OPTIMAL CONTRACTS FOR LOSS AVERSE CONSUMERS

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Abstract. We enrich the standard model of optimal contract design between a monopolist and a continuum of potential buyers under asymmetric information by assuming that consumers have reference-dependent preferences and loss aversion. In our model, consumers are endowed with quasi-linear utilities over the quality parameter of a good sold by a monopolist. The total valuation for quality is composed of the standard consumption valuation, which is affected by privately known types, and an additional gain-loss valuation that depends on deviations of purchased quality from the reference point. Potential buyers are loss-averse, so that deviations from the reference point are evaluated differently depending on whether they are gains or losses. We maintain Kőszegi and Rabin's (2006) basic framework and let gains and losses relative to the reference point be evaluated in terms of changes in the consumption valuation. However, we consider different ways in which reference quality levels are formed. In particular, we consider reference qualities as a monotone function of types. The presence of reference points creates downward kinks in the total valuation for the good, as a function of the privately known types, which renders the standard techniques based on revenue equivalence moot. Nonetheless, we apply a generalization of the standard envelope techniques to derive, for monotone reference functions, the optimal selling contract between the monopolist and loss averse consumers. We find that, depending on how reference points are formed, there is substantial difference between optimal contracts for loss averse consumers and optimal contracts for loss neutral buyers, both in terms of expected revenue generated to the monopolist and quality–price offered to each type of consumer.

1. Introduction

This paper studies how a revenue maximizing firm solves the problem of optimal contract design in the presence of consumers with heterogeneous tastes, asymmetric information, and reference-dependent preferences. Building on the classic monopoly pricing models under asymmetric information developed by Mussa and Rosen (1978) and Maskin and Riley (1984), we introduce consumers whose preferences exhibit loss aversion in the product attribute dimension. In our model, consumers are endowed with quasi-linear utilities over the quality parameter of a good sold by a monopolist. Moreover, the total valuation for quality is composed of the standard consumption valuation, which is affected by privately known types, and an additional gain-loss valuation that depends on deviations of purchased quality from the reference point. Deviations from the reference point are evaluated differently depending on whether they are gains or losses, with losses counting more heavily than comparable gains.

Loss aversion in riskless environments was introduced by Thaler (1980) and Tversky and Kahneman (1991), and has been since validated, with few exceptions, in various experimental setting.[1] In this paper we adopt instead the line of inquiry pioneered by della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), and Heidhues and Kőszegi (2008), among others,

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[1]For a recent account, see for example Novemsky and Kahneman (2005).

and study how revenue maximizing firms deal with optimal contract design in a market context where consumers have systematic deviations from traditional preferences.

In our model, consumers with quasi-linear utilities and heterogeneous tastes derive utility from the consumption of a unit of a good of different qualities and from comparison of actual consumption with a reference point. We let consumers with different tastes differ in terms of their reference quality levels as well. As a first step in our model, we assume that reference points are formed via an increasing reference function. The way we model the firm's behavior is standard: a monopolist with a convex cost function maximizes expected revenues, given that consumers' tastes are private information. The monopolist is, on the other hand, aware of the loss aversion biases and of the consumers' reference points, and takes these facts into consideration. From a technical point of view, the presence of reference points creates downward kinks in the total consumer's valuation for the good, as a function of the privately known types, which renders the standard contract theory techniques based on revenue equivalence moot. Nonetheless, we apply a generalization of the standard envelope techniques to derive, for monotone reference functions, the optimal menu of posted contracts between the monopolist and loss averse consumers. We find that, depending on how reference points are formed, there is substantial difference between optimal contracts for loss averse consumers and optimal contracts for loss neutral buyers, both in terms of expected revenue generated to the monopolist and quality–price offered to each type of consumer.

Our paper is not the first to combine loss aversion and asymmetric information in a market context with a revenue-maximizing firm. However, we depart from previous work in behavioral contract theory in several important ways. First, while we follow Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2010) in modelling the consumer's gain-loss function in terms of the difference between valuation at consumption and valuation at the reference point, we differ in terms of how comparisons with the reference point occur. Specifically, we assume that reference points are state-dependent (in a manner to be explained below) and that consumption in each state is compared with the reference point for that state alone. If a consumer has high marginal valuation for a good in state H and low marginal valuation in state L, then actual consumption in state H is to be compared with the (possibly high) reference point for state H and *not* with the (possibly low) reference point for state L.

Second, while in our model, again as in the models of Kőszegi and Rabin (2006) and Heidhues and Kőszegi (2010), a reference point is interpreted as the recent expectation of consumption outcome, we do not impose a priori rational expectations in consumers' behavior. We endow consumers with a reference formation process that depends on consumers' types (states), which are private information. Aside from a technical continuity condition, the reference point formation process is only assumed to be weakly increasing so that a higher consumer's type (i.e., a higher marginal consumption valuation) corresponds to a weakly higher reference point. Thus, our model allows us to explore different ways in which reference points are generated and their non-trivial implications in terms of optimal contract design. These implications are, in principle, testable. We consider reference point formation processes that could be influenced partly by advertising campaigns, fashion, etc. Thus our model bears some resemblance to the aspirational considerations of Gilboa and Schmeidler (2001). The dynamic version of our model considers an adaptive process in which reference points are formed using past observations of quality levels as well as past reference points. This adaptive reference formation seems to be consistent with empirical evidence from the marketing literature that suggests that reference points respond to memory-based stimuli; see for instance Briesch, Krishnamurthi, Mazumdar, and Raj (1997).

Third, we model reference points in terms of expected or aspirational consumption levels that affect the valuation for the object offered by the revenue-maximizing monopolist. We depart from recent work in the area, for instance Herweg and Mierendorff (2011) and Spiegler (2011), that specifies reference points in terms of prices. Thus, our model combines private consumer heterogeneity in the valuation function for the object, captured by the one-dimensional type space, with a reference formation process that delivers type-dependent reference consumption levels. This specification goes in line with empirical evidence from the marketing literature that suggests that loss aversion on quantity/quality attributes is at least as important as, if not more important than, loss aversion in monetary transactions; see Hardie, Johnson, and Fader (1993). This point is also suggested by some experimental data as reported recently by Novemsky and Kahneman (2005).

Fourth, our model, while parsimonious in terms of deviations from standard preferences, allows us to derive optimal contracts that differ substantially from the optimal contract without reference-dependent preferences (even under complete information). As we shall demonstrate, the optimal contract design problem of the revenue-maximizing firm is heavily influenced by the reference formation process. In general, for a range of intermediate types, the shape of the optimal quality schedule will be determined by the shape of the reference quality function. Since we consider weakly monotone increasing, continuous reference functions, the optimal quality schedule may exhibit flat parts among increasing parts for intermediate types. The additional flexibility in contract design could partially explain complex contract schemes (e.g., three part tariff schemes) that have become increasingly popular among wireless phone providers, Internet providers, and other subscription services; see Lambrecht, Seim, and Skiera (2007) among others. The optimal quality/quantity provided by the monopolist is increasing for low types, constant for intermediate types, and then reverts to being increasing for higher types. Moreover, depending on the reference function, the optimal quality/quantity schedule may exhibit two or more flat parts, which seems to be consistent with two-tier tariff systems used by electricity providers, as reported by Reiss and White (2005).

The rest of the paper is organized as follows. Section 2 contains the formal details of our model and the analysis of the complete information case. In Section 3, we first study the benchmark case of loss neutral consumers, in which the optimal menu of contracts coincides with the standard optimal menu once the effect of the gain-loss function is accounted for in the computation of the virtual surplus. We then present the analysis for the loss aversion case. As mentioned, optimal contracts for loss averse consumers depend on the reference function. For types with high or low reference quality levels (compared to the quality level that maximizes virtual surplus when the gain-loss parameter and the loss aversion parameter are taken into account), the optimal quality schedule for the monopolist maximizes virtual surplus. For types with reference quality in the intermediate region, the optimal quality schedule coincides with the reference function. Note that in this static setting we let optimal contracts differ from reference qualities, as long as they are individually rational and incentive compatible. That is, a consumer's actual consumption need not coincide with her expectations, given by the reference point, as long as the menu of contracts is incentive feasible: optimal contracts may not be self-confirming.

In Section 4, we consider a long-lived myopic monopolist who faces a sequence of short-lived consumers to explore the reference point formation process. We show that if consumers use an adaptive reference formation process, which in essence takes the average of actual consumption and reference levels of the previous generation, then the steady-state optimal quality schedule and the steady-state reference function coincide. Thus, in the long term, the optimal menu of contracts is self-confirming. It is interesting to note that the shape of the long-term contracts will be heavily influence by the reference function of the first generation. Section 5 offers some concluding remarks. Omitted proofs from the text are gathered in Section 6.

2. Monopolist's Problem under Reference-Dependent Preferences

In this section we lay out the notation and main assumptions of our basic model. We build on standard monopoly pricing models studied by Mussa and Rosen (1978) and Maskin and Riley (1984), among others. The main difference is that we consider consumers who derive utility from consumption and, in addition, from comparisons between actual consumption and a reference point. Following the insights of Tversky and Kahneman (1991), we consider consumers who evaluate negative deviations from the reference point more heavily than similar positive deviations; i.e., we consider loss-averse consumers.

2.1. **The firm.** A revenue-maximizing monopolist produces a commodity of different characteristics, which are captured by the "product attribute" parameter $q \geq 0$. This parameter can be interpreted as either a one-dimensional measure of quality (speed of connection offered by an Internet provider, exclusive features of a luxury good) or a measure of quantity (minutes offered by a mobile phone provider, electricity units offered by a local power supplier). We pay tribute to Mussa and Rosen (1978) and maintain the first interpretation, thus referring to q as the *quality attribute*. The cost of producing one unit of the good with quality $q \geq 0$ is represented by $c(q)$. The cost function c, defined on $Q = [0, M]$ for some $0 < M < +\infty$, is increasing and $\mathcal{C}^1$, with $c(0) = 0$. The firm's problem is to design a revenue maximizing menu of posted quality–price pairs to offer to potential buyers with differentiated demands. Demand heterogeneity is captured by the state parameter $\theta \in \Theta = [\theta_L, \theta_H]$, where $0 \leq \theta_L < \theta_H < +\infty$. We refer to $\theta \in \Theta$ as the type of a consumer. Types are private information, so that the monopolist knows the distribution F, with full support on $[\theta_L, \theta_H]$ and density function $f > 0$, but not the actual realization of types. Thus, the monopolist maximizes expected revenue.

Assumption 1. The *inverse hazard rate* $h\colon \Theta \to \mathbb{R}$, defined for all $\theta \in \Theta$ by

$$h(\theta) \equiv (1 - F(\theta))/f(\theta),$$

is non-negative, $\mathcal{C}^1$ and decreasing on Θ.

2.2. **Consumers.** There is a mass of small consumers with quasi-linear preferences who have unit demands for the good offered by the monopolist but heterogeneous tastes in terms of product attributes. A θ-type consumer has a *consumption valuation* for quality $q \geq 0$ given by

$$\theta m(q)$$

The function m defined on $Q = [0, M]$ is strictly increasing and $\mathcal{C}^1$, with $m(0) = 0$. We let M be sufficiently large so that for every θ, the quality level that maximizes consumption surplus is strictly less than M. We also impose the next assumption.

Assumption 2. The functions c and m satisfy the following:

(a) $\lim_{q \to 0}\big(m'(q) - c'(q)\big) > 0$;
(b) m is concave on Q;
(c) c is strongly convex on Q.

Recall that function $f\colon \mathbb{R} \to \mathbb{R}$ is strongly convex if there exists a number $\rho > 0$ such that $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha)f(y) - (\rho/2)\alpha(1-\alpha)|x-y|^2$, for all $x, y \in \mathbb{R}$, for all $\alpha \in (0,1)$. For twice differentiable functions, strong convexity is obtained whenever $f''(x) \geq \rho > 0$, for all x

(see Montrucchio (1987) for instance). This specification of the consumption surplus $\theta m(q) - c(q)$ includes, among others, the monopoly model of Mussa and Rosen (1978) with linear consumption valuation and quadratic costs. The extra condition of strong convexity ensures that the quality schedule that maximizes (virtual) consumption surplus is Lipschitz continuous (see Section 3.1), which is needed in the analysis of long term contracts given in Section 4. An alternative to Assumption 2 to accommodate Maskin and Riley's (1984) monopoly model is to impose strong concavity of m and convexity of c.

We depart from standard models in contract theory and assume that, aside from classic consumption preferences, potential buyers exhibit reference-dependent preferences for the product attribute (but not for monetary transactions). Specifically, in addition to the consumption valuation $\theta m(q)$, a consumer assesses quality q relative to a reference quality level $r \geq 0$ via the function $n(q, \theta, r)$. Following Kőszegi and Rabin (2006), we let gains and losses derived from q relative to r be evaluated in terms of changes in the consumption valuation, so that the *gain-loss valuation* is given by:

$$n(q, \theta, r) \;=\; \mu\, \theta\big(m(q) \;-\; m(r)\big),$$

where μ is equal to η if $m(q) \geq m(r)$ and to $\eta\lambda$ if $m(q) \leq m(r)$ instead. We make two alternative behavioral assumptions on the gain-loss coefficient η and the loss-aversion coefficient λ.

Assumption 3. Consumers with reference-dependent preferences exhibit:

(a) *loss aversion*, if $0 < \eta < \eta\lambda < 1$;
(b) *loss neutrality*, if $0 < \eta = \eta\lambda < 1$.

Assumption 3 states that consumers are either loss averse (i.e. $\lambda > 1$) or loss neutral (i.e., $\lambda = 1$), but evaluate changes in the product attribute relative to their reference points as being less important than the direct effect of quality consumption. Most of the paper focuses on the loss aversion case; we use loss neutrality as benchmark.

While in some economic contexts it may be reasonable to assume that all potential buyers share the same reference point, one shall not rule out a priori that in other contexts buyers with different marginal consumption valuation have different reference quality levels. Some evidence of this fact comes from the marketing literature; see Bell and Lattin (2000). Moreover, reference qualities encompass different concepts: a reference point r may reflect subjective expectations of consumption that may be influenced by past experiences as well as by current aspirational considerations, which in turn are affected by fashion, advertising campaigns, social influences, etc. As a first step, we consider reference points specific to each θ-type consumer and represent the reference formation process via the function r endowed with the following properties.

Assumption 4. The *reference function* $r\colon \Theta \to Q$ for consumers with reference-dependent preferences is weakly increasing, Lipschitz continuous, and piecewise differentiable on Θ.

The weak monotonicity of the reference formation function should be noncontroversial. From a technical point of view, one can weaken Assumption 4 to require only that the reference function r be Lipschitz continuous on Θ. The derivation of our results are simplified by, but do not depend on, the extra requirement of piecewise differentiability. Let $T = \{\theta_k \in \Theta \mid k = 1, \dots, K\}$ denote the elements of Θ where the function r fails to be differentiable. Given the reference function $r\colon \Theta \to Q$, the *total valuation* for the θ-type consumer from product quality $q \in Q$ is:

$$v(q, \theta) \;\equiv\; \theta m(q) \;+\; \mu\, \theta\big(m(q) \;-\; m(r(\theta))\big).$$

The utility that the θ-type consumer obtains from buying one unit of the good with product quality $q \geq 0$ at price $p \geq 0$, given her reference quality level $r(\theta) \geq 0$, is expressed as $v(q, \theta) - p$, for all $\theta \in \Theta$. Note that her overall utility from consuming zero quality at a null price is $-\eta\lambda\theta m(r(\theta))$. We take this as the value of the outside option for the θ-type consumer.

Assumption 5. The outside option $u_0(\theta)$ of a θ-type consumer with reference-dependent preferences is given by:

$$u_0(\theta) \;=\; -\,\eta\lambda\theta m(r(\theta)), \qquad \text{for all } \theta \in \Theta.$$

The interpretation of our framework is the following. Consumers and monopolist interact for a period of time. Before interaction starts, both consumers and monopolist know the state-space Θ and the distribution of signals F. At the beginning of the period, each consumer receives a type (state signal) that affects her consumption valuation for the good. After types are privately observed, but before the monopolist can post any menu of contracts, consumers form their reference quality levels via the function r. One can consider different ways in which reference qualities are formed, but for now we only impose Assumption 4 on the reference function (we shall come back to this issue in Section 4). Once reference levels are established, the monopolist offers a menu of posted contracts to maximize expected revenue. At this stage, each θ-type consumer self-selects the quality-price combination that maximizes her total utility given her reference level $r(\theta)$, even if the chosen quality does not coincide with the reference quality, as long as the total utility derived from the contract higher than the value of her outside option. Throughout the paper, it is assumed that the monopolist knows the functional forms of the consumption valuation m, the gain-loss valuation n, and the reference function r, but types are private information. Before analyzing optimal contract design under incomplete information, we briefly consider the case of an informed monopolist.

2.3. **Monopolist's problem under complete information.** When consumers' types are known, the monopolist can make specific take-it-or-leave-it offers to each θ-type consumer. For a consumer with reference point $r(\theta)$, the monopolist sets a price $p(\theta) = \theta m(q(\theta)) + \mu\theta\big(m(q(\theta)) - m(r(\theta))\big) - u_0(\theta)$, and chooses a quality level $q(\theta)$ that maximizes profits at θ:

$$(1+\mu)\theta m(q) \;-\; c(q) \;+\; (\eta\lambda - \mu)\theta m(r(\theta)). \tag{1}$$

Let q^e_μ be the (unique) efficient quality schedule under reference-dependent preferences:

$$q^e_\mu(\theta) \;\equiv\; \operatorname{argmax}\{(1+\mu)\theta m(q) \;-\; c(q) \mid q \in Q\},$$

for $\mu \in \{\eta, \eta\lambda\}$, for all $\theta \in \Theta$. Since m is strictly increasing, notice that $q^e_\eta(\theta) < q^e_{\eta\lambda}(\theta)$ for all $\theta > 0$. Under loss neutrality and complete information, the monopolist offers to each type θ a quality level $q^{fb}(\theta) = q^e_\eta(\theta)$, and a price $p^{fb}(\theta) = (1+\eta)\theta m(q^e_\eta(\theta))$ that extracts all consumer surplus. This revenue-maximizing quality-price combination is independent of the reference level $r(\theta)$. Any reduction in the consumer's willingness to pay due to the reference point is effectively cancel by the value of the the outside option.

Suppose now that consumers are loss averse, so that μ can be either η or $\eta\lambda$, depending on whether the chosen quality level exceeds or comes short of the reference level. If $r(\theta)$ is sufficiently low (respectively, sufficiently high) compared to the efficient quality level $q^e_\eta(\theta)$ (respectively, $q^e_{\eta\lambda}(\theta)$), then the monopolist sets quality at its efficient level and charges the corresponding price to leave the θ-type consumer with zero surplus. If the reference point $r(\theta)$ lies in intermediate ranges, any deviation from the reference quality will hurt the firm's profits. Providing a quality level higher than the reference level will position the monopolist in the downward sloping portion of its profit function since the gain-loss coefficient is $\mu = \eta$, so the monopolist has incentives to reduce quality level. A similar effect takes place when quality level q is below the reference point $r(\theta)$, only now the profit function considers $\mu = \eta\lambda$. This leads us to the following observation.

Proposition 1. *Under Assumptions 2, 3(a), 4 and 5, the complete information optimal menu of contracts for consumers with reference-dependent preferences consists of quality–price pairs* $\{(q^{FB}(\theta), p^{FB}(\theta)) \mid \theta \in \Theta\}$ *such that:*

$$q^{FB}(\theta) = \begin{cases} q^e_\eta(\theta) & : \ r(\theta) < q^e_\eta(\theta), \\ r(\theta) & : \ q^e_\eta(\theta) \le r(\theta) \le q^e_{\eta\lambda}(\theta), \\ q^e_{\eta\lambda}(\theta) & : \ q^e_{\eta\lambda}(\theta) < r(\theta); \end{cases}$$

$$p^{FB}(\theta) = (1+\mu)\theta m(q^{FB}(\theta)) + (\eta\lambda - \mu)\theta m(r(\theta)),$$

where in the expression for prices one has $\mu = \eta$ *if* $q^{FB}(\theta) > r(\theta)$ *and* $\mu = \eta\lambda$ *if* $q^{FB}(\theta) \le r(\theta)$*.*

While mathematically trivial, Proposition 1 stresses the relevance of the reference process r for optimal contract design under loss aversion even in the complete information case, as it entirely determines the shape of the first-best quality schedule for types with $q^e_\eta(\theta) \le r(\theta) \le q^e_{\eta\lambda}(\theta)$. Since the reference function can indeed be very general, as long as it is Lipschitz continuous and monotone, first-best contracts may take various forms. On the other hand, for low and high reference points relative to the efficient qualities, the first best quality level $q^{FB}(\theta)$ differs from expectations or aspirational levels as captured by $r(\theta)$. This divergence between actual consumption and expectations is allowed to happen as long as the consumer's payoff satisfies the participation constraint specified by Assumption 5. In such case, under complete information, the θ-type consumer will carry out with the contract designed for her, since this contract is individually rational. Thus, we do not limit the way beliefs about future consumption are formed to rational expectations, but we impose individual rationality as stated in the participation constraints for each type. On the other hand, a monopolist may want to influence the reference function r, as a higher reference quality level never decreases the monopolist profits.

Corollary 1. *The monopolist's net revenue from every θ-type customer under complete information and loss aversion is increasing in the reference quality level $r(\theta)$, for all $r(\theta) \le q^e_{\eta\lambda}(\theta)$, and constant for all $r(\theta) > q^e_{\eta\lambda}(\theta)$.*

Suppose the monopolist can publicly announce (but not necessarily commit to) a quality schedule prior to the market interaction. If these announcements are costless for the monopolist, and if each θ-type consumer can perfectly observe her quality announcement and form rational expectations on the reference quality accordingly, then from Proposition 1 and Corollary 1 it follows that the monopolist announces $q^e_{\eta\lambda}(\theta)$ for each $\theta \in \Theta$. These announcements are credible, in the sense that consumers form quality expectations $r(\theta) = q^e_{\eta\lambda}(\theta)$ that coincide with their actual quality purchases, and maximize revenue among all possible announcements. This conclusion does not hold if there are costs involved in advertising different quality levels, or if consumers do not observe their announcements and thus form reference levels that differ from the levels intended by the monopolist.

3. Optimal Contract Design under Reference-Dependent Preferences

In this section we consider the asymmetric information case, where consumers' types are private information. Since the monopolist knows the distribution but not the realization of types, its problem is to design menu of contracts that maximizes expected revenue subject to the incentive compatibility and individually rationality constraints.

3.1. **Optimal contracts under loss neutrality.** In this subsection we consider as benchmark the case of reference-dependent preferences without loss aversion biases; i.e., here we impose Assumption 3(b). For a fixed reference function $r\colon \Theta \to Q$, the total valuation for the θ-type consumer is simply

$$v(q,\theta) \;=\; (1+\eta)\theta m(q) \;-\; \eta\theta m(r(\theta)), \tag{2}$$

for all $\theta \in \Theta$ and all $q \in Q$. The problem of the monopolist under incomplete information can be stated as follows: design a menu of posted contracts $\{(q(\theta), p(\theta)) \mid \theta \in \Theta\}$ that maximizes

$$\Pi^{sb} \;=\; \int_{\theta_L}^{\theta_H} \big[p(\theta) \;-\; c(q(\theta))\big] f(\theta)\, d\theta$$

subject to the informational constraints:

$$v(q(\theta),\theta) \;-\; p(\theta) \;\geq\; v(q(\hat\theta),\theta) \;-\; p(\hat\theta), \qquad \text{for all } \theta, \hat\theta \in \Theta; \tag{3}$$

$$v(q(\theta),\theta) \;-\; p(\theta) \;\geq\; -\,\eta\theta\, m(r(\theta)), \qquad \text{for all } \theta \in \Theta. \tag{4}$$

The incentive constraints expressed in (3) contain the total valuation v for the case of loss neutrality, as formulated in Equation 2. The participation constraints in (4) capture Assumption 5 for $\lambda = 1$. A menu of contracts $\{(q(\theta), p(\theta)) \mid \theta \in \Theta\}$ is said to be *incentive feasible* if it satisfies all (3) and (4) constraints.

Since the subset T has finitely many elements and the total valuation function $\theta \mapsto v(q,\theta)$ is differentiable with respect to types on $\Theta \setminus T$ with bounded derivatives, one can modify the standard Envelope Theorem arguments in Milgrom and Segal (2002) and Williams (1999) to obtain an expression for the indirect utility associated to an incentive feasible menu of contracts. Thus, any incentive compatible price schedule can be written as $p(\theta) = v(q(\theta),\theta) - \int [\partial v(q(\tilde\theta),\tilde\theta)/\partial\tilde\theta] d\tilde\theta - U(\theta_L)$, where $U(\theta_L)$ is the indirect utility of the lowest type. Setting this indirect utility equal $-\eta\theta_L m(r(\theta_L))$ to maximize revenue and using Equation 2, we have

$$p(\theta) \;=\; (1+\eta)\theta m(q(\theta)) \;-\; \int_{\theta_L}^{\theta} (1+\eta) m(q(\tilde\theta))\, d(\tilde\theta)\,, \qquad \text{for all } \theta \in \Theta.$$

We now replace this expression into the profit function and integrate by parts. Let $\phi\colon \Theta \to \mathbb{R}$ the *marginal virtual consumption valuation*:

$$\phi(\theta) \;\equiv\; \theta \;-\; h(\theta), \qquad \text{for all } \theta \in \Theta. \tag{5}$$

From Assumption 1, ϕ is $\mathcal{C}^1$ and increasing on Θ. The monopolist's problem is to choose quality levels $\{q(\theta) \mid \theta \in \Theta\}$ to maximize

$$\Pi^{sb} \;=\; \int_{\theta_L}^{\theta_H} \Big[(1+\eta)\phi(\theta) m(q(\theta)) \;-\; c(q(\theta))\Big] f(\theta)\, d\theta,$$

subject to the incentive and participation constraints.

Let $\Theta_0 \subseteq \Theta$ be such that $\phi(\theta) \leq 0$ for all $\theta \in \Theta_0$. If $\Theta_0 \neq \emptyset$, by the monotonicity of the marginal virtual consumption valuation one has $\Theta_0 = [\theta_L, \theta_0]$ for some $\theta_L \leq \theta_0 \leq \theta_H$, so that $\phi(\theta) > 0$ for each type $\theta > \theta_0$. Let q^*_μ be the unique quality schedule that maximizes virtual consumption surplus under reference-dependent preferences; that is, for $\mu \in \{\eta, \eta\lambda\}$, for all $\theta \in \Theta$ with $\phi(\theta) > 0$,

$$q^*_\mu(\theta) \;\equiv\; \operatorname{argmax}\{(1+\mu)\phi(\theta) m(q) \;-\; c(q) \mid q \in Q\}. \tag{6}$$

Inspection to the last expression for profits in terms of the virtual surplus under loss neutrality yields to the optimal menu of contracts.

Proposition 2. *Suppose that Assumptions 1, 2, 3(b), 4 and 5 hold. Then the optimal menu of incentive feasible contracts for the monopolist consists of pairs* $\{(q^{sb}(\theta), p^{sb}(\theta)) \mid \theta \in \Theta\}$ *such that the following is satisfied:*

$$q^{sb}(\theta) \;=\; \begin{cases} 0 & : \; \theta \leq \theta_0, \\ q^*_\eta(\theta) & : \; \theta > \theta_0; \end{cases}$$

$$p^{sb}(\theta) \;=\; (1+\eta)\theta m(q^{sb}(\theta)) \;-\; \int_{\theta_0}^{\theta} (1+\eta) m(q^{sb}((\tilde{\theta}))\, d\tilde{\theta}.$$

Proof. It is clear that the quality schedule q^{sb} derived above maximizes expected revenue Π^{sb}. In Section 6, we show that this menu of posted quality-price combinations is indeed incentive compatible. To argue individually rationality, one uses $q^{sb}(\theta)$ and $p^{sb}(\theta)$ to compute the consumer's indirect utility:

$$U(\theta) \;=\; v(q^{sb}(\theta), \theta) \;-\; p^{sb}(\theta) \;=\; \int_{\theta_L}^{\theta} (1+\eta) m(q^{sb}(\tilde{\theta}))\, d\tilde{\theta} \;-\; \eta\theta m(r(\theta)). \tag{7}$$

This immediately implies that (4) is satisfied for all $\theta \in \Theta$, as desired. □

Proposition 2 shows that the presence of reference-dependent preferences without systematic loss aversion biases does not alter the logic or the main features of optimal contracts derived for consumption preferences alone. Low types receive zero quality levels and make zero payments, and higher types are allocated a quality level that maximizes virtual surplus. In this case, as in the case of loss neutral consumers under complete information, optimal contracts are independent of the reference function. From the Lipschitz continuity of the quality schedule q^*_μ, one has also the following.

Corollary 2. *The optimal quality schedule* q^{sb} *under incomplete information and loss neutrality is Lipschitz continuous on* Θ.

3.2. **Optimal contracts for loss averse consumers.** We now focus on the framework with reference-dependent preferences and loss aversion to study optimal contract design. Notice that in this setting one cannot apply the standard Envelope Theorem techniques to express expected profits in terms of the virtual valuation, as proposed fist by Myerson (1981), and more recently by Williams (1999), Milgrom and Segal (2002), and Berger, Müller, and Naeemi (2010), among others. This is because the standard integral representation of the indirect utility function (and thus revenue equivalence) may fail.

Indeed, while for any quality level $q \in Q$ the consumption valuation $\theta m(q)$ is linear in types, the gain-loss valuation $n(q, \cdot, r(\cdot))$, expressed in terms of the reference function r, is neither convex nor everywhere differentiable with respect to types. In particular, differentiability may fail for a subset of types of positive measure. To see this, fix a quality level $\hat{q} \in Q$ and consider again the total valuation $\theta \mapsto v(\hat{q}, \theta)$

$$v(\hat{q}, \theta) \;=\; \theta m(\hat{q}) \;+\; \mu\, \theta\big(m(\hat{q}) \;-\; m(r(\theta))\big), \tag{8}$$

where $\mu = \eta$ if $m(\hat{q}) \geq m(r(\theta))$ and $\mu = \eta\lambda$ if $m(\hat{q}) \leq m(r(\theta))$. Given the piecewise differentiability of r, there are two potential sources of non-convexity and/or non-differentiability in the valuation $v(\hat{q}, \cdot)$. First, it is the fact that the reference function itself may have non-convex kinks. Second, it is the fact that even in differentiable segments of r, the valuation function may change suddenly depending on whether $\hat{q} \leq r(\theta)$ or $\hat{q} > r(\theta)$. While the first problem is notationally cumbersome, it will have no effect on the standard Envelope Theorem formulation, as the types $\theta_k \in T$ at which the reference function exhibits a kink are finitely many (thus, as in Section 3.1 one could use the standard approach). Keeping that in mind, we focus on the second and more significant source of non-convexity and non-differentiability and avoid discussion of kinks in the valuation function due to kinks in the reference function (our treatment is however sufficiently general to encompass the failure of differentiability in the reference function r, as long as r is Lipschitz continuous).

For a fixed quality level $\hat{q} \in Q$, define the right and left subderivatives of the function $\theta \mapsto v(\hat{q}, \theta)$ evaluated at $\hat{\theta} \in \Theta$, respectively, as:

$$\overline{\mathrm{d}}v(\hat{q},\hat{\theta}) \;\equiv\; \liminf_{\delta \downarrow 0} \frac{v(\hat{q},\hat{\theta}+\delta) \,-\, v(\hat{q},\hat{\theta})}{\delta}, \quad \text{and}$$

$$\underline{\mathrm{d}}v(\hat{q},\hat{\theta}) \;\equiv\; \limsup_{\delta \uparrow 0} \frac{v(\hat{q},\hat{\theta}+\delta) \,-\, v(\hat{q},\hat{\theta})}{\delta}.$$

Since $v(\hat{q}, \cdot)$, as defined in Equation 8, is Lipschitz continuous with uniformly bounded Lipschitz constant, these subderivatives exist for all $\theta \in \Theta$ and all $\hat{q} \in Q$ (in fact, except for types in the finite set T, the right and left subderivatives coincide with the right and left derivatives, but we keep the first notation for expositional convenience). We define now the *subderivative correspondence* $\theta \rightrightarrows S(\hat{q}, \theta)$ associated with quality level $\hat{q}$ as:

$$S(\hat{q},\theta) \;\equiv\; \big\{ s \in \mathbb{R} \mid \overline{\mathrm{d}}v(\hat{q},\theta) \,\le\, s \,\le\, \underline{\mathrm{d}}v(\hat{q},\theta) \big\}.$$

The subderivative correspondence is by definition closed-valued. It is empty-valued at types where the right subderivative of $v(\hat{q}, \cdot)$ is greater than the left subderivative, and single-valued when these two expressions are equal.

One can easily find the following expressions for each $\theta \in \Theta \setminus T$:

$$S(\hat{q},\theta) \;=\; \begin{cases} (1+\eta)m(\hat{q}) \,-\, \big(\eta\,\theta m(r(\theta))\big)' & : \;\hat{q} > r(\theta), \\ (1+\eta\lambda)m(\hat{q}) \,-\, \big(\eta\lambda\,\theta m(r(\theta))\big)' & : \;\hat{q} < r(\theta), \\ \big[m(r(\theta)) - \eta\lambda\,\theta m'(r(\theta))r'(\theta)\,,\, m(r(\theta)) - \eta\,\theta m'(r(\theta))r'(\theta)\big] & : \;\hat{q} = r(\theta). \end{cases} \tag{9}$$

Notice that if there are types $\theta, \hat{\theta}$, $\theta \neq \hat{\theta}$, satisfying $r(\theta) = r(\hat{\theta}) = \hat{q}$, then since r is piecewise differentiable and weakly increasing, it follows that $r'(\theta) = r'(\hat{\theta}) = 0$. Thus, the subderivative correspondence $S(\hat{q}, \cdot)$ is single-valued on $\Theta \setminus T$ except when there exists a unique type θ for which $r(\theta) = \hat{q}$. Clearly, one can find the values of $S(\hat{q}, \theta)$ for θ in T as well by considering the left and right subderivatives of the piecewise differentiable Lipschitz function r.

The problem of the monopolist can now be stated as follows: design a menu of posted contracts $\{(q(\theta), p(\theta)) \mid \theta \in \Theta\}$ that maximizes expected profits

$$\Pi^{SB} \;=\; \int_{\theta_L}^{\theta_H} \big[p(\theta) \,-\, c(q(\theta))\big]\, f(\theta)\, d\theta$$

subject to the informational constraints:

$$v(q(\theta),\theta) \,-\, p(\theta) \;\ge\; v(q(\hat{\theta}),\theta) \,-\, p(\hat{\theta}), \qquad \text{for all } \theta,\hat{\theta} \in \Theta; \tag{10}$$

$$v(q(\theta),\theta) \,-\, p(\theta) \;\ge\; -\,\eta\lambda\theta\, m(r(\theta)), \qquad \text{for all } \theta \in \Theta. \tag{11}$$

The incentive and participation constraints capture in (10) and (11) contain the total valuation v as formulated in Equation 8. Thus, we consider Assumptions 3(a), 4 and 5 in our formulation of feasibility constraints for the optimization problem of the firm. We stress the fact that the monopolist choice of quality schedule affects the subderivative correspondence. To be more precise, the monopolist may find optimal to select a quality schedule $q(\cdot)$ such that $q(\theta) = r(\theta)$ for a subset of Θ of positive measure. If for these types one has $r'(\theta) > 0$, then the subderivative correspondence will be multi-valued for a non-negligible subset of Θ. This implies that the valuation v may fail to be convex or differentiable in types on a non-negligible subset of the type space Θ. To overcome these difficulties, we employ the results reported in Carbajal and Ely (2011) to characterize incentive compatible contracts based on an integral monotonicity condition and a generalization of the Mirrlees representation of the indirect utility.

Given a quality schedule $\theta \mapsto q(\theta)$ and associated subderivative correspondence $\theta \rightrightarrows S(q(\theta), \theta)$, we use $\theta \mapsto s(q(\theta), \theta) \in S(q(\theta), \theta)$ to indicate an integrable selection of the subderivative correspondence (whenever one such selection exists). Say that $\theta \rightrightarrows S(q(\theta), \theta)$ is a *regular correspondence* if it is non empty-valued almost everywhere on Θ, closed-valued, measurable and integrably bounded. Given the incentive feasible menu of contracts $\{(q(\theta), p(\theta) \mid \theta \in \Theta\}$ satisfying the informational constraints (10) and (11), its indirect utility is

$$U(\theta) \;=\; v(q(\theta), \theta) \;-\; p(\theta), \qquad \text{for all } \theta \in \Theta.$$

Proposition 3. *Suppose that Assumptions 1, 2, 3(a), 4 and 5 hold. The quality schedule $q\colon \Theta \to Q$ designed by a monopolist is incentive feasible with associated indirect utility U if, and only if, the subderivative correspondence $S(q(\cdot), \cdot)$ is regular and admits an integrable selection $\theta \mapsto s(q(\theta), \theta)$ for which the following conditions are satisfied:*

(a) *Integral monotonicity: for all $\theta, \hat{\theta} \in \Theta$,*

$$v(q(\hat{\theta}), \hat{\theta}) \;-\; v(q(\hat{\theta}), \theta) \;\geq\; \int_{\theta}^{\hat{\theta}} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta} \;\geq\; v(q(\theta), \hat{\theta}) \;-\; v(q(\theta), \theta).$$

(b) *Generalized Mirrlees representation: for all $\theta \in \Theta$,*

$$U(\theta) \;=\; U(\theta_L) \;+\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}.$$

(c) *Endogenous participation: for all $\theta \in \Theta$,*

$$U(\theta_L) \;+\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta} \;+\; \eta\lambda\theta\, m(r(\theta)) \;\geq\; 0.$$

Proof. It follows from Theorem 1 in Carbajal and Ely (2011). □

To find the incentive feasible, revenue maximizing menu of contracts, we use the generalized Mirrlees representation to express payments in terms of the integral of the subderivative:

$$p(\theta) \;=\; v(q(\theta), \theta) \;-\; U(\theta_L) \;-\; \int_{\theta_L}^{\theta} s(q(\tilde{\theta}), \tilde{\theta})\, d\tilde{\theta}, \qquad \text{for all } \theta \in \Theta, \tag{12}$$

where the selection $s(q(\cdot), \cdot)$ is associated to the indirect utility U. We shall use (12) and the fact that $U(\theta_L) = -\eta\lambda\theta_L m(r(\theta_L))$ to write expected profits. Integration by parts leads to:

$$\Pi^{SB} \;=\; \int_{\theta_L}^{\theta_H} \Big[(1+\mu)\theta m(q(\theta)) \;-\; \mu\theta m(r(\theta)) \;+\; \eta\lambda\theta_L m(r(\theta_L)) \;-\; h(\theta) s(q(\theta), \theta) \;-\; c(q(\theta)) \Big] f(\theta)\, d\theta.$$

The first three terms of the integrand in the above equation denote the direct impact on surplus due to the presence of reference dependent preferences. The next term, involving the inverse hazard rate and the integrable selection $s(q(\theta), \theta)$, captures the additional flexibility gained by the monopolist due to potential downward kinks in the consumer's valuation function, as both the value that μ takes in the integrand of the expected profits and the subderivative correspondence $S(q(\cdot),\cdot)$ from which to draw the selection $s(q(\cdot),\cdot)$ depend on the choice of $q(\cdot)$. The last term captures the fact that the outside option for the lowest type may not be zero and depends on her reference point.

From Equation 9 and the fact that $h \geq 0$ by Assumption 1, we can express any revenue maximizing selection as

$$s(q(\theta),\theta) \;=\; (1+\eta)m(q(\theta)) \;-\; \big(\eta\,\theta m(r(\theta))\big)'$$

for all θ such that $r(\theta) < q(\theta)$, and

$$s(q(\theta),\theta) \;=\; (1+\eta\lambda)m(q(\theta)) \;-\; \big(\eta\lambda\, m(r(\theta))\big)'$$

for all θ such that $r(\theta) \geq q(\theta)$. We obtain:

$$\begin{aligned}\Pi^{SB} \;=\; \int_{\theta_L}^{\theta_H} \Big[&(1+\mu)\phi(\theta)m(q(\theta)) \;-\; c(q(\theta)) \;-\; \mu\,\theta m(r(\theta)) \;+\; \eta\lambda\theta_L m(r(\theta_L)) \\ &+\; h(\theta)\big(\mu\,\theta m(r(\theta))\big)'\Big] f(\theta)\, d(\theta),\end{aligned} \tag{13}$$

with $\mu = \eta$ whenever $r(\theta) < q(\theta)$ and $\mu = \eta\lambda$ otherwise. From Proposition 3, the problem of the monopolist can now be stated as follows. Choose the quality schedule $q^{SB}\colon \Theta \to Q$ to maximize Π^{SB} in Equation 13, subject to the integral monotonicity condition and the endogenous participation constraint. Here we present the solution to this problem ignoring the constraints. In Section 6, we argue that this solution is incentive feasible.

Claim 1. *For all $\theta \in \Theta_0$ such that $\phi(\theta) \leq 0$, the optimal quality schedule is $q^{SB}(\theta) = 0$.*

Proof. Fix any $\theta \in \Theta_0$ and assume $r(\theta) > 0$ (the case of $r(\theta) = 0$ is similar). Choosing a quality level $q^{SB}(\theta) = 0$ performs better than choosing an alternative quality level $0 < q \leq r(\theta)$, as the value of $\mu = \eta\lambda$ does not change and the first term of the integrand in Equation 13 becomes negative. Now consider the alternative quality level $\hat{q} > r(\theta) > 0$, so that $\mu = \eta$. The difference between profits at $q^{SB}(\theta) = 0$ and profits at $\hat{q}$ is given by:

$$-(1+\eta)\phi(\theta)m(\hat{q}) \;+\; c(\hat{q}) \;-\; (\eta\lambda - \eta)m(r(\theta))\phi(\theta) \;+\; (\eta\lambda - \eta)\theta m'(r(\theta))r'(\theta).$$

This expression is nonnegative, as $\phi(\theta) \leq 0$, $m'(q) \geq 0$ and $r'(\theta) \geq 0$. □

Thus, just as in the case of standard consumption preferences, a revenue maximizing monopolist assigns quality zero to types with negative marginal virtual consumption valuation. Next we argue that for high reference quality levels, the optimal quality schedule maximizes virtual surplus under reference-dependent preferences, whereas for intermediate reference quality levels, optimal quality coincides with the reference point.

Claim 2. *For each type $\theta \in \Theta \setminus \Theta_0$ for which $r(\theta) > q^*_{\eta\lambda}(\theta)$ holds, the optimal quality schedule selects $q^{SB}(\theta) \;=\; q^*_{\eta\lambda}(\theta)$.*

Proof. Fix a type $\theta > \theta_0$ with $r(\theta) > q^*_{\eta\lambda}(\theta)$. From Equation 6, the unique maximizer of the integrand in the profit function of (13) with $\mu = \eta\lambda$ is precisely $q^*_{\eta\lambda}(\theta)$. Let then $q^{SB}(\theta) = q^*_{\eta\lambda}(\theta)$. Any deviation to an alternative quality level $q \leq r(\theta)$ will only hurt profits as it decreases the first term of the objective function in (13) without changing the rest of the expression. Now consider a deviation to a quality level $\hat{q} \geq r(\theta)$, which changes μ in the objective function from $\eta\lambda$ to η. From Assumption 2 and $\hat{q} \geq r(\theta) > q^*_{\eta}(\theta)$, it follows that the optimal deviation in this case is $\hat{q} = r(\theta)$. The difference between profits at $r(\theta)$ and $\mu = \eta\lambda$, and profits at $r(\theta)$ and $\mu = \eta$ is given by:

$$(\eta\lambda \;-\; \eta)h(\theta)\theta m'(r(\theta)r'(\theta) \;\geq\; 0.$$

It follows that profits at $q^*_{\eta\lambda}(\theta)$ and $\mu = \eta\lambda$ are strictly greater than profits at $r(\theta)$ with $\mu = \eta\lambda$, which are greater than profits at $r(\theta)$ with $\mu = \eta$. This proves the claim. $\square$

Claim 3. *For each type $\theta \in \Theta \setminus \Theta_0$ for which $q^*_\eta(\theta) \le r(\theta) \le q^*_{\eta\lambda}(\theta)$ holds, the optimal quality schedule is $q^{SB}(\theta) = r(\theta)$.*

Proof. Let $\theta > \theta_0$ satisfy the conditions of the claim, and let $q^{SB}(\theta) = r(\theta)$. Consider the quality level $\hat{q}$, such that $\hat{q} > r(\theta) \ge q^*_\eta(\theta)$. The integrand of Equation 13 has $\mu = \eta$. Clearly, profits at type θ are strictly decreasing in quality as long as $\hat{q} > r(\theta)$, so that there is no downward profitable deviation. One can use a similar argument to show that there is no upward profitable deviation from $q^{SB}(\theta) = r(\theta)$, and therefore this is the optimal quality schedule. $\square$

We now find the optimal quality schedule for types θ with reference points below $q^*_\eta(\theta)$.

Claim 4. *For all $\theta \in \Theta \setminus \Theta_0$ with $0 \le r(\theta) < q^*_\eta(\theta)$, the optimal quality schedule selects either $q^{SB}(\theta) = q^*_\eta(\theta)$ or $q^{SB}(\theta) = r(\theta)$.*

Proof. Fix a type $\theta > \theta_0$ with $0 \le r(\theta) < q^*_\eta(\theta)$. From first order conditions, the unique maximizer of the integrand in (13) with $\mu = \eta$ is $q^*_\eta(\theta)$. Any deviation to a quality level $\hat{q} > r(\theta)$ will not change μ from η to $\eta\lambda$ and thus will only decrease profits. Among deviations from $q^*_\eta(\theta)$ to quality levels $\hat{q} \le r(\theta)$ that change μ to $\eta\lambda$ in (13), the one with highest profits is $\hat{q} = r(\theta)$. This change may potential be beneficial, as profits at $r(\theta)$ and $\eta\lambda$ are greater than profits at $r(\theta)$ and η. The difference between profits at $r(\theta)$ with associated $\mu = \eta\lambda$ and profits at $q^*_\eta(\theta)$ with associated $\mu = \eta$ is given by the expression:

$$\begin{aligned} \triangle(r(\theta), q^*_\eta(\theta)) \;&=\; (\eta\lambda - \eta)h(\theta)\theta m'(r(\theta)r'(\theta) \\ &-\; \big\{(1+\eta)\phi(\theta)m(q^*_\eta(\theta)) - c(q^*_\eta(\theta)) \;-\; (1+\eta)\phi(\theta)m(r(\theta)) + c(r(\theta))\big\}. \end{aligned} \tag{14}$$

The sign of the above expression depends on the difference between gains associated with a change the parameter μ from η to $\eta\lambda$ in the profit function, and the gains in virtual surplus derived from shifting quality from $r(\theta)$ to $q^*_\eta(\theta)$. This proves the claim. $\square$

Consider a subset of types $\Theta_\eta = [\underline{\theta}_\eta, \overline{\theta}_\eta]$ on which the inequality $r(\theta) \le q^*_\eta(\theta)$ holds, with $r(\underline{\theta}_\eta) = q^*_\eta(\underline{\theta}_\eta)$. From Claim 4, the optimal quality schedule on Θ_η is either $q^*_\eta(\theta)$ or $r(\theta)$. If the reference formation function r is flat on Θ_η, then clearly one has $q^{SB}(\theta) = q^*_\eta(\theta)$ as Equation 14 becomes non-positive for the entire subinterval. Suppose r is strictly increasing, with $r(\overline{\theta}_\eta) = q^*_\eta(\overline{\theta}_\eta)$. Then, in principle, one may have an optimal quality schedule that assigns $r(\theta)$ for types at both the low and high ends of Θ_η, and $q^*_\eta(\theta)$ for types in the middle of the subinterval Θ_η. Such quality schedule violates monotonicity.

Claim 5. *Consider a quality schedule $q\colon \Theta \to Q$ such that for types $\theta < \hat{\theta}$ in Θ (sufficiently closed to each other), it assigns $q(\theta) = q^*_\eta(\theta) > r(\theta)$ and $q(\hat{\theta}) = r(\hat{\theta}) < q^*_\eta(\theta)$. Then q is not incentive compatible.*

Proof. From Proposition 3, it suffices to show that q violates monotonicity:

$$v(q(\hat{\theta}), \hat{\theta}) \;-\; v(q(\hat{\theta}), \theta) \;<\; v(q(\theta), \hat{\theta}) \;-\; v(q(\theta), \theta).$$

The left-hand side of the above equation is given by

$$v(q(\hat{\theta}), \hat{\theta}) \;-\; v(q(\hat{\theta}), \theta) \;=\; \hat{\theta}m(r(\hat{\theta})) \;-\; (1+\eta)\theta m(r(\hat{\theta})) \;+\; \eta\theta m(r(\theta)),$$

whereas its right-hand side is

$$v(q(\theta), \hat{\theta}) \;-\; v(q(\theta), \theta) \;=\; (1+\eta)\hat{\theta}m(q^*_\eta(\theta)) \;-\; \eta\hat{\theta}m(r(\hat{\theta})) \;-\; \theta m(q^*_\eta(\theta)).$$

Thus, we obtain

$$\begin{aligned}&\big(v(q(\hat\theta),\hat\theta) - v(q(\hat\theta),\theta)\big) \;-\; \big(v(q(\theta),\hat\theta) - v(q(\theta),\theta)\big)\\ &\quad= \;\big(m(r(\hat\theta)) \,-\, m(q^*_\eta(\theta))\big)\big((1+\eta)\hat\theta - \theta\big) \,-\, \eta\theta\big(m(r(\hat\theta)) \,-\, m(r(\theta))\big) \;<\; 0,\end{aligned}$$

as desired. □

One concludes from Claim 4 and Claim 5 that if there exists an interval of types Θ_η such that $r(\theta) \leq q^*_\eta(\theta)$ for all $\theta \in \Theta_\eta$, with equality at the low (and possibly high) endpoint, the optimal quality schedule corresponds to one of the following three cases: (1) either it assigns $q^*_\eta(\theta)$ for all types in Θ_η; or (2) it assigns $r(\theta)$ for all types in Θ_η; or (3) there exists a cutoff type $\theta_c \in \Theta_\eta$ such that $q^{SB}(\theta) = r(\theta)$ for all $\theta \leq \theta_c$, and $q^{SB}(\theta) = q^*_\eta(\theta)$ for all $\theta > \theta_c$. The existence and position of the cutoff type θ_c depends on the details of the model (i.e., marginal virtual consumption valuation, distribution of types, reference function r, and the value of the gain-loss and loss-aversion coefficients).

We now construct the optimal quality schedule $q^{SB}\colon \Theta \to Q$ that maximizes the monopolist's expected profits. To simplify notation, let Θ_η, Θ_r, and $\Theta_{\eta\lambda}$ denote subsets of $\Theta \setminus \Theta_0$ for which the following holds:

$$\begin{aligned}&\text{for all } \theta \in \Theta_\eta: && r(\theta) \;\leq\; q^*_\eta(\theta),\\ &\text{for all } \theta \in \Theta_r: && q^*_\eta(\theta) \;\leq\; r(\theta) \;\leq\; q^*_{\eta\lambda}(\theta),\\ &\text{for all } \theta \in \Theta_{\eta\lambda}: && q^*_{\eta\lambda}(\theta) \;\leq\; r(\theta).\end{aligned}$$

By the continuity and monotonicity of r and q^*_μ, these subsets are either empty or closed intervals. The next assumption ensures that Θ is partitioned in at most four subintervals.

Assumption 6. For $\mu \in \{\eta, \eta\lambda\}$, the function $\theta \to \operatorname{sgn}(r(\theta) - q^*_\mu(\theta))$ is weakly monotonic on the set of types $\Theta \setminus \Theta_0$.

Our last assumption rules out the functions r and q^*_μ crossing more than once on the relevant part of the type space. This single crossing condition can be relaxed to a finite crossing condition, but doing so complicates the notation and the arguments without providing additional insights to the results. Optimal contracts for loss averse consumers are derived next.

Proposition 4. *Suppose that Assumptions 1, 2, 3(a), 4, 5 and 6 hold. Consider the contract design problem of a monopolist facing loss averse consumers. The optimal menu of incentive feasible contracts consists of pairs $\{(q^{SB}(\theta), p^{SB}(\theta)) \mid \theta \in \Theta\}$ such that*

$$q^{SB}(\theta) \;=\; \begin{cases} 0 & :\; \theta \in \Theta_0,\\ q^*_{\eta\lambda}(\theta) & :\; \theta \in \Theta_{\eta\lambda},\\ r(\theta) & :\; \theta \in \Theta_r,\; \theta \in \Theta_\eta,\; \theta \leq \theta_c\ (\theta_c \in \Theta_\eta),\\ q^*_\eta(\theta) & :\; \theta \in \Theta_\eta,\; \theta > \theta_c\ (\theta_c \in \Theta_\eta),\end{cases} \tag{15}$$

$$p^{SB}(\theta) \;=\; v(q^{SB}(\theta),\theta) \,+\, \eta\lambda\theta_L m(r(\theta_L)) \,-\, \int_{\theta_L}^{\theta} s(q^{SB}(\tilde\theta),\tilde\theta)\, d\tilde\theta, \tag{16}$$

where the selection $s(q^{SB}(\theta),\theta) \,=\, (1+\mu)m(q^{SB}(\theta)) - \big(\mu\,\theta m(r(\theta))\big)'$ has $\mu = \eta\lambda$ for $q^{SB}(\theta) \leq r(\theta)$ and $\mu = \eta$ otherwise. Moreover, the optimal quality schedule q^{SB} is weakly increasing, piecewise differentiable and continuous except possibly at $\theta_c \in \Theta_\eta$.

Proof. The construction of the optimal quality schedule $\theta \mapsto q^{SB}(\theta)$ described above follows from Claims 1 to 5, while the derivation of the optimal pricing schedule $\theta \mapsto p^{SB}(\theta)$ comes from the Mirrlees representation in Proposition 3. Clearly, q^{SB} is weakly increasing, piecewise differentiable, and continuous except possibly at $\theta_c \in \Theta_\eta$. By Assumption 6, this can happen at most once. To prove that this menu of contracts is indeed incentive feasible, we must show that conditions (a) and (c) in Proposition 3 are both satisfied. Here we show (c) and leave (a) for the Section 6. For all $\theta \in \Theta$, one has

$$\eta\lambda\big(\theta m(r(\theta)) - \theta_L m(r(\theta_L))\big) \;+\; \int_{\theta_L}^{\theta} s(q^{SB}(\tilde\theta),\tilde\theta)\,d\tilde\theta$$
$$\geq\; \eta\lambda\big(\theta m(r(\theta)) - \theta_L m(r(\theta_L))\big) \;+\; \int_{\theta_L}^{\theta}(1+\mu)q^{SB}(\tilde\theta)\,d\tilde\theta \;+\; \int_{\theta_L}^{\theta}\big(\eta\lambda\,\tilde\theta m(r(\tilde\theta))\big)'\,d\tilde\theta \;\geq\; 0.$$

Thus, the participation constraints are satisfied for all types. □

An immediate implication from Proposition 4 is that lower types with zero quality allocation receive zero rent, as in the standard monopoly pricing model, while higher types receive a positive rent that is weakly increasing in types. Note also that while the optimal quality schedule q^{SB} under loss aversion exhibits common elements with the optimal schedule under loss neutrality q^{sb} given in Proposition 2, the actual shape of q^{SB} very much depends on the reference function. On the one hand, the informational constraints have an effect on contract design similar to the case without loss aversion, as the optimal menu of contracts distorts qualities for lower types and allocates qualities for higher types according to the marginal virtual surplus, which is modified to consider reference dependent preferences. On the other hand, it is possible that for a subset of types of positive measure, the optimal quality schedule be determined entirely by the reference function. Since r is assumed to be weakly increasing, this implies that optimal quality schedules may exhibit flat parts among increasing parts for intermediate and higher types. Thus, Proposition 4, which isolates the effect of loss aversion biases onto optimal contract design under asymmetric information, stresses both new possibilities and limits of the model. It also highlights the relevance of the reference point formation process. We explore these possibilities next.

3.3. **Application: optimal contracts under linear valuation and quadratic cost.** In this subsection, we make additional simplifying assumptions to explicitly derive optimal contracts for loss averse consumers. Types are uniformly distributed on $\Theta = [0, \theta_H]$, the function m is linear and the cost function c is quadratic: $m(q) = q$ and $c(q) = q^2/2$, for all $q \geq 0$. Thus, the marginal virtual consumption valuation is $\phi(\theta) = 2\theta - \theta_H$, with $\theta_0 = \theta_H/2$.

In this setup, the optimal quality schedule for loss neutral consumers with reference-dependent preferences is given by

$$q^{sb}(\theta) \;=\; \begin{cases} 0 & :\; 0 \leq \theta \leq \theta_H/2, \\ (1+\eta)(2\theta - \theta_H) & :\; \theta_H/2 < \theta \leq \theta_H, \end{cases}$$

This quality schedule is implemented by a price scheme that has $p^{sb}(\theta) = 0$ for all $\theta \leq \theta_H/2$ and $p^{sb}(\theta) = (1+\eta)^2(\theta^2 - \theta_H^2/4)$ for all $\theta > \theta_H/2$. The monopolist's expected revenues from loss neutral consumers are $\Pi^{sb} = (1+\eta)^2\theta_H^2/12$. We also note that the prior expected quality level is $\mathrm{E}_\theta[q^{sb}(\theta)] = (1+\eta)\theta_H/4$.

Suppose now that after learning her type but before observing the menu of contracts offered by the firm, a θ-type consumer forms quality expectations or aspiration levels according to a reference point formation function r_n. We study three ways in which consumers' quality aspirations are constructed, each of which incorporates a different assumption in terms of how consumers use their private information. Implicitly, we are assuming that consumers, while aware of their reference-dependent preferences, do not anticipate loss aversion:

$$r_1(\theta) = q^{sb}(\theta), \qquad \text{for all } \theta \in \Theta, \tag{17}$$

$$r_2(\theta) = (1+\eta)\theta_H/4, \qquad \text{for all } \theta \in \Theta, \tag{18}$$

$$r_3(\theta) = \tfrac{1}{2}q^{sb}(\theta) + \tfrac{1}{2}(1+\eta)\theta_H/4, \qquad \text{for all } \theta \in \Theta. \tag{19}$$

Under r_1, consumers correctly utilize their private information to update prior believes about quality schedules to coincide with optimal schedules offered by a monopolist under loss neutrality. Under r_2, consumers do not use their private information and thus the reference quality level equals the prior belief about quality schedule under loss neutrality. The reference function r_3 considers consumers who partially update their quality expectations based on their private information. As we shall see, each reference function generates a different optimal quality schedule in the presence of loss aversion.

3.3.1. *Optimal contract under r_1.* Consider the reference function r_1 defined in (17). Let $\Theta_0 = [0, \theta_H/2]$ and $\Theta_r = [\theta_H/2, \theta_H]$. From Proposition 4, the optimal quality schedule is

$$q_1^{SB}(\theta) = r_1(\theta) = q^{sb}(\theta), \qquad \text{for all } \theta \in \Theta.$$

In this case, the optimal quality schedule is consistent with consumers' interim expectations. Importantly, while the loss aversion bias does not affect the optimal quality schedule, it nonetheless positively impacts expected profits, as the monopolist can adjust prices. Indeed, the optimal pricing scheme under r_1 is $p_1^{SB}(\theta) = 0$ for all $\theta \leq \theta_H/2$, and $p_1^{SB}(\theta) = (1+\eta)(1+\eta\lambda)(\theta^2 - \theta_H^2/4)$ for $\theta > \theta_H/2$, and thus expected revenues under loss aversion and r_1 are given by $\Pi_1^{SB} = (1+\eta)(1+2\eta\lambda-\eta)\theta_H^2/12$.

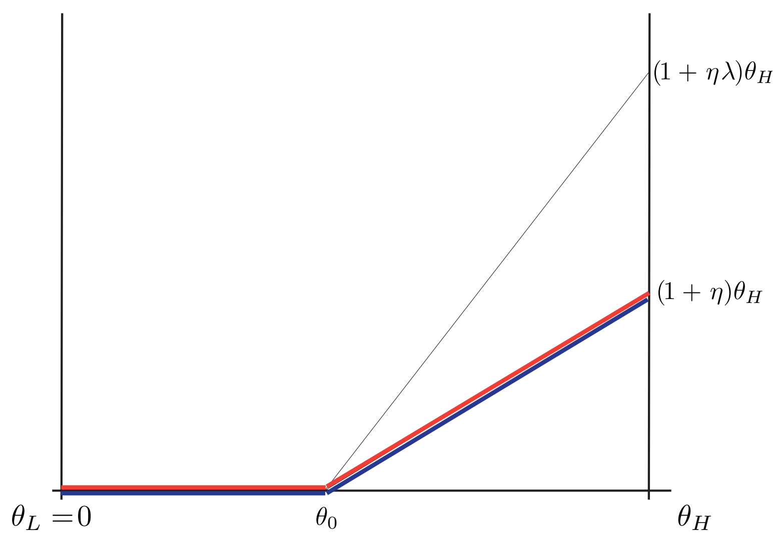

FIGURE 1. Optimal quality schedule q_1^{SB} for reference function r_1.

3.3.2. *Optimal contract under r_2.* Consider the constant reference function r_2 given in (18). Denote by $\underline{\theta}_2$ and $\overline{\theta}_2$ the following types:

$$\underline{\theta}_2 \;=\; \frac{5+4\eta\lambda+\eta}{8(1+\eta\lambda)}\theta_H \;<\; \frac{5}{8}\theta_H \;=\; \overline{\theta}_2.$$

We construct the following partition of the type space: $\Theta_0 = [0, \theta_0]$ is as before; $\Theta_{\eta\lambda} = [\theta_0, \underline{\theta}_2]$ for which one has that $(1+\eta)\theta_H/4 = r_2(\theta) \geq (1+\eta\lambda)\phi(\theta) = (1+\eta\lambda)(2\theta - \theta_H)$ for all $\theta \in \Theta_{\eta\lambda}$; $\Theta_r = [\underline{\theta}_2, \overline{\theta}_2]$ for which one has that $(1+\eta\lambda)(2\theta-\theta_H) = (1+\eta\lambda)\phi(\theta) \geq r_2(\theta) \geq (1+\eta)\phi(\theta) = (1+\eta)(2\theta-\theta_H)$, for all $\theta \in \Theta_r$; and $\Theta_\eta = [\overline{\theta}_2, \theta_H]$ for which one has that $r_2(\theta) \leq (1+\eta)\phi(\theta) = (1+\eta)(2\theta - \theta_H)$ for all types $\theta \in \Theta_\eta$. Since $r'(\theta) = 0$ everywhere, from Proposition 4 and Equation 14, we obtain the optimal quality schedule q_2^{SB} for the reference function r_2:

$$q_2^{SB}(\theta) \;=\; \begin{cases} 0 & : \; \theta \in \Theta_0, \\ (1+\eta\lambda)(2\theta - \theta_H) & : \; \theta \in \Theta_{\eta\lambda}, \\ \theta_H/4 & : \; \theta \in \Theta_r, \\ (1+\eta)(2\theta-\theta_H) & : \; \theta \in \Theta_\eta. \end{cases}$$

In this case, the quality levels purchased by consumers differ from their reference points except for types in the midrange interval Θ_r. As mentioned before, a divergence between actual consumption and expectations (in this static application) is allowed, as long as the optimal menu of contracts satisfies individual rationality.

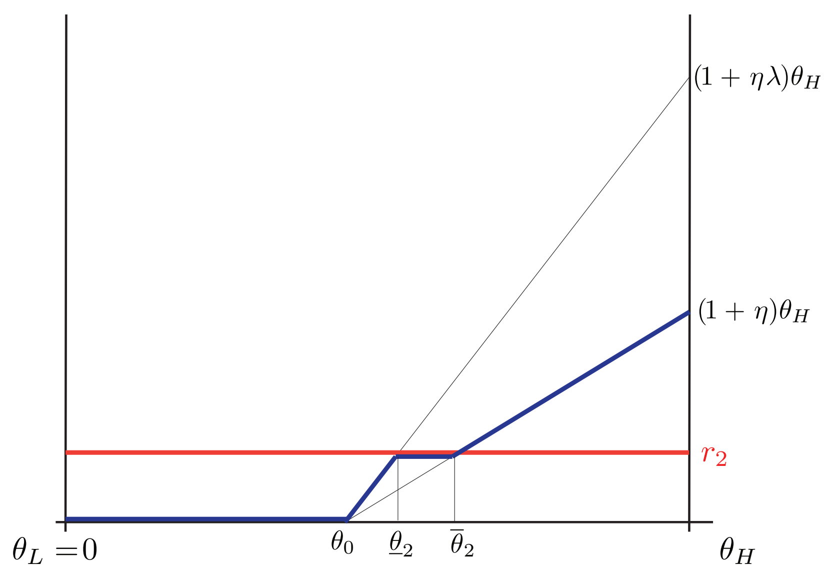

FIGURE 2. Optimal quality schedule q_2^{SB} for reference function r_2.

3.3.3. *Optimal contract under r_3.* Consider finally the reference function r_3 given in (19). As in the previous case, one can immediately see that the type space is partitioned into four subintervals $\Theta_0 = [0, \theta_0]$, $\Theta_{\eta\lambda} = [\theta_0, \underline{\theta}_3]$, $\Theta_r = [\underline{\theta}_3, \overline{\theta}_3]$ and $\Theta_\eta = [\overline{\theta}_3, \theta_H]$, on each of which the relationship between the reference function r_3, and the functions $(1+\eta)\phi$ and $(1+\eta\lambda)\phi$, is as defined before. Here, the cutoff types $\underline{\theta}_3, \overline{\theta}_3$ are given by

$$\underline{\theta}_3 \;=\; \frac{5+8\eta\lambda-3\eta}{8(1+2\eta\lambda-\eta)}\theta_H \;<\; \frac{5}{8}\theta_H \;=\; \overline{\theta}_3.$$

Notice $\underline{\theta}_3 < \underline{\theta}_2$ and $\overline{\theta}_3 = \overline{\theta}_2$. Notice also that $r'(\theta) > 0$ on Θ_η. Thus, we shall use Equation 14 to evaluate the shape of the optimal quality schedules in the subinterval Θ_η. For each $\theta \in \Theta_\eta$, the difference between profits at $r_3(\theta)$ and $\mu = \eta\lambda$ and profits at $q^*_\eta(\theta)$ and $\mu = \eta$ is:

$$\triangle(r_3(\theta), q^*_\eta(\theta)) \;=\; (1+\eta)(\eta\lambda-\eta)\theta(\theta_H-\theta) \;-\; \frac{(1+\eta)^2(\theta-\overline{\theta}_3)^2}{2}.$$

One has that $\triangle(r_3(\theta), q^*_\eta(\theta)) > 0$ for types sufficiently closed to $\overline{\theta}_3$, and $\triangle(r_3(\theta_H), q^*_\eta(\theta_H)) < 0$. Since in this case $\triangle$ as a function of types is continuous and strictly decreasing on Θ_η, there exists a unique cutoff type θ_c in the interior of Θ_η such that $\triangle_3(\theta_c) = 0$. For types in Θ_η to the left of θ_c, the optimal quality schedule coincides with the reference quality levels, even though these reference points lie below the marginal virtual valuation $(1+\eta)\phi$. This is because the monopolist gains more flexibility in terms of contract design, as it now can put $\mu = \eta\lambda$ in the selection $s(q_3^{SB}(\cdot), \cdot)$ that is used to provide incentives. With these observations, we construct the optimal quality schedule q_3^{SB} under the reference function r_3:

$$q_3^{SB}(\theta) \;=\; \begin{cases} 0 & : \; 0 \le \theta \le \theta_0, \\ (1+\eta\lambda)(2\theta-\theta_H), & : \; \theta_0 < \theta \le \underline{\theta}_3, \\ (1+\eta)(\theta - 3\theta_H/8) & : \; \underline{\theta}_3 < \theta \le \theta_c, \\ (1+\eta)(2\theta-\theta_H) & : \; \theta_c < \theta \le \theta_H. \end{cases}$$

As with r_2, the optimal menu of contracts associated with r_3 involves quality levels that differ from the reference quality levels for consumers with low and high types.

4. Self-Confirming Contracts

Our results so far illustrate two related points. First, optimal contract design in the presence of loss aversion is heavily influenced by the reference function. Indeed, while the broad insights of contract theory prevail — i.e., the allocation of qualities is monotone and downward distorted for all but the highest type — for intermediate types optimal contracts can exhibit various new features, including discontinuities and flat parts. Second, an optimal menu of contracts may require differences between the quality schedule offered by the firm and the reference quality level expected by consumers. As long as contracts are incentive compatible and individually rational, a θ-type consumer will purchase the quality-price pair designed for her, even if the quality level differs from her expectations.

To study these issues further, we introduce the following concept. Given a reference function $r\colon \Theta \to Q$, a quality schedule $q\colon \Theta \to Q$ is said to be *self-confirming with respect to* r if the quality levels offered by the monopolist coincide with consumers' expectations: $q(\theta) = r(\theta)$ for all θ in Θ. An optimal incentive feasible quality schedule need not be self-confirming. Note that assuming that reference-dependent consumers are Bayesian updaters, in the sense of $r(\theta) = 0$ for all $\theta \le \theta_0$ and $r(\theta) = q^*_\eta(\theta)$ for all $\theta > \theta_0$ if consumers are unaware of their loss aversion bias — or $r(\theta) = q^*_{\eta\lambda}(\theta)$ if instead they are aware of it — is a sufficient but not necessary condition for an optimal contract to be self-confirming. Our notion of a self-confirming quality schedule is stronger than the personal equilibrium notion of Kőszegi and Rabin (2006) and Heidhues and

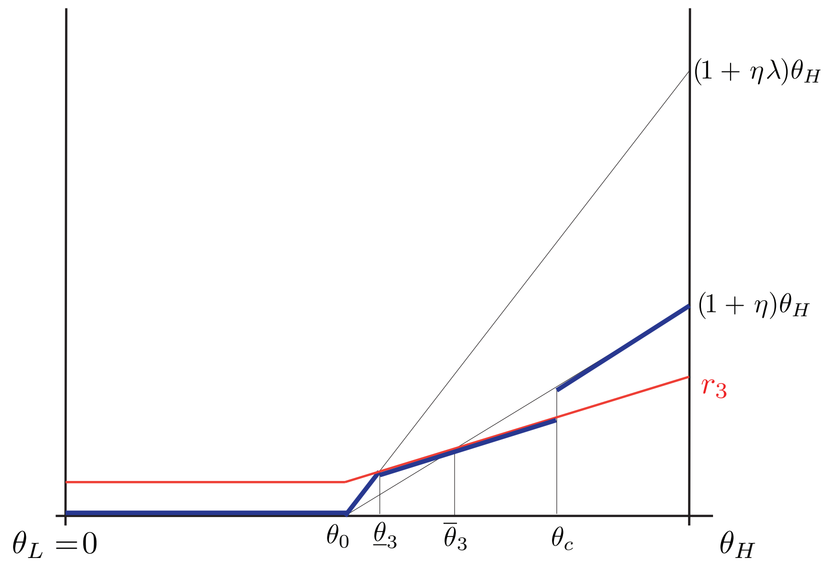

FIGURE 3. Optimal quality schedule q_3^{SB} for reference function r_3.

Kőszegi (2010), as the latter involves agreement in expectations between a consumption plans and reference points.

Proposition 5. *Let Assumptions 1, 2, 3(a), 4, 5 and 6 be satisfied. If $r\colon \Theta \to Q$ satisfies*

$$r(\theta) \;=\; 0 \text{ for } \theta \in \Theta_0, \qquad \text{and} \qquad q^*_\eta(\theta) \;\le\; r(\theta) \;\le\; q^*_{\eta\lambda}(\theta) \text{ for } \theta \in \Theta \setminus \Theta_0,$$

then the optimal incentive feasible menu of contracts $\{(q^{SB}(\theta), p^{SB}(\theta)) \mid \theta \in \Theta\}$ is self-confirming with respect to r.

Proof. It follows immediately from Proposition 4. □

So far we have derived optimal contracts for loss averse consumers with an exogenously given reference function in a static setting, and we have shown that an optimal quality schedule may or may not be self-confirming. We now explore the issue of reference point formation in a dynamic environment and show that, if the consumer's reference formation process is adaptive, then the steady state the optimal menu of contract is self-confirming. For simplicity, we consider a long-lived myopic monopolist who faces a sequence of small, short-lived consumers with heterogeneous preferences and loss aversion. Thus, we modify the static framework of Section 2 as follows.

Consumers are born in each period $t = 1, 2, \ldots$ of time and live for two periods. In the first period of her life, a consumer can choose to engage in a market transaction with the monopolist. In the second period the consumer is a passive player. At the beginning of period t, each consumer born in the current period receives a type $\theta_t \in \Theta = [\theta_L, \theta_H]$ drawn according to the distribution F. Consumers born in period $t = 1$ are endowed with an exogenous reference function $r_1\colon \Theta \to Q$ that satisfies Assumption 4. Consumers born in period $t \ge 2$ form reference quality levels based on an adaptive process that considers both observations of past quality schedules offered by the monopolist and reference quality level of the previous generation (the adaptive process is formally described below). At every $t \ge 1$, after each consumer has observed her type θ_t and formed a reference point $r_t(\theta_t)$, the monopolist posts a menu of short-term contracts $\{(q_t(\theta_t), p_t(\theta_t)) \mid \theta_t \in \Theta\}$ to be executed at the end of the period. Since the monopolist is myopic and faces short-lived consumers, it designs incentive feasible short term contracts to maximize per period profits. Since the reference function r_1 for the first period is exogenous, the optimal menu of quality-price pairs for first-period consumers is given by Proposition 4, where we let $\Theta_{\eta\lambda} = \Theta_{\eta\lambda(1)}$ and so forth.

Consider the following adaptive reference point formation process. A θ-type consumer born in period $t \geq 2$ becomes informed of the reference quality a consumer of the same type born in period $t-1$, say by word of mouth. She also observes the actual quality offered by the monopolist to the θ-type consumer of the previous generation, and uses this information to adjust the past period reference level. Formally, suppose that $r_{t-1}\colon \Theta \to Q$ satisfies Assumption 4 and $\operatorname{sgn}(r_{t-1} - q^*_\mu)$ satisfies Assumption 6, for $\mu \in \{\eta, \eta\lambda\}$. Thus, there is at most one interval $\Theta_{\eta(t-1)} \subseteq \Theta$ on which the inequality $r_{t-1}(\theta) \leq q^*_\eta(\theta)$ holds, so that q^{SB}_{t-1} may be discontinuous at most at a single type $\theta_{c(t-1)} \in \Theta_{\eta(t-1)}$. If such is the case, then one has $q^{SB}_{t-1}(\theta) = r_{t-1}(\theta) \leq q^*_\eta(\theta)$ for all $\theta \in \Theta_{\eta(t-1)}, \theta \leq \theta_{c(t-1)}$, and $q^{SB}_{t-1}(\theta) = q^*_\eta(\theta) > r_{t-1}(\theta)$ for $\theta \in \Theta_{\eta(t-1)}, \theta > \theta_{c(t-1)}$. Choose a type $\theta_{d(t-1)} \in \Theta_{\eta(t-1)}$ with $\theta_{d(t-1)} > \theta_{c(t-1)}$, and an increasing, $\mathcal{C}^1$ function $\sigma_{t-1}\colon \Theta \to Q$ satisfying $\sigma_{t-1}(\theta) = 0$ for all $\theta \in [\theta_L, \theta_{c(t-1)}]$, $\sigma_{t-1}(\theta) = 1$ for all $\theta \in [\theta_{d(t-1)}, \theta_H]$, and $0 < \sigma_{t-1}(\theta) < 1$ for all θ in the open interval $(\theta_{c(t-1)}, \theta_{d(t-1)})$. Choose $0 < \alpha_t < 1$ and let the reference function $r_t\colon \Theta \to Q$ for period $t \geq 2$ be formed as follows:

$$r_t(\theta) \;=\; \alpha_t r_{t-1}(\theta) \;+\; (1-\alpha_t) q^{SB}_{t-1}(\theta), \tag{20}$$

for $\theta_t \in [\theta_L, \theta_{c(t-1)}] \cup [\theta_{d(t-1)}, \theta_H]$, and

$$r_t(\theta) \;=\; (1-\sigma_{t-1}(\theta)) r_{t-1}(\theta_{c(t-1)}) \;+\; \sigma_{t-1}(\theta)\big[\alpha_t r_{t-1}(\theta_{d(t-1)}) + (1-\alpha_t) q^{SB}_{t-1}(\theta_{d(t-1)})\big], \tag{21}$$

for $\theta_t \in (\theta_{c(t-1)}, \theta_{d(t-1)})$.

That is, the reference function r_t is a weighted average of the past reference function r_{t-1} and the past quality schedule q^{SB}_{t-1} everywhere except for types immediately to the right of $\theta_{c(t-1)}$, where it becomes an average of past reference and quality schedule at $\theta_{c(t-1)}$ and at $\theta_{d(t-1)}$. We choose $\{\theta_{d(t-1)}, \alpha_t\}_{t=2}^\infty$ and the sequence of functions $\{\sigma_{t-1}\}_{t=2}^\infty$ to be such that the derivative σ'_{t-1} is uniformly bounded for all $t \geq 2$. The approximation with the smooth σ_t functions is required to impose continuity on the reference function r_t. There is no need for this approximation if the optimal quality schedule in period $t-1$ is continuous on Θ. This adaptive reference formation process seems to be consistent with empirical evidence from marketing research that suggests that reference points are best operationalized using memory-based models; see for instance Hardie, Johnson, and Fader (1993) and Briesch, Krishnamurthi, Mazumdar, and Raj (1997) among others. Notice that r_t defined by expressions (20) and (21) satisfies Assumption 4 and Assumption 6, as the functions q^*_μ and σ_t are both Lipschitz continuous. Thus, we can apply Proposition 4 again to obtain per period optimal contracts.[2] Since for all $t \geq 2$, $\Theta_{\mu(t)} = \Theta_{\mu(t-1)}$, $\Theta_{r(t)} = \Theta_{r(t-1)}$, and further $\theta_{c(t-1)} \leq \theta_{c(t)}$, one can see that the reference function r_t and its associated optimal quality schedule q^{SB}_t satisfy the following relations:

$$\begin{aligned}
q^{SB}_{t-1}(\theta) \;&=\; q^{SB}_t(\theta) \;\leq\; r_t(\theta) \;\leq\; r_{t-1}(\theta), &&\text{for all } \theta \in \Theta_0 \cup \Theta_{\eta\lambda(1)};\\
q^{SB}_{t-1}(\theta) \;&=\; q^{SB}_t(\theta) \;=\; r_t(\theta) \;=\; r_{t-1}(\theta), &&\text{for all } \theta \in \Theta_{r(1)};\\
r_{t-1}(\theta) \;&\leq\; r_t(\theta) \;\leq\; q^{SB}_t(\theta) \;\leq\; q^{SB}_{t-1}(\theta), &&\text{for all } \theta \in \Theta_{\eta(1)}.
\end{aligned}$$

[2]In particular, from Equation 14, we can choose $\theta_{d(t-1)}$, σ_{t-1} and α_t to be such that the optimal quality schedule in period t coincides with r_t, for types $\theta_{c(t-1)} \leq \theta \leq \theta_{d(t-1)}$.

Let $\boldsymbol{q}(\theta) = \lim_{t\to\infty} q_t^{SB}(\theta)$ and $\boldsymbol{r}(\theta) = \lim_{t\to\infty} r_t(\theta)$ be the steady-state quality level and reference point for each θ-type consumer. From the above analysis, we can conclude that $\boldsymbol{q}$ constitutes an optimal, self-confirming quality schedule with respect to $\boldsymbol{r}$.

Proposition 6. *Consider the optimal contract design problem of a myopic monopolist facing a sequence of short-lived consumers with reference-dependent preferences. Suppose that Assumptions 1, 2, 3(a), and 5 hold for all periods $t \geq 1$. Suppose that the reference function $r_1 \colon \Theta \to Q$ is exogenously given and satisfies Assumption 4 and 6, and that for $t \geq 2$ the reference function $r_t \colon \Theta \to Q$ is formed according to Equation 20 and Equation 21. Then, for each $t \geq 1$, the optimal menu of incentive feasible contracts consists of pairs $\{(q_t^{SB}(\theta_t), p_t^{SB}(\theta_t)) \mid \theta_t \in \Theta\}$ given by (15) and (16), for $\Theta_{\mu(t)} = \Theta_{\mu(1)}$, $\mu = \eta, \eta\lambda$, $\Theta_{r(t)} = \Theta_{r(1)}$ and $\theta_{c(t)} \in \Theta_{\eta(1)}$. Moreover, the steady-state quality schedule $\boldsymbol{q}$ and reference function $\boldsymbol{r}$ satisfy:*

$$
\begin{aligned}
\boldsymbol{r}(\theta) &= \boldsymbol{q}(\theta) = q_1^{SB}(\theta), && \textit{for all } \theta \in \Theta \setminus \Theta_{\eta(1)};\\
r_1(\theta) \leq \boldsymbol{r}(\theta) &= \boldsymbol{q}(\theta) \leq q_1^{SB}(\theta), && \textit{for all } \theta \in \Theta_{\eta(1)}.
\end{aligned}
$$

The steady-state menu of contracts $\{(\boldsymbol{q}(\theta), \boldsymbol{p}(\theta)) \mid \theta \in \Theta\}$, where $\boldsymbol{p}$ is obtained using (16), is optimal and self-confirming with respect to $\boldsymbol{r}$ among all incentive feasible contracts.

Proof. From the above, it follows that $q_t^{SB}(\theta)$ converges to $\boldsymbol{q}(\theta)$ and $r_t(\theta)$ converges to $\boldsymbol{r}(\theta)$, with $\boldsymbol{q}(\theta) = \boldsymbol{r}(\theta)$ for all $\theta \in \Theta$. We argue in Section 6 that the steady state reference function $\boldsymbol{r}$ is Lipschitz continuous, which this suffices to apply the arguments in Proposition 4 to compute the optimal quality schedule given $\boldsymbol{r}$. Clearly, this coincides with $\boldsymbol{r}$ for all $\theta \leq \theta_{c(1)}$. For a type θ to the right of $\theta_{c(1)}$, there cannot be a profitable deviation for the monopolist for types with $\boldsymbol{r}(\theta) = r_1(\theta)$ or types with $\boldsymbol{r}(\theta) = q_1^{SB}(\theta)$. For a type θ with $r_1(\theta) < \boldsymbol{r}(\theta) < q_1^{SB}(\theta)$, suppose $\boldsymbol{r}(\theta)$ is not the optimal quality schedule. Then, from Claim 4, one must have that the monopolist offers such type a quality $\hat{q} = q_1^{SB}(\theta)$. This implies that for sufficiently large t, the optimal quality schedule is not $q_t^{SB}(\theta) = r_t(\theta) < q_1^{SB}(\theta)$, a contradiction. Thus, the optimal menu of incentive feasible long term contracts is $\{(\boldsymbol{q}(\theta), \boldsymbol{p}(\theta)) \mid \theta \in \Theta\}$, where $\boldsymbol{p}$ is obtained from the Mirrlees representation in Proposition 3. □

Notice that the long term optimal quality schedule is continuous on Θ, even when short term quality contracts have points of discontinuity.

Corollary 3. *The optimal long term quality schedule $\boldsymbol{q} \colon \Theta \to Q$ is Lipschitz continuous and weakly increasing on Θ.*

Proof. It follows from the fact that the long term quality schedule has $\boldsymbol{q}(\theta) = \boldsymbol{r}(\theta)$ for all $\theta \in \Theta$, which is Lipschitz continuous and weakly increasing by construction. □

We stress that the long-term optimal quality schedule $\boldsymbol{q}$ is largely influence by the first-period reference function. Thus, the effects of the aspirational reference levels of the first generation are persistent, even when in the long term contracts are self-confirming. This raises the question of what, if any, should the firm do if it were able to influence consumers' expectations. To answer this question, we study how the reference function impact the monopolist expected profits.

From Equation 13, the direct impact of r on the firm's revenues can be expressed as follows:

$$
\int_{\theta_L}^{\theta_H} \Big[-\mu\theta m(r(\theta)) + \eta\lambda\theta_L m(r(\theta_L)) + h(\theta)\big(\mu\theta m(r(\theta))\big)' \Big] f(\theta)\, d\theta.
$$

Let $[\theta_c, \theta_d]$ be a subinterval of Θ in which $r(\theta) \leq q^{SB}(\theta)$ holds. Integrating the above expression by parts, we obtain the following expression for expected profits:

$$\begin{aligned}\Pi^{SB} \;&=\; \int_{\theta_L}^{\theta_H}\Big[(1+\mu)\phi(\theta)m(q^{SB}(\theta)) \;-\; c(q(\theta))\Big]f(\theta)\,d(\theta) \\ &\quad+\; (\eta\lambda-\eta)\Big[\theta_c m(r(\theta_c))(1-F(\theta_c)) \;-\; \theta_d m(r(\theta_d))(1-F(\theta_d))\Big]\end{aligned}$$

It is difficult to evaluate how a change in the reference function r impacts expected profits, as it involves changes in the (endogenous) optimal quality schedule and the interval $[\theta_c, \theta_d]$, and the value of this changes also depends on the function m, the distribution F, and the slope of the reference function. However, if $r(\theta) \geq q^{SB}(\theta)$ holds for all types θ, then the second part of the previous equation vanishes and thus the impact of r on profits is unambiguous: a monotone transformation of r weakly increases expected revenue, as it increases the consumption virtual surplus (for $\mu = \eta\lambda$).

5. Concluding Remarks

In this paper, we study optimal contract design by a revenue maximizing monopolist who faces consumers with heterogeneous tastes, reference-dependent preferences and loss aversion. Our paper follows the line of work pioneered by della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), Kőszegi and Rabin (2006), and Heidhues and Kőszegi (2008), among others, in studying the optimal responses of profit maximizing firms in a market context with consumers who have systematic biases.

We found that, while the general insights of contract theory and mechanism design prevail, namely, the optimal incentive compatible quality schedule is monotone and presents downward distortions from the efficient qualities for all but the highest type, for intermediate types the optimal quality schedule is determined by the reference function. Moreover, we showed that this dependance is persistent in the long run, if consumers of latter generations have adaptive reference formation processes. Thus, depending on how potential buyers (of the first generation) form their expectations of quality consumption, the optimal contracts may exhibit various new features. This can partially explain complex contracts often found in cable, telecommunications and electricity industries.

Our research stresses the importance of understanding how the initial reference points are formed. Consumers' aspirational consumption levels may be influenced by fashion and mode cycles, by social and peer pressure, by imitation effects, but also by marketing campaigns. If expectations of consumption have their minimal level constituted by optimal quality levels for loss neutral consumers (i.e., if consumers are Bayesian updaters but remain ignorant at large of their loss aversion biases), then the monopolist is better off trying to increase quality expectations as much as possible. We do not as yet have a clear answer as of how a monopolist should try to influence consumers' reference functions in the general case. This, we believe, important topic is left for future research.

6. Omitted Proofs

Proof of Proposition 1. Fix a type $\theta \in \Theta$ and suppose that $r(\theta) < q^e_\eta(\theta)$. First order conditions applied to Equation 1 with $\mu = \eta$ yield to a unique maximum at $q^e_\eta(\theta)$ with profits equal to

$$\Pi^{FB}(\theta | r(\theta) < q^e_\eta(\theta)) \;=\; (1+\eta)\theta m(q^e_\eta(\theta)) \;-\; c(q^e_\eta(\theta)) \;+\; (\eta\lambda-\eta)\theta m(r(\theta)).$$

Choosing a different quality level $q > r(\theta)$ does not change the objective function of the monopolist problem and strictly reduces profits. Choosing an alternative quality level $\hat{q} \leq r(\theta)$ shifts the parameter μ in the objective function in (1) to $\mu = \eta\lambda$. Since $r(\theta) < q^e_{\eta\lambda}(\theta)$, the monopolist would choose $\hat{q} = r(\theta)$ with associated profits equal to $(1+\eta\lambda)\theta m(r(\theta)) - c(r(\theta))$. Subtracting this last expression from profits at $q^e_\eta(\theta)$, we obtain:

$$(1+\eta)\theta m(q^e_\eta(\theta)) \,-\, c(q^e_\eta(\theta)) \,-\, (1+\eta)\theta m(r(\theta)) \,+\, c(r(\theta) \,>\, 0.$$

Showing that the revenue maximizing quality level is $q^e_{\eta\lambda}(\theta)$ when $r(\theta) > q^e_{\eta\lambda}(\theta)$ is similar and therefore omitted. In this case, profits are equal to

$$\Pi^{FB}(\theta|r(\theta) > q^e_{\eta\lambda}(\theta)) \;=\; (1+\eta\lambda)\theta m(q^e_{\eta\lambda}(\theta)) \,-\, c(q^e_{\eta\lambda}(\theta)).$$

Now suppose that for the θ-type consumer, $q^e_\eta(\theta) \leq r(\theta) \leq q^e_{\eta\lambda}(\theta)$. Choosing a quality level $\hat{q} > r(\theta) \geq q^e_\eta(\theta)$ leaves us with $\mu = \eta$ in Equation 1, and thus we are in the strictly decreasing part of the profit function. Hence the monopolist will put $\hat{q} = r(\theta)$. Similarly, choosing $\hat{q} < r(\theta) \leq q^e_{\eta\lambda}(\theta)$ yields to $\mu = \eta\lambda$ in (1), so that the monopolist is now in the strictly increasing section of the profit function. It follows that, in this case, the revenue maximizing quality level is $r(\theta)$, which generates profits equal to

$$\Pi^{FB}(\theta|q^e_\eta(\theta) \leq r(\theta) \leq q^e\eta\lambda(\theta)) \;=\; (1+\eta\lambda)\theta m(r(\theta)) \,-\, c(r(\theta)).$$

This completes the proof. □

Proof of Corollary 1. It is clear from the expressions $\Pi^{FB}(\theta|\cdot)$ in the above proof that profits under complete information and loss aversion are strictly increasing in the reference quality level $r(\theta)$ for all $r(\theta) \leq q^e_{\eta\lambda}(\theta)$, and independent of $r(\theta)$ for all $r(\theta) > q^e_{\eta\lambda}(\theta)$, as desired. □

Proof of Proposition 2. We have already argued that the menu of contracts $\{(q^{sb}(\theta), p^{sb}(\theta) \mid \theta \in \Theta\}$ is individually rational and maximizes expected revenue Π^{sb}. We show here that it is also incentive compatible. Clearly, one can use Proposition 3 with loss neutrality replacing loss aversion, in which case the subderivative correspondence is single-valued on Θ:

$$s(q^{sb}(\theta),\theta) \;=\; (1+\eta)m(q^{sb}(\theta)) \,-\, \eta\big(m(r(\theta)) \,+\, \theta m'(r(\theta))r'(\theta)\big).$$

Using the above expression, one immediately sees that the Mirrlees representation coincides with the indirect utility computed in Equation 7.

To verify that the integral monotonicity condition is satisfied, fix any two types $\theta', \theta'' \in \Theta$, with $\theta' < \theta''$. We now compute the difference

$$v(q^{sb}(\theta''),\theta'') \,-\, v(q^{sb}(\theta''),\theta') \;=\; \int_{\theta'}^{\theta''} (1+\eta)m(q^{sb}(\theta''))\, d\tilde{\theta} \,-\, \big(\eta\theta'' m(r(\theta'')) - \eta\theta' m(r(\theta'))\big),$$

and the value of the integral of the subderivative

$$\int_{\theta'}^{\theta''} s(q^{sb}(\tilde{\theta}),\tilde{\theta})\, d\tilde{\theta} \;=\; \int_{\theta'}^{\theta''} (1+\eta)m(q^{sb}(\tilde{\theta}))\, d\tilde{\theta} \,-\, \big(\eta\theta'' m(r(\theta'')) - \eta\theta' m(r(\theta'))\big).$$

From these equations and the monotonicity of q^{sb}, one sees that

$$v(q^{sb}(\theta''),\theta'') \,-\, v(q^{sb}(\theta''),\theta') \;\geq\; \int_{\theta'}^{\theta''} s(q^{sb}(\tilde{\theta}),\tilde{\theta})\, d\tilde{\theta}.$$

The remaining inequality is verified in a similar way. □

Proof of Corollary 2. Define the function W_μ on $Q \times \overline{\Theta \setminus \Theta_0}$ by

$$W_\mu(q,\theta) \;=\; (1+\mu)\phi(\theta)m(q) \;-\; c(q).$$

This function is continuous in q for each θ. From Assumption 2, it is also strongly concave in q for each θ. Let $\theta, \hat{\theta}$ be arbitrary types in $\overline{\Theta \setminus \Theta_0}$. Then, since m is $\mathcal{C}^1$ in Q, there exists a number $L > 0$ such that $|W_{\mu_q}(q,\theta) - W_{\mu_q}(q,\hat{\theta})| = |(1+\mu)m'(q)||\phi(\theta) - \phi(\hat{\theta})| \leq L|\theta - \hat{\theta}|$, for all $q \in Q$. It follows from Montrucchio (1987), Theorem 3.1, that the quality schedule $\theta \mapsto q^*_\mu(\theta) = \text{argmax}_{q \in Q} W(q,\theta)$ is Lipschitz continuous on $\overline{\Theta \setminus \Theta_0}$. Readily one obtains that q^{sb} is Lipschitz continuous on Θ. □

Proof of Proposition 6. First note that one has $\boldsymbol{r}(\theta) = \boldsymbol{q}(\theta) = 0$, for all $\theta \in \Theta_0$; $\boldsymbol{r}(\theta) = \boldsymbol{q}(\theta) = q^*_{\eta\lambda}(\theta)$, for $\theta \in \Theta_{\eta\lambda(1)}$; and $\boldsymbol{r}(\theta) = \boldsymbol{q}(\theta) = r_1(\theta)$, for types $\theta \in \Theta_{r(1)}$. Second, notice that we can choose the sequences $\{\theta_{d(t-1)}, \alpha_t, \sigma_{t-1}\}_{t=2}^\infty$ so that for all $t \geq 2$, $\theta_{c(t)} = \theta_{d(t-1)} \geq \theta_{c(t-1)}$, and therefore $q^{SB}_{t+1}(\theta) = q^{SB}_t(\theta) = r_t(\theta) = r_{t+1}(\theta) \leq q^*_\eta(\theta)$, for every $\theta \in \Theta_{\eta(1)}$, $\theta \leq \theta_{c(t)}$. Since Θ is closed and bounded and we can choose the σ_t functions to have a uniformly bounded derivative, it follows from this construction that $\boldsymbol{q}(\theta) = \boldsymbol{r}(\theta)$ for all $\theta \in \Theta_{\eta(1)}$, and moreover $\boldsymbol{r}$ is Lipschitz continuous in $\Theta_{\eta(1)}$, as $\boldsymbol{r}(\theta)$ is equal to either $r_1(\theta)$, or $q^*_\eta(\theta)$, or $\beta_t + \gamma_t\sigma_t(\theta)$ for some $t \geq 2$ and some real numbers β_t, γ_t. □

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