Financial Transaction Tax and Financial Market Stability with Diverse Beliefs

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Abstract

This paper studies the impact of a financial transactions tax on the trading volume and asset price volatility in a model with heterogeneous beliefs. To model heterogeneous beliefs we follow Kurz (1994, 1997) and restrict the class of beliefs to the subset of rational beliefs. We study a tax on bond and asset purchases. The simulated model shows that the introduction of a transaction tax results in a lower trading volume and therefore in less liquid financial markets. Because of the decreased liquidity the volatility of the stock market increases. We also study the welfare effects of a financial transaction tax and the simulation results also show that there is only a small change in welfare.

JEL Classification Numbers: G12, G18
Keywords: Transaction Tax, Financial Regulation, Heterogeneous Beliefs

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1 Introduction

The recent banking and financial crisis has reignited the public debate about a financial transactions tax as proposed by Keynes (1936) for the stock market or Tobin (1978, 1996) for the foreign-exchange market. The proponents of a financial transaction tax claim that a financial transaction would reduce 'harmful' speculation, reduce the volatility of asset prices and it would generate considerable tax revenues for the government or supranational institutions. Whereas opponents of a transaction tax argue that it would increase the cost of capital, lead to illiquid markets, and distorts portfolio and capital structure decisions\(^1\).

Recently the European Commission proposed the introduction of a financial transaction tax in the European Union. This transaction tax is supposed to be levied on stocks, bonds and derivatives. One objective of this transaction tax is “creating appropriate disincentives for transactions that do not enhance the efficiency of financial markets thereby complementing regulatory measures to avoid future crises.”\(^2\), i.e. reducing speculative trades and improving financial market stability.

Opponents of a transaction tax argue that the high trading volume observable on financial markets does not cause price volatility and is in fact stabilizing (Friedman (1953)). They argue that the introduction of a transaction tax would lower liquidity and therefore trades do have a larger impact on prices which in turn implies that the volatility increases. Thus, from the point of view of opponents, a financial transaction tax would actually destabilize financial markets.

\(^1\)An overview of the debate is given by the surveys of Habermeier and Kirilenko (2003) and McCulloch and Pacillo (2011).

\(^2\)see the press release of the European Commission from 28/09/2011, available at http://europa.eu/rapid/press-release_IP-11-1085_en.htm. Another stated objective of the proposed tax is to generate tax revenue, although even proponents of a transaction tax such as Tobin argue that tax revenue should not be a key objective of such a tax.
In contrast to the view of the opponents, proponents of a financial transaction tax argue that “in the absence of any consensus on fundamentals the markets are dominated ... by traders in the game of guessing what other traders are going to think” (Tobin (1978) p. 158). Or, to put it in other words most traders on financial markets are not interested in long-term investing and do not buy assets by looking at the ‘fundamental value’ but participate in a Keynesian beauty contest and hope to make profits by correctly predicting the actions of other traders which in turn causes excessive volatility of prices.

There is little disagreement in the literature that heterogeneous beliefs increases the trading volume (see e.g. Harris and Raviv (1993) or Scheinkman and Xiong (2003)) and price volatility (see e.g. Harrison and Kreps (1978), Kurz et al. (2005a), Kurz and Motolese (2011), Basak (2005)). In models with heterogeneous beliefs, the true value of assets are unknown and traders act according to their beliefs while taking the beliefs of other traders into account. This resembles the argument made by proponents of a transaction tax on the causes of volatility. This might lead one to believe that the introduction of a transaction tax in an economy with heterogeneous beliefs reduces trading volume and price volatility, however reducing market liquidity might offset the reduced volatility from reduced speculative trade and actually increase the market volatility. Thus, ex-ante it is not clear whether a financial transaction tax increases or decreases price volatility.

In this paper we study the implications of a financial transaction tax on the stock and bond market for asset price volatility and welfare in an asset pricing model with heterogeneous beliefs. We restrict the beliefs to be rational beliefs in the sense of Kurz (1994, 1997) which requires the beliefs to be compatible with the empirical data. This rationality requirement is weaker than the one for rational expectations. Rational ex-
pectations requires the households to know the true probabilities in the economy while in an economy with rational beliefs households only know the empirical distribution. If the households also believe that the economy is not stationary, then households’ beliefs might not converge. Hence, households might disagree.\(^3\)

The simulation results show that a financial transaction tax has ambiguous effect on the volatility of asset prices and interest rates. In general, transaction taxes on different markets have the same qualitative results in the sense that the introduction of a tax on either market pushes the volatility of stocks into the same direction. However the taxes have different quantitative effects in the sense that the magnitude of the change is different. This quantitative differences stem from the different restriction on the agents’ behavior taxes on different markets have. In particular, a transaction tax on the stock market impedes the ability of the agents to buy or sell stocks. Whereas a transaction tax on the bond impedes the ability of the agents to finance asset purchases or to save money from selling stocks.

If the true probability distribution is unknown, as it is standard in models with heterogeneous beliefs, the standard approach fails to measure economic welfare as they would be measured under the subjective beliefs of the agents which are wrong. Under the rational beliefs principle however, the agents agree upon the empirical distribution of prices and dividends and it has been proposed by Nielsen (2004) to use the empirical distribution to measure welfare. We follow his approach and find that the introduction of a transaction tax can have positive welfare effects. Under rational beliefs an agent will make large losses if he bets in the wrong direction. A transaction tax prevents the agent to make such costly mistakes and reduces his losses and thus increases welfare.

\(^3\)The argument that non-stationarity leads to possible disagreements among agents has also been made by Lehrer and Smorodinsky (1996).
There have been several studies on the impact of transaction taxes and transaction costs on stock markets. In a static model Hara (2012) shows that transaction costs have the adverse effect of an increasing trading volume whereas Subrahmanyan (1996) studies the impact of transaction taxes in a model with asymmetric information and argues that with competitive traders the trading volume declines.

Our model differs from noise trader models such as Song and Zhang (2005), Xu (2010) or Lendvai et al. (2013). In models with noise traders a subset of trader can have arbitrarily beliefs, whereas in our model the rational beliefs principle restricts the beliefs to a subset of beliefs. Buss et al. (2013) study how various regulatory measures affect stock market volatility. However, in their paper the difference of opinion stems from the fact that the agents in the economy overrate the value of a public signal whereas in our paper the source of heterogeneity is the restricted learnability of the underlying stochastic process.

The rest of this paper is structured as follows. In section 2 we discuss the model and the transaction tax, section 3 defines rational beliefs and explains the simulation model. Section 4 discusses the impact of the financial transaction tax on market volatility and in Section 5 studies the impact of beliefs on the efficacy of transaction taxes. In section 6 we discuss the welfare effects of financial transaction taxes and section 7 concludes the paper.
2 The Model

There are two representative households in the economy. Given his belief $Q_t^j$, a household selects portfolios and consumption plans to solve the problem

$$\max_{c^j_t, \theta^j_t, \theta^0_{t,t}} E_Q^j \left[ \sum_{s=t}^{\infty} \beta^{s-t} \frac{1}{1 - \gamma} (c^j_s)^{1-\gamma} | H_t \right]$$

(1)

With $\beta$ as the subjective discount factor of the household, $c^j_t$ as the consumption of household $j \in \{1, 2\}$ at time $t$ and $H_t$ as the history of prices and dividends at time $t$. The holdings of agent $j$ in the risky asset is denoted by $\theta^j_t$ and the one in the risk-free bond is denoted by $\theta^0_{j,t}$. The risky-asset has a net-supply of 1, while the bond is in zero net-supply.

Furthermore, as in Burnside (1998) we assume that the empirical process of the log-dividends follows an AR(1)-process of the form:

$$x_{t+1} = (1 - \lambda)x^* + \lambda_x x_t + \rho_{t+1}^x, \quad \rho_{t+1}^x \sim \mathcal{N}(0, \sigma_x)$$

(2)

and $d_t = e^{x_t}$. It should be kept in mind that equation (2) is what the agents in the economy observe empirically. Under the rational beliefs principle the observed process does not have to be the true process.

In this model trading on financial markets is subject to a transaction tax. The transaction tax is levied on the value traded in a market in period $t$. Following Heaton and Lucas (1996) and Subrahmanyan (1996) we assume that the tax-base of the transaction-
tax is quadratic. In particular, we have

\[ {\text{tax}}^S_t = \frac{\tau^S}{2} (q_t \theta^j_t - q_{t-1} \theta^j_{t-1})^2, \]
\[ {\text{tax}}^B_t = \frac{\tau^B}{2} (q^b_t \theta^j_{0,t})^2, \]

(3) (4)

with \( \tau^S \) and \( \tau^B \) representing the tax-level and \( q_t \) the price of the risky asset and \( q^b_t \) the price of the bond. The quadratic form of the tax-base stems from several reasons. First, it is to avoid subsidy on selling stocks or bonds. Second, a quadratic transaction tax also captures the effect that with higher taxes it will be more difficult to find a trading partner for large blocks of shares and thus the trader incurs some extra costs in finding a trading partner.

One should note that there is a difference between the tax on asset purchases and the tax on bond purchases. The tax on the stock market is a transaction tax on the secondary market, because the asset is long-lived and the sale and purchase of the stock on the secondary market is taxed. On the other hand, the transaction tax on the bond market is a tax on the primary market.

Matheson (2011) reports that transaction taxes on bond market are usually levied on the primary market and not on the secondary market. However, the newly proposed transaction tax by the European Commission is a tax on the secondary market for bonds as well. A transaction tax for bonds on the secondary market would take the following form:

\[ {\text{tax}}^B_t = \frac{\tau^B}{2} (q^b_t \theta^j_{0,t} - \theta^j_{0,t-1})^2 \]

(5)

As most transaction taxes on bonds are on the primary market and not on the secondary
market, we will use (4) and not (5) for our simulation study.

Additionally we assume that the revenues from the transaction tax are not redistributed among the households, i.e. the administrative costs to collect taxes eat up the whole income from taxation. Given the transaction tax, the budget constraint of the household is now given by

\[ c_j^t + \theta_j^t q_t + \theta_{0,t}^j q_t + tax^S_t + tax^B_t = e_j^t + \theta_{t-1}^j (v_t + q_t) + \theta_{0,t-1}^j. \]  

(6)

with \( e_j^t \) as the households’ income in period \( t \).

The euler equations for stock and bondholdings are therefore:

\[
(c_t^j)^{-\gamma} (q_t + \tau^S (q_t)^2 (\theta_t^j - \theta_{t-1}^j)) = \beta \mathbb{E}_{Q_t} \left[ (c_{t+1}^j)^{-\gamma} (q_{t+1} + d_{t+1} + \tau^S (q_{t+1})^2 (\theta_{t+1}^j - \theta_t^j)) \mid H_t \right],
\]

\[
(c_t^j)^{-\gamma} (q_t + \tau^B (q_t)^2 (\theta_{t,0}^j)) = \beta \mathbb{E}_{Q_t} \left[ (c_{t+1}^j)^{-\gamma} \mid H_t \right].
\]

The bond and the stock-market are cleared for all \( t = 1, 2, 3... \) with the following equilibrium conditions:

\[
\theta_1^t + \theta_2^t = 1, \tag{7}
\]

\[
\theta_{0,t}^1 + \theta_{0,t}^2 = 0. \tag{8}
\]

3 Rational Beliefs

We are now turning to the discussion of the structure of beliefs. Instead of modelling the learning process we follow Kurz et al. (2005a,b) and assume that the beliefs are part of the economic primitives. Although this assumption seems to be inferior compared to models with learning it has been argued by Jouini and Napp (2007) that in models with
learning the beliefs are not ‘more endogeneous’ because of the separability of learning and the optimization problem of the agent. Thus assuming that beliefs are exogeneous is not as restrictive as it might seem at first glance.

3.1 Individual Beliefs and the Market State of Belief

To study the rational beliefs we will introduce three different types of beliefs:

1. $g_j^t$ denotes the state of belief of $j$ as known only by the agent.

2. $z_t = (z_1^t, z_2^t)$ denotes the market state of belief. While the market state of belief is publicly observable, the model consistency condition is not recognized by agent $j$.

3. $z_{t+1}^j = (z_{t+1}^1, z_{t+1}^2)$ is the forecast of agent $j$ of the market state of belief at the future date $t + 1$.

Let the usual state space of agent $j$ consist of endogeneous and exogeneous variables be $s_j^t$. Here we extend the state-space by adding an additional state-variable, called the agent $j$ state of belief generated by agent $j$. It represents his date $t$ subjective view of date $t + 1$ and is denoted by $g_j^t \in G^j$. With this variable we can express the conditional probably as $P(s_{t+1}^j, g_{t+1}^j | s_t^j, g_t^j)$. Furthermore, $g_j^t$ is privately perceived by agent $j$. We assume that $g_t^j$ follows a process of the form

$$g_{t+1}^j = \lambda_z g_t^j + \lambda_x (x_t - x^*) + \tilde{\rho}_{t+1}^j, \tilde{\rho}_{t+1}^j \sim N(0, \tilde{\sigma}_{g}^2) \tag{9}$$

The variable $g_t^j$ can be interpreted as an assessment variable as in Kurz and Schneider (1996) and it can be shown that the assessment variable fully pins down the conditional
beliefs. For example, an agent could believe that the empirical distribution is the true
distribution. In this case the variable $g^j_t$ has to be constant. This implies not only $\tilde{\rho}^g_j$
has zero variance but also that $\lambda^g_j, \lambda_g$ and $g^j_t$ are all zero. The random variable $\tilde{\rho}^g_t$ can
be correlated across agents which reflects some communication among agents (see e.g. Nakata (2007)).

In equilibrium, asset prices depend on the distribution of beliefs. Similar to the
assumption that a competitive firm cannot affect prices, we assume that agents cannot
affect endogeneous variables, i.e. they take prices and their beliefs as given.

We are now discussing how agents in the model forecast prices. First, we define
the ’market state of belief’ as a vector $z_t = (z^1_t, z^2_t)$. The model consistency condition
$z_t = g_t$ is not recognized by the agents. With the prices in the economy depending now
on the market state of belief we add additional uncertainty to the economy. If the usual
state-variables are denoted by $s_t$, we can define a price-map as follows:

$$
\begin{bmatrix}
q^i_t \\
q^b_t
\end{bmatrix} = \Phi(s_t, z^1_t, z^2_t). 
$$

(10)

However, with prices depending on the market-state of belief we get another impli-
cation of the extended market state of belief. To forecast the prices in the economy,
households also have to forecast the market state of belief in the economy. And al-
though all households use (10) to forecast prices, forecasts will be different because the
forecast of $(s_{t+1}, z^1_{t+1}, z^2_{t+1})$ will depend on his own state of belief $g^j_t$.

Because of the observability of $z_t$ we augment the empirical distribution of the div-
idends with the market states of belief. We assume that the empirical distribution is an
AR-process of the form

\[ x_{t+1} = (1 - \lambda_x)x^* + \lambda_xx_t + \rho_{t+1}^x \]  \hspace{1cm} (11)

\[ z_{t+1}^1 = \lambda_z z_t^1 + \lambda_z^1 (x_t - x^*) + \rho_{t+1}^z \]  \hspace{1cm} (12)

\[ z_{t+1}^2 = \lambda_z z_t^2 + \lambda_z^2 (x_t - x^*) + \rho_{t+1}^z \]  \hspace{1cm} (13)

\[
\begin{pmatrix}
\rho_{t+1}^x \\
\rho_{t+1}^z \\
\rho_{t+1}^z
\end{pmatrix}
\sim
\mathcal{N}
\begin{pmatrix}
0, & \begin{bmatrix}
\sigma_x^2, & 0, & 0 \\
0, & 1, & \sigma_{z_1z_2} \\
0, & \sigma_{z_1z_2}, & 1
\end{bmatrix}
= \Sigma
\end{pmatrix}, \text{ i.i.d}
\]  \hspace{1cm} (14)

We rewrite (11), (12) and (13) in a more compact notation, i.e. let \( w_t = (x_t - x^*, z_t^1, z_t^2) \), \( \rho_t = (\rho_t^x, \rho_t^z, \rho_t^z) \) and denote by \( A \) the \( 3 \times 3 \) matrix of (11), (12) and (13). Thus, we have

\[ w_{t+1} = Aw_t + \rho_{t+1} , \rho_{t+1} \sim \mathcal{N}(0, \Sigma). \]  \hspace{1cm} (15)

The unconditional covariance of \( w \) is denoted by \( V = E(ww') \) and it is computed as the solution of the equation

\[ V = AVA' + \Sigma. \]  \hspace{1cm} (16)

### 3.2 General Structure of Beliefs

We first define the perception model of an agent. The transition functions representing the households’ conditional probability belief is the perception model. Thus, to determine the households forecasts the perception model is used. The households’ beliefs can deviate from the empirical distribution which can be interpreted as overconfidence.
of the households.

We denote the date $t+1$ variables as perceived by agent $j$ as $w_{t+1}^j = (x_{t+1}^j, z_{t+1}^{1j}, z_{t+1}^{2j})$ and $\Gamma$ is the stationary measure implied by (11)-(13). We also have $\Psi_{t+1}(g_{t+1}^j)$, a three-dimensional vector of date $t+1$ random variables conditional upon $g_{t+1}^h$.

**Definition 1.** A perception model in the economy under study has the general form

$$w_{t+1}^j = Aw_t + \Psi_{t+1}(g_t^j), \quad (17)$$

together with (11)-(13). Since $E_{\Gamma}[w_{t+1}|H_t] = Aw_t$, we can write (17) as follows

$$w_{t+1}^j - E_{\Gamma}[w_{t+1}|H_t] = \Psi_{t+1}(g_t^j). \quad (18)$$

There is documented evidence in the psychological literature that people exhibit overconfidence (see e.g. Larwood and Whittaker (1977), Svenson (1981), or Alicke (1985)). In our model households can be overconfident in the sense that their subjective beliefs deviate from the empirical probabilities. This in contrast to overconfidence models such as Daniel et al. (2001) which assume agents believe a public signal contains more information than it actually does. $\Psi(g_t^j)$ is modelled by using a random variable $\eta_{t+1}^j(g_t^j)$:

$$\Psi_{t+1}(g_t^j) = \begin{pmatrix} \lambda_g^x \eta_{t+1}^j(g_t^j) + \tilde{\rho}_{t+1}^j \\ \lambda_g^1 \eta_{t+1}^j(g_t^j) + \tilde{\rho}_{t+1}^{z1} \\ \lambda_g^2 \eta_{t+1}^j(g_t^j) + \tilde{\rho}_{t+1}^{z12} \end{pmatrix}, \tilde{\rho}_{t+1} \sim N(0, \Omega_{\rho \rho}). \quad (19)$$
We can now express the perception model of agent $j$ as follows:

$$x^j_{t+1} = (1 - \lambda_x)x^* + \lambda_x x_t + \lambda_x^j \eta^j_{t+1}(g^j_t) + \tilde{\rho}^j_{t+1}, \quad (20)$$

$$z^1_{t+1} = \lambda_x z^1_t + \lambda_x^j (x_t - x^*) + \lambda_x^{z1} \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{z1}_{t+1}, \quad (21)$$

$$z^2_{t+1} = \lambda_x z^2_t + \lambda_x^j (x_t - x^*) + \lambda_x^{z2} \eta^j_{t+1}(g^j_t) + \tilde{\rho}^{z2}_{t+1}, \quad (22)$$

$$g^j_{t+1} = \lambda_x g^j_t + \lambda_x^j (x_t - x^*) + \tilde{\rho}^j_{t+1}. \quad (23)$$

And $\tilde{\rho}^j_{t+1} = (\tilde{\rho}^{x}_{t+1}, \tilde{\rho}^{z1}_{t+1}, \tilde{\rho}^{z2}_{t+1}, \tilde{\rho}^j_{t+1})$ is i.i.d. Normal with mean zero. Let $\Omega_{wgj} = (\text{cov}(\tilde{\rho}^{x}_{t+1}, \tilde{\rho}^j_{t+1}), \text{cov}(\tilde{\rho}^{z1}_{t+1}, \tilde{\rho}^j_{t+1}), \text{cov}(\tilde{\rho}^{z2}_{t+1}, \tilde{\rho}^j_{t+1}))$, then the covariance matrix $\Omega^j$ is given by

$$\Omega^j = \begin{pmatrix} \Omega^j_{pp}, & \Omega^j_{wpgj}^\top \\ \Omega^j_{wgp}, & \sigma^2_{g^j} \end{pmatrix}. \quad (24)$$

We are now turning to the description of the random variable $\eta^j_{t+1}(g^j_t)$. We define $\eta^j_{t+1}(g^j_t)$ as follows:

$$p(\eta^j_{t+1}|g^j_t) = \begin{cases} 
\phi_1(g^j_t) f(\eta^j_{t+1}) & \text{if } \eta^j_{t+1} \geq a \\
\phi_2(g^j_t) f(\eta^j_{t+1}) & \text{if } \eta^j_{t+1} < a 
\end{cases}, \quad (25)$$

with $\eta^j_{t+1}$ and $\tilde{\rho}^j_{t+1}$ are independent and $f(\eta) = [1/\sqrt{2\pi}] e^{-\eta^2/2}$. The functions $(\phi_1, \phi_2)$ are then defined as follows:

$$\phi(g^j) = \frac{1}{1 + e^{b(g^j-a)}}, \text{ and define } G \equiv E_g \phi(g^j), \quad (26)$$

$$\phi_1(g^j) = \frac{\phi(g^j)}{G}, \phi_2(g^j) = 2 - \phi_1(g^j). \quad (27)$$
The parameter $b$ measures the intensity of fat tails. Fat tails in the empirical distribution of returns have been attributed to resolve the equity premium puzzle (Rietz (1988), Barro (2006)) and have also been documented empirically (see e.g. Fama (1963) or Fama (1965)). While in models with rare events fat tails of the return distribution are exogenously given they arise endogenously in our model because of the beliefs of the agents. Furthermore, the parameter $a$ denotes the asymmetry of the distribution of beliefs, i.e. if $a = 0$ the beliefs are symmetric and if $a \neq 0$ the beliefs are asymmetric.

Given our description above, we can define bull and bear states as follows

**Definition 2.** Let $Q_j$ be the probability belief of agent $j$. Then $q_j^t$ is said to be

- **a bear state** for agent $j$ if $E_{Q_j}[x_{t+1}^j|g_j^t, H_t] < E(\Gamma(x_{t+1}|H_t));$

- **a bull state** for agent $j$ if $E_{Q_j}[x_{t+1}^j|g_j^t, H_t] > E(\Gamma(x_{t+1}|H_t)).$

### 3.3 Restriction of Beliefs

First, we define the implications of Rational Beliefs in the context of our simulation model:

**Definition 3.** A perception model as defined in (18) is a Rational Belief if the agent’s model $w_{t+1} = Aw_t + \Psi_t + \Gamma_t(g_t^t)$ has the same empirical distribution as $w_{t+1} = Aw_t + \Psi_t + \Gamma_t.$

The interpretation of this definition is straightforward, i.e. although the conditional expectations of an agent might be different from the empirical distribution, in the long run his beliefs cannot be arbitrary. Thus, the rational beliefs principle implies that the parameters determining the beliefs have to be restricted. The following Theorem due to Kurz et al. (2005a,b) gives us these restrictions on the beliefs:
Theorem 1. Let the beliefs of an agent be a Rational Belief. Then the belief is restricted as follows:

(i) For any vector of parameters \((\lambda_x, \lambda_y, b)\) the Variance-Covariance matrix \(\Omega^j\) is fully defined and not subject to choice.

(ii) The condition that \(\Omega^j\) is a positive definite matrix establishes a feasibility region for the vector \((\lambda_x, \lambda_y, b)\). In particular, it requires \(|\lambda_x| \leq \sigma_x, |\lambda_y| \leq 1\).

(iii) \(\Psi_{t+1}(g^j_t)\) cannot exhibit serial correlation and this restriction pins down the vector

\[
\Omega_{xg^j} = [\text{cov}(\hat{\rho}_t^{x}, \hat{\rho}_t^{g^j}), \text{cov}(\hat{\rho}_t^{z_1}, \hat{\rho}_t^{g^j}), \text{cov}(\hat{\rho}_t^{z_2}, \hat{\rho}_t^{g^j})]
\]

4 Numerical Results

4.1 Calibration

To set the parameters for the simulation we follow Kurz et al. (2005a) and set the parameters of the perception model as close as possible to maximum value as implied by the rationality conditions. For the dividend process, we set \(x^* = 0.01773\) and \(\lambda_x = -0.117\) and \(\sigma_x = 0.03256\). This calibration of the empirical dividend process is consistent with the one used by Mehra and Prescott (1985). For the preferences of the households we set \(\beta = 0.96\) and \(\gamma = 2\). Furthermore, households’ income \(e^j_t\) will be constant to 3. Setting the income constant means that households trade only because of the differences in beliefs but not because of risk-sharing.

In the non-stochastic steady-state the portfolio holdings of the agents are indetermi-
Table 1: The parameters of the economic fundamentals and the beliefs of the agents.

<table>
<thead>
<tr>
<th>Beliefs</th>
<th>$\lambda^x_0$</th>
<th>$\lambda^z_0$</th>
<th>$\lambda^z_a$</th>
<th>$b$</th>
<th>$a$</th>
<th>$\sigma^{z_1 z_2}$</th>
<th>$\lambda^z_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy I</td>
<td>-0.027</td>
<td>0.200</td>
<td>0.900</td>
<td>-6</td>
<td>-0.25</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>Economy II</td>
<td>0.027</td>
<td>-0.200</td>
<td>0.900</td>
<td>-6</td>
<td>-0.25</td>
<td>0.9</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Because of Definition 2 the difference between the two economies is the frequency of bull and bear states. Bear states are more frequent in Economy I, whereas in Economy II bull states are more frequent.
For the simulation study the tax on bonds and stocks are in the range of 0% and 1% and taxes are increased by 5 basis points. This range of taxes is in line with most existing transaction taxes. Furthermore, the transaction tax on stocks or bonds are usually different.

An important aspect of our study is the trading volume in the economy and we will use the trading volume as a proxy for liquidity on the market, i.e. a lower trading volume implies a less liquid market.\(^4\) The trading volume in the stock and bond market is defined as follows:\(^5\):

\[
V = \frac{1}{T} \sum_{t=1}^{T} |\theta^j_t - \theta^j_{t-1}|, \quad (28)
\]

\[
V^b = \frac{1}{T} \sum_{t=1}^{T} |\theta^j_{0,t}|. \quad (29)
\]

The economy is approximated using a second order perturbation using the software pertsolve by Jin (2003).

### 4.2 Transaction Tax on Stocks

First, we are studying the impact of a financial transaction tax on the trading volume in the stock market. The main results are as follows:

**Summary of Results 1.** *The introduction of a transaction tax reduces the trading volume in stocks and bonds, whereas for the volatility of the Price/Dividend-ratio and interest rates we have the following:*

\(^4\)Although the trading volume is only a crude measure for liquidity other liquidity measures based on the market microstructure literature imply a positive relation ship between the trading volume and the liquidity of financial markets. Furthermore, empirical studies on the impact of transaction taxes typically focus on the trading volume.

\(^5\)see Lo and Wang (2010) for various definitions of trading volumes
Figure 1: Effects of a Financial Transaction Tax on the Stock Market on Trading Volume and Volatility

- The volatility of the Price/Dividend-ratio increases in Economy I and is nearly unaffected in Economy II;

- The volatility of the interest rates decreases in Economy I and increases in Economy II.

In figure 1(a) we see that in both economies, the trading volume on the stock market decreases with the transaction tax. However, the difference between these two economies is how much a transaction tax results in a decline of the trading volume. For Economy I, the trading volume reduces by about 60% with a tax of 1%. On the other hand in Economy II trading volume decreases only by about 40% with a 1% transaction tax.
The impact of the financial transaction tax on the bond market is shown in figure 1(b). Again, the trading volume in the stock and the bond market declines for both economies. Also, the reduction in trading volume is larger for Economy I than for Economy II. The larger reduction in the trading volume for Economy I comes from the fact that the agents in this model are more often pessimistic, i.e. they expect that the returns on holding the risky asset are lower than the empirical returns. Define the returns on buying the asset in period $t$ and receiving dividends and selling it in period $t + 1$ as $r_t = \frac{v_{t+1} + q_{t+1} - (q_t - 2c)}{q_t + c}$ and let $c$ be the transaction costs the agents face for trading. For the sake of the argument keep the price $q_t$ fixed and the differences in the optimistic and pessimistic economy is the expected dividend $v_{t+1}$. Now we see that in the pessimistic economy increasing the transaction tax has a larger effect on the expected returns than in the optimistic economy. Hence, the trading volume in Economy I drops more than in Economy II.

Thus, a transaction tax reduces the trading volume in both markets. Which in turn implies that there is no substitution across assets. It is also different to the argument by Heaton and Lucas (1996) who argue that transaction costs on one market affects the trading on the other market as agents shift their attention to the market without taxes.\footnote{A similar point has been made by Levine and Zame (2002) who argue that for the case of CRRA-utility it is possible to achieve the pareto-optimal allocation in an economy with only a risk-free asset.}

The volatility of the Price-Dividend Ratio is shown in figure 1(c). And one can see that there are now significant differences between the two economies. In Economy I, the volatility increases whereas in Economy II the volatility decreases. Furthermore, the size of the effect is also different, as the volatility in Economy I increases by more than 0.2% with a transaction tax of 1%. Whereas the volatility in Economy II is nearly unaffected.
The change in volatility in this model has two sources. The first source is the ‘speculative’ effect, i.e. trading volume and volatility are higher because of differences in beliefs and a transaction tax would reduce volatility. The second effect is the liquidity effect, i.e. a reduction in trading volume increases volatility and a transaction tax that reduces trading volume would increase volatility. In a model with heterogeneous beliefs this two effects affect the volatility in two different directions. In Economy I, the liquidity effect dominates and thus the volatility increases whereas in Economy II both effects are of the same strength and thus cancel each other out and the volatility is unaffected.

This result is also in contrast to the result by Heaton and Lucas (1996) who had no significant effect on the price volatility with transaction costs only on the stock market because with a tax on only one market the households will almost exclusively trade in one market. However, in our model households withdraw from trading in both markets.

On the other hand, figure 1(d) shows that in Economy II the volatility of the interest increases with a transaction tax whereas the volatility of the interest rate decreases with the transaction tax. Furthermore, the effect of the transaction tax on the bond market is also much stronger than on the stock market. In particular, the volatility declines by about 10% in Economy I and increases by about 15% in Economy II.

Volatility on the bond market is also affected by the speculation and the liquidity effect. In contrast to the stock market the effects are reversed, i.e. in Economy I the introduction of a transaction tax reduces interest rate volatility and in Economy II the volatility of the interest rate increases.

The decrease in trading volume and the simultaneous increase in volatility in Economy I is qualitative similar to empirical results. Empirical estimations of the elasticity of trading volume with transaction costs on stock markets varies from 0 (Hu (1998)) to −1.7 (Lindgren and Westlund (1990)). With an elasticity of −1 meaning that if trans-
action costs increase by 50% the trading volume has to decrease by 50% as well. Thus, our model produces a reduction in trading volume which is consistent with the empirical evidence. Furthermore, the observed change in the volatility is within the empirically observed range. While Hu (1998) finds that transaction costs do not affect volatility Hau (2006) observes that increasing the tick size on the French stock market increases transaction costs by 20% and volatility by 30%.

4.3 Transaction Tax on Bonds

We are now turning to the discussion of the impact of a financial transaction tax on the bond market and we have the following results:

**Summary of Results 2.** A transaction tax on the bond market results in a decreasing trading volume, whereas for the volatility of the price/dividend-ratio and the interest rates we have:

- The volatility of the price/dividend-ratio increases in Economy I and decreases in Economy II;
- The volatility of the interest-rate decreases in Economy I and increases in Economy I.

As opposed to the transaction tax on the equity market, a transaction tax on the bond market has the same quantitative effect on the trading volume in Economy I and Economy II. In particular, the trading volume on the stock and bond market decreases with the transaction tax (figures 2(a) and 2(b)). Additionally, a transaction tax on the bond market affects the trading volume in both economies equally, i.e. in both economies and both markets the trading volume is reduced by about 60% with a 1% transaction tax.
We also see in figures 2(c) and 2(d) that the introduction of a transaction tax on the bond market has qualitative similar effects as a transaction tax on the stock market. The differences lies in the quantitative effects on the volatility. In particular the change in the volatility of the Price/Dividend-Ratio and the interest rates is larger with the transaction tax on the bond market. For example in Economy I, the volatility of the Price/Dividend-Ratio increases by about 0.15% with a 1% transaction tax on the stock market whereas it increases about 0.3% with a 1% transaction tax on the bond market.

These differences stem from the fact that agents decisions are restricted differently by different taxes. A transaction tax on the bond market affects the ability of the agents to borrow money to purchase the stock by changing the post-tax interest rate of the households. A household that wants to save money faces now a lower interest rate
whereas a household that wants to borrow money faces a higher interest rate. Thus the higher the tax the higher the spread between the post-tax interest rates of borrowers and lenders.

Consider for example that the economy is hit by a positive shock, i.e. agents get more optimistic. In the Economy I, the economy in which households are bearish more than 50% of the time, the intensity of optimism is higher than in Economy II. Now, with a transaction tax on the bond market the expected price of the risky asset has to rise even stronger to satisfy the Euler-equations. On the other hand the impact of a negative shock is less pronounced, because the distribution of beliefs in Economy II is less skewed than the distribution in Economy I. Hence, the volatility increases in Economy I and decreases in Economy II.

4.4 Simultaneous Transaction Tax on Stocks and Bonds

So far, the discussion was only about the cases in which a transaction tax was on one market. Now, we are turning to the case in which a transaction tax is on both markets, i.e. the stock and the bond market. The key results of this exercise are as follows:

**Summary of Results 3.** The introduction of a transaction tax on the stock market and the bond market has the following effects:

1. A transaction tax on any market always reduces the trading volume.

2. In Economy I a transaction tax on the stock market does not reduce the volatility in the economy.

3. In Economy II a transaction tax on the bond market always reduces the volatility of the P/D-Ratio whereas a transaction tax on the stock market only reduces
the volatility of the P/D-Ratio if there is a transaction tax on the bond market.

Furthermore, the volatility of the interest does not always decrease.

Figure 3(a)) shows the trading volume on the stock market in Economy I. While we still have a small decrease and an increase later on in the stock market, if we increase the transaction tax on equities, the increase becomes smaller if the transaction tax on bonds is increased. In addition, the trading volume on the bond market decreases with an increase in the transaction tax on the bond market, regardless of the transaction tax on the stock market. Furthermore, the trading volume increases with the transaction tax on
We now look at the volatility on the stock and the bond market. In Economy I, the volatility of the stock market increases with a transaction tax on the stock or bond market (figure 4(a)) whereas in Economy II the volatility of the P/D-Ratio is nearly unaffected by a transaction tax on the stock market, regardless of the transaction tax on the bond market.

While the volatility of the interest rate in Economy I declines with the transaction tax on bonds, the volatility of the P/D-Ratio in Economy II is nearly unaffected by the transaction tax on the bond market (see figures 4(c)).
tax on the stock or bond market (figure 4(b)) this is not case for Economy II. As long as there is no transaction tax on the stock market, the volatility of the interest rate increases with the tax on the bond market. If, on the other hand the tax on the stock market is 1% then the volatility of the interest rate declines with the tax on the bond market (see figure 4(d))

5 Comparative Statics: Beliefs and the Effects of Transaction Taxes

Thus far, we have seen that the introduction of a transaction tax has ambiguous effects on market volatility, i.e. the volatility can go up or down depending whether the liquidity effect or the speculative effect dominates. Thus we are now investigating how the volatility depends on the parameters for the beliefs.

5.1 Transaction Tax on the Stock Market

In figures 5 and 6 the impact of the different belief parameters on the volatility of the Price/Dividend-Ratio are shown. The key result of this exercise are as follows:

Summary of Results 4. A change in the beliefs parameters \((a, b, \lambda_g^z, \lambda_g^x)\) has the following effects on the volatility of the P/D-ratio:

1. In Economy I the volatility of the P/D ratio changes monotonically with the parameters \((a, \lambda_g^z, \lambda_g^x)\) and non-monotonically with the parameter \(b\).

2. in Economy II no parameter has a monotonic effect on the volatility of the P/D-ratio.
In figures 5(a) and 6(a) we graph the volatility of the P/D-ratio under different parameters for \( a \). For Economy I we can see a monotonic pattern, i.e. the change in the volatility is largest for \( a = -0.05 \) whereas it is smallest for \( a = -0.4 \). While for Economy II, the increase in volatility is the largest when \( a = -0.05 \) Thus for an economy with a more asymmetric belief distribution than the baseline economy we see that the change in volatility is smaller than the change in the volatility of the baseline economy. For an economy with a distribution of beliefs which is close to a symmetric distribution the liquidity effect of the transaction tax becomes stronger. On the other hand, for an economy with a very asymmetric distribution it cannot be concluded that the speculative effect of a transaction tax becomes more dominant.

The volatility of the P/D-ration under different parameters for \( b \) is shown in figures
In both economies a change in the intensity of the fat tails has a non-monotonic effect on the volatility. A very low $b$ means that the households’ beliefs have large fat tails thus gives the households more incentive to speculate on a favorable outcome. In an economy where bullish expectations are rare, the households could gain a lot from the dividends. Whereas in an economy in which bearish expectations are rare households will have an incentive to sell the stock to not get hit by a crash. As the net-supply of the stock is positive the equilibrium price has to adjust. In particular it has to become smaller so that the agents will hold the asset, hence the volatility increases.

From equations (20)-(22) it can be seen that the parameters $(\lambda_g^x, \lambda_g^z)$ also measure the intensity of the distribution, hence there is some substitution between the parameter $b$ and $(\lambda_g^x, \lambda_g^z)$. Which in turn implies a similar interpretation of the effects of a change
of $\lambda^z_2$ on the volatility.

![Graphs showing changes in parameters and P/D-Ratio](image)

(a) Changes in $a$ and the P/D-Ratio  
(b) Changes in $b$ and the P/D-Ratio  
(c) Changes in $\lambda^z_2$ and the P/D-Ratio  
(d) Changes in $\lambda^z_4$ and the P/D-Ratio

Figure 7: Price-Dividend Ratio under different parameters for beliefs in Economy I

5.2 Transaction Tax on the Bond Market

We are now studying the relationship between a financial transaction tax on the bond market and beliefs. The graphs for Economy I and Economy II are shown in figures 7 and 8. We have the following the key results

**Summary of Results 5.** *For the changes in the beliefs parameters $(a, b, \lambda^z_2, \lambda^z_4)$ the volatility of the P/D ratio changes monotonically in Economy I and Economy II with a change in $(a, \lambda^z_2, \lambda^z_4)$ and non-monotonically with $b$.*

From figures 7(a),(c), (d) and 8(a),(c), (d) it can be seen that the effects of a change in parameters are antisymmetric $(a)$ or symmetric $(\lambda^z_2, \lambda^z_4)$. For the parameter $a$ we
Figure 8: Price-Dividend Ratio under different parameters for beliefs in Economy II

see that in Economy I the change in volatility declines and in Economy II the change in volatility increases. Hence implying that with a large asymmetry in the distribution of beliefs a transaction tax has less effects on the volatility of the P/D-ratio. Similar conclusion can be drawn for the parameters \((\lambda^x, \lambda^z)\), i.e. with a less persistent personal state of belief (measured by \(\lambda^x\)) or forecast about the market state of belief (measured by \(\lambda^z\)) the less is the P/D-ratio affected by a transaction tax on the bond market.

The only exception is the parameter \(b\), although there is some substitutability between \(b\) and \((\lambda^x, \lambda^z)\), the changes are not monotonic. In particular large fat tails gives the households an incentive to trade because of the gains that can be made from those trades. As only the access to the bond market is restricted but not the access to the stock market, a household in Economy I can still buy the stock if he foregoes consumption to
buy the stock. Which in turn means that the transaction tax on the bond market has less impact on equilibrium prices in the presence of fat tails.

6 Welfare Effects of a Financial Transaction Tax

So far, we have only discussed the impact of a financial transaction tax on the volatility in the economy but left out whether the introduction of a financial transaction tax improves the welfare or not.

6.1 Measuring the Economic Welfare under Heterogeneous Beliefs

To measure the impact of a financial transaction tax on welfare we use an ex-post welfare measure, because under heterogeneous beliefs households do not know the true probability distribution in the economy and hence do hold incorrect beliefs. This rules out the standard criterion, which is an ex-ante criterion. In particular, the welfare under the ex-ante measure would be measured as follows:

\[
V = E_{Q^n} \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{(c^j_t)^{1-\gamma}}{1-\gamma} \right) \right],
\]

i.e. the welfare would be measured under the households’ subjective beliefs which might be different from the true probabilities. Thus, this measure doesn’t give us an accurate picture of the welfare effects of a financial transaction tax.\(^7\)

In this paper we use the ex-post welfare optimum as introduced by Hammond (1981)\(^7\)

\(^7\)The recent interest in models with heterogeneous beliefs has lead to some research how welfare could be measured under heterogeneous beliefs. Other proposals for welfare measures are given by Brunnermeier et al. (2012), Gilboa and Schmeidler (2012) and Blume et al. (2013).
in which the outcome is evaluated under the planners probability distribution and not the beliefs of the individual agent. And although nobody in the economy knows the true probability distribution they agree on the empirical distribution. Thus, we follow the argument by Nielsen (2009) and use the stationary measure as for all other distributions there will be no agreement among the households. Hence, the ex-post welfare of household \( j \) is given by

\[
V = E_t \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{(c^j_t)^{1-\gamma}}{1 - \gamma} \right) \right].
\]  

(31)

To calculate the welfare we simulate 1000 times 600 years of the economy and discard the first 100 years for every simulation.

6.2 The Welfare Effects of a Financial Transaction Tax

The welfare effects of a financial transaction tax can be summarized as follows:

Summary of Results 6. The introduction of a financial transaction tax has the following effects on welfare:

1. The change in welfare is very small \(< |0.01|\%\).

2. A financial transaction tax improves the welfare in Economy I.

3. A transaction tax on the stock market reduces the welfare in Economy II whereas a transaction tax on the bond market does not affect welfare.

Figures 9(a) and 9(b) show the welfare effects of a financial transaction tax in Economy I and Economy II and we can clearly see that the introduction of a financial transaction tax always improves the welfare in the economy.
The reason that the introduction of a financial transaction tax improves the welfare in Economy I is that the households now trade less on the financial markets. If they now trade less they will make less mistakes, i.e. because of their overconfidence they might assign a higher probability to a positive dividend compared to the empirical distribution and hence end up buying too many assets. If the economy ends up now in a state with a low dividend payout the households lose. Thus, preventing households from trading on the stock market improves the welfare.

In Economy II a transaction tax on the bond market does not affect the welfare in the economy while a transaction tax on the stock market decreases the welfare in the economy, because the interest in the economy increases. That means that households face now a higher risk in the economy for transferring wealth across time.

The reason for the small welfare effects of a financial transaction tax on welfare is that only 15% of the households income comes from the financial market. Even if a transaction tax reduces the wrong trades of the households, it only affects a small proportion of the income. Hence the small effects on welfare.
6.3 Implications for Public Policy

Thus far, we have seen that the introduction of a financial transaction tax on the stock market does not necessarily reduce the stock-market volatility. In fact, depending on the structure of beliefs of the households, a financial transaction tax on the stock market or bond market can have the adverse effect of increasing the stock market volatility. The downside of the result is however that the structure of beliefs is not observable. Hence, a policy maker who tries to introduce a transaction tax on the stock market does not know ex-ante whether such a policy will increase or decrease the volatility on financial markets.

Additionally, a policymaker might be thinking about introducing a transaction tax on both markets. However, the effects of a transaction tax on both markets are similar to a transaction tax on one market. Thus, whether the volatility is increased or decreased depends on the structure of beliefs.

In Economy I the introduction of a transaction tax always had positive welfare effects and the structure of beliefs in this economy, i.e. short waves of intense optimism and long waves of moderate pessimism is supported by the observations made by Shilling (1992). Hence, one could argue that although a transaction tax might not stabilize financial markets or prevent potential bubbles it still increases the welfare because of the reduced trade in the economy. And although the trade in Economy II was also reduced we saw no improvements in the welfare. However, this could be attributed to the fact that the income from the transaction tax was lost and with a more efficient government one would expect positive welfare effects.
7 Conclusion

In this paper we studied the impact of a financial transactions tax on the volatility of asset prices and the welfare in the economy. The model is able to replicate some important empirical facts such as that the introduction of a transaction tax reduces the trading volume but increases the asset-price volatility. In addition, the welfare in the economy also increases.

The model in this paper can be extended into several directions. First, the model in this paper has only one stock market. In the presence of several stock markets one would be able to study the migration of traders from a financial market with a transaction tax towards a stock market without a financial transaction tax. This is an issue for smaller financial centres. For example Umlauf (1993) discusses the experience of the swedish stock market where a large fraction of the trade in stocks migrated to London. In addition, there might be potential spill-over effects from introducing a financial transaction tax on only one market.

Furthermore, the model presented in this paper is a pure asset-pricing model and the connection between the financial side of the economy and the real side of the economy is not explored here, in particular the effect of a transaction tax on business cycles. Studying the implying of a financial transaction on the business cycle would also provide another fruitful direction of research.

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