
&= \frac{1}{36b}\left[4(a-c-k_\alpha)^2 + 4(a-c-k_\beta)^2 + 7(k_\alpha - k_\beta)^2\right].
\end{aligned}$$

(ii) Consumer surplus, $S^D = \frac{1}{2}\left[a - p^D\right] \cdot \left(x_\alpha^D + x_\beta^D\right)$:

$$\begin{aligned}
S^D &= \frac{1}{2}\left[a - \frac{1}{6}(4a + 2c + k_\alpha + k_\beta)\right] \cdot \left(\frac{1}{6b}(a - c + k_\beta - 2k_\alpha) + \frac{1}{6b}(a - c + k_\alpha - 2k_\beta)\right) \\
&= \frac{1}{72b}\left[2(a-c) - (k_\alpha + k_\beta)\right]^2.
\end{aligned}$$

(iii) Welfare, $W^D = \Pi^D + S^D$:

$$W^D = \frac{1}{72b}\left[20(a-c)^2 - 20(k_\alpha + k_\beta)(a-c) + 10\left((k_\alpha)^2 + (k_\beta)^2\right) + 13(k_\alpha - k_\beta)^2\right].$$

■

Proof of Proposition 4.4.

Part (i): *Consumer surplus*

$$S^D = \frac{1}{72b}\left[(a-c-k_\alpha) + (a-c-k_\beta)\right]^2 > \frac{1}{32b}(a-c-k_\alpha)^2 = S^M \iff a - c > 2k_\beta - k_\alpha.$$

Part (ii): *Aggregate profits*

$$\begin{aligned}
\Pi^D &= \pi^D + \pi_\alpha^D + \pi_\beta^D \\
&= \frac{1}{36b}\left[4(a-c-k_\alpha)^2 + 4(a-c-k_\beta)^2 + 7(k_\alpha - k_\beta)^2\right] > \frac{3}{16b}(a-c-k_\alpha)^2 = \Pi^M \\
&\iff 16(a-c-k_\beta)^2 + 28(k_\alpha - k_\beta)^2 > 11(a-c-k_\alpha)^2.
\end{aligned}$$

Part (iii): *Welfare*

$$\begin{aligned}
W^D &= \frac{1}{72b}\left[20(a-c)^2 - 20(k_\alpha + k_\beta)(a-c) + 10\left((k_\alpha)^2 + (k_\beta)^2\right) + 13(k_\alpha - k_\beta)\right] \\
&> \frac{7}{32b}(a-c-k_\alpha)^2 = W^M \\
&\iff 136(a-c)^2 + 368(a-c)k_\alpha + 320(k_\beta)^2 + 416(k_\alpha - k_\beta) > 640(a-c)k_\beta + 184(k_\alpha)^2.
\end{aligned}$$

■

Proof of Lemma 4.5. Denoting aggregate supply of n players in the downstream market by $X^n := \sum_{k=1}^{n} x_k = X^n_{-j} + x_j$, where $X^n_{-j} = \sum_{\substack{k=1 \\ j \neq k}}^{n} x_k$ is the supply of the other active traders, one can rewrite the best reply of trader j as

$$x_j = r_j(X^n - x_j) = \frac{a - bX^n + bx_j - q_j - k_j}{2b}$$

which can be solved to yield

$$x_j = R_j(X^n) := \max\left\{0, \frac{a - bX^n - q_j - k_j}{b}\right\}. \tag{A-1}$$

Summing over the n active traders one has

$$X^n := \sum_{j=1}^{n} x_j = \sum_{j=1}^{n} \frac{a - bX^n - q_j - k_j}{b},$$

which can be solved for X^n to yield the Nash equilibrium aggregate supply in the downstream market,

$$X^C(q) = \frac{1}{(n+1)b}\left(na - \sum_{j=1}^{n}(q_j + k_j)\right).$$

The associated equilibrium price

$$p^C(q) = a - bX^C(q) = \frac{1}{(n+1)}\left(a + \sum_{k=1}^{n}(q_k + k_k)\right),$$

individual traders' supply

$$x_j^C(q) = \max\left\{0, \frac{1}{(n+1)b}\left(a + \sum_{\substack{k=1 \\ k \neq j}}^{n}(q_k + k_k) - n(q_j + k_j)\right)\right\}$$

and profit

$$\pi_j^C(q) = \left[p^C(q) - q_j - k_j\right] x_j^C(q) = \frac{1}{(n+1)^2 b}\left(a + \sum_{\substack{k=1 \\ k \neq j}}^{n}(q_k + k_k) - n(q_j + k_j)\right)^2.$$

is easily derived by substituting into the respective equations. ■

Proof of Proposition 4.6. Differentiating with respect to the supply prices q_j, one obtains the n first-order conditions

$$\begin{aligned}\frac{\partial \pi(q)}{\partial q_j} &= x_j^C(q) + \sum_{k=1}^{n} (q_k - c) \frac{\partial x_k^C(q)}{\partial q_j} = 0 \\ \iff q_j &= \frac{1}{2n}\left[\left(a + c + \sum_{k=1}^{n} k_k - (n+1)k_j\right) + 2\sum_{\substack{k=1 \\ k\neq j}}^{n} q_k\right].\end{aligned}$$

These n linear equations can be solved for $q^C = \left(q_1^C, ..., q_n^C\right)$. Let $Q := \sum_{k=1}^{n} q_k$, then

$$q_j = \frac{1}{2n}\left[\left(a + c + \sum_{k=1}^{n} k_k - (n+1)k_j\right) + 2\sum_{k=1}^{n} q_k - 2q_j\right]$$

can be solved for q_j,

$$q_j = \frac{1}{2(n+1)}\left[\left(a + c + \sum_{k=1}^{n} k_k - (n+1)k_j\right) + 2Q\right]. \tag{A-2}$$

Summing over q_j

$$Q = \sum_{j=1}^{n} q_j = \frac{1}{2(n+1)}\sum_{j=1}^{n}\left(a + c + \sum_{k=1}^{n} k_k - (n+1)k_j\right) + \frac{n}{n+1}Q$$

and solving for Q, one obtains

$$Q^C = \frac{1}{2}\left(n(a+c) - \sum_{j=1}^{n} k_j\right).$$

Finally, substituting into Equation (A-2) yields the optimal supply price for Trader j,

$$q_j = \frac{1}{2(n+1)}\left[\left(a + c + \sum_{k=1}^{n} k_k - (n+1)k_j\right) + 2Q\right] = \frac{1}{2}(a + c - k_j).$$

Substituting $q_j^C = \frac{1}{2}(a + c - k_j)$ into the Nash equilibrium solution given in Lemma 4.5 allows us to compute the optimal allocation from the point of view of the producer:

(i) $X(q)$:

$$X^C = \frac{1}{(n+1)b}\left(na - \sum_{j=1}^{n}\left(\frac{1}{2}(a + c - k_j) + k_j\right)\right) = \frac{1}{2(n+1)b}\left(n(a-c) - \sum_{j=1}^{n} k_j\right).$$

(ii) $p(q)$:

$$p^C = \frac{1}{(n+1)}\left(a + \sum_{k=1}^{n}(q_k + k_k)\right) = \frac{1}{2(n+1)}\left((n+2)a + nc + \sum_{k=1}^{n} k_k\right).$$

(iii) $x_j(q)$:

$$\begin{aligned} x_j^C &= \max\left\{0, \frac{1}{(n+1)b}\left(a + \sum_{\substack{k=1\\k\neq j}}^{n}(q_k+k_k) - n(q_j+k_j)\right)\right\} \\ &= \max\left\{0, \frac{1}{2(n+1)b}\left(a - c + \sum_{j=1}^{n}k_j - (n+1)\,k_j\right)\right\}. \end{aligned}$$

(iv) $\pi_j(q)$:

$$\pi_j^C = \frac{1}{(n+1)^2 b}\left(a + \sum_{\substack{k=1\\k\neq j}}^{n}(q_k+k_k) - n(q_j+k_j)\right)^2 = \frac{1}{4(n+1)^2 b}\left(a - c + \sum_{j=1}^{n}k_j - (n+1)\,k_j\right)^2.$$

(v) $\pi(q)$:

$$\begin{aligned} \pi^C &= \sum_{j=1}^{n}(q_j - c)\,x_j^C(q) \\ &= \sum_{j=1}^{n}\left(\frac{1}{2}(a+c-k_j) - c\right)\frac{1}{2(n+1)b}\left(a - c + \sum_{j=1}^{n}k_j - (n+1)\,k_j\right) \\ &= \frac{1}{4(n+1)b}\left[\sum_{j=1}^{n}(a-c-k_j)^2 + n\sum_{j=1}^{n}k_j^2 - \left(\sum_{k=1}^{n}k_k\right)^2\right]. \end{aligned}$$

■

Proof of Proposition 4.7 In a Cournot equilibrium a trader j in the downstream market will not supply if and only if

$$\begin{aligned} x_j^C &= \max\left\{0, \frac{1}{2(n+1)b}\left(a - c + \sum_{k=1}^{n}k_k - (n+1)\,k_j\right)\right\} = 0 \\ &\iff \frac{1}{n+1}\left(a - c + \sum_{k=1}^{n}k_k\right) \leq k_j. \end{aligned}$$

Hence, if there is a trader m such that

$$k_{m+1} > k_m \geq \frac{1}{n+1}\left(a - c + \sum_{k=1}^{n}k_k\right) > k_{m-1}$$

holds, then all traders $j \geq m$ will not supply in the downstream market, while all traders $j < m$ will be trading the producer's product. ■

6 Acknowledgements

We would like to thank Monika Schnitzer and Markus Reisinger for valuable comments and Thomas Merkl for excellent research assistance.

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